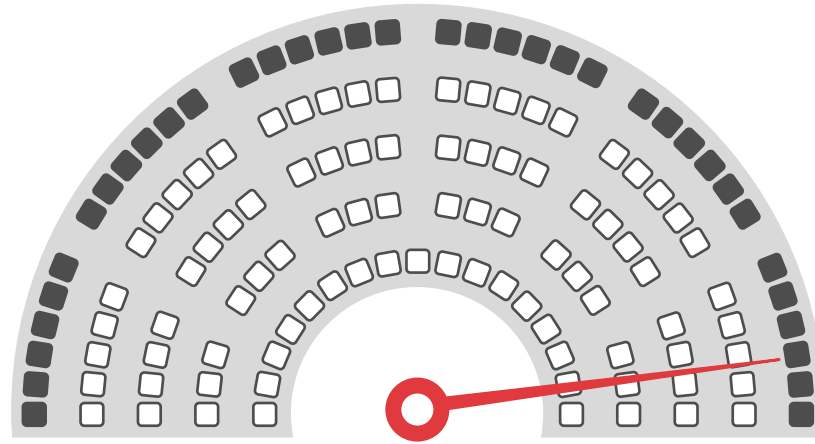




Optimized Democracy

Fall 2026 | Lecture 9

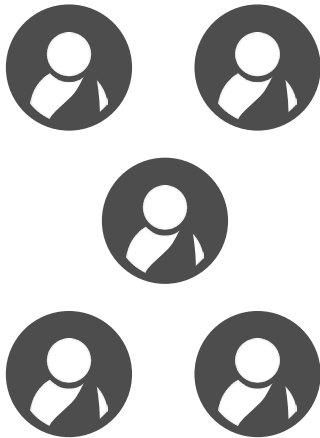
Liquid Democracy

Ariel Procaccia | Harvard University

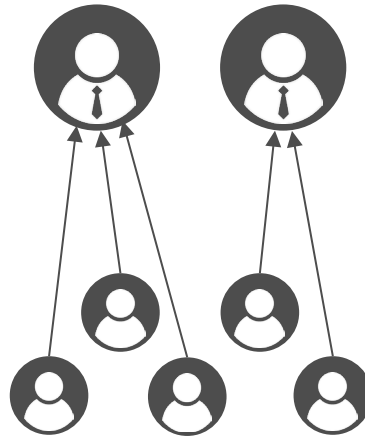
FORMS OF DEMOCRACY



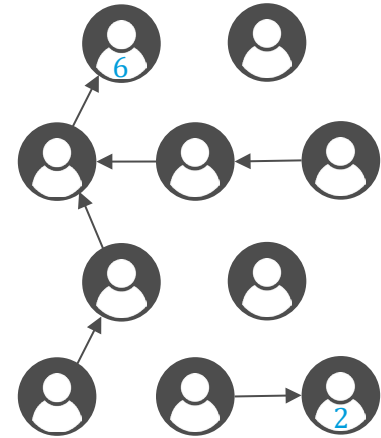
Direct
democracy



Representative
democracy



Liquid
democracy



LIQUID DEMOCRACY SYSTEMS



LiquidFeedback
Germany
Since 2010



DemocracyOS
Argentina
Since 2012



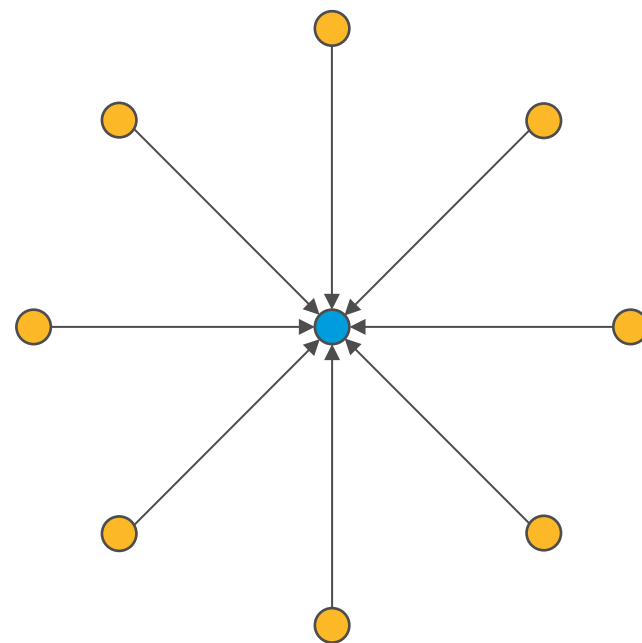
Flux
Australia
Since 2016

AN EPISTEMIC MODEL

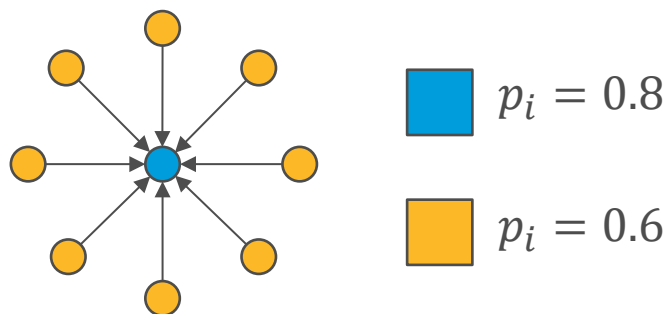
- Underlying labeled directed graph $G = (V, E, \mathbf{p})$ on n vertices, where V is the set of voters, and $(i, j) \in E$ if i knows j
- There are two alternatives, correct and incorrect
- Decisions are made based on majority vote
- Each voter i has a **competence level** p_i , which is their probability of voting correctly
- i **approves** j if $(i, j) \in E$ and $p_j > p_i + \alpha$
- Denote i 's approved neighbors by $A_G(i)$

LIQUID VS. DIRECT DEMOCRACY

- Consider a star with n vertices; leaves have $p_i = 0.4$, center has $p_i = 0.8$, and $\alpha < 0.4$
- Direct democracy: By the Condorcet Jury Theorem, probability that majority is correct $\rightarrow 0$ as $n \rightarrow \infty$
- Under liquid democracy, all leaves delegate, and the probability of correctness is 0.8



LIQUID VS. DIRECT DEMOCRACY



Poll 1

Which system would be more accurate if we raised the competence levels of the leaves to 0.6 and set $\alpha < 0.2$?

- Liquid Democracy
- Direct Democracy
- It's a tie!



DELEGATION MECHANISMS

- Can we give liquid democracy an edge via smarter delegation?
- A **delegation mechanism** observes G and the approval relation, and outputs for each $i \in V$ a probability distribution over $A_G(i) \cup \{i\}$ that represents the probability that i delegates their vote to each approved neighbor or votes directly
- Denote the probability that delegation mechanism M makes a correct decision on G by $P_M(G)$

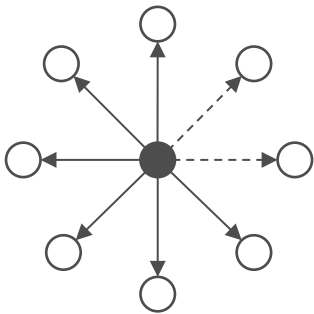
DELEGATION MECHANISMS

- $P_M(G)$ is defined via the following process:
 1. Apply M to G
 2. Sample the probability distribution for each vertex to obtain an acyclic delegation graph, where each sink i of the delegation graph has weight equal to the number of vertices with directed paths to i , including i
 3. Each sink i votes for the correct alternative with probability p_i
 4. A decision is made based on weighted majority

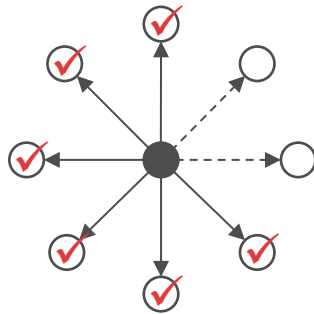
LOCAL DELEGATION MECHANISMS

In a **local** delegation mechanism, the distribution of each vertex i depends only on $\{j \in V: (i, j) \in E\}$ and $A_G(i)$

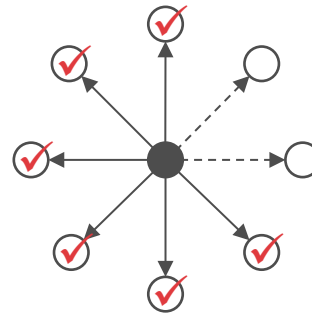
Examples:



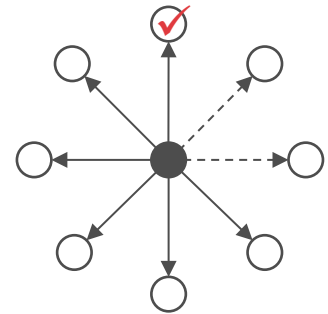
Direct voting: no delegation



Delegate to a random approved neighbor



Delegate to a random approved neighbor if most neighbors are approved



Delegate to a specific approved neighbor

FIRST, DO NO HARM

- Define $\text{gain}(M, G) = P_M(G) - P_D(G)$, where D is direct voting
- Mechanism M satisfies the **do no harm (DNH)** property if for every graph G , $\text{gain}(M, G) \geq 0$
- Mechanism M satisfies the **positive gain (PG)** property if there exist $\gamma > 0$ and graph G such that $\text{gain}(M, G) \geq \gamma$
- **Theorem:** For any $\alpha \in [0,1)$, there is no local delegation mechanism that satisfies the DNH and PG properties



Foto: dapd

Fotostrecke

Photo Gallery: Germany's Pirates Promote Digital Democracy

 **3 Bilder**

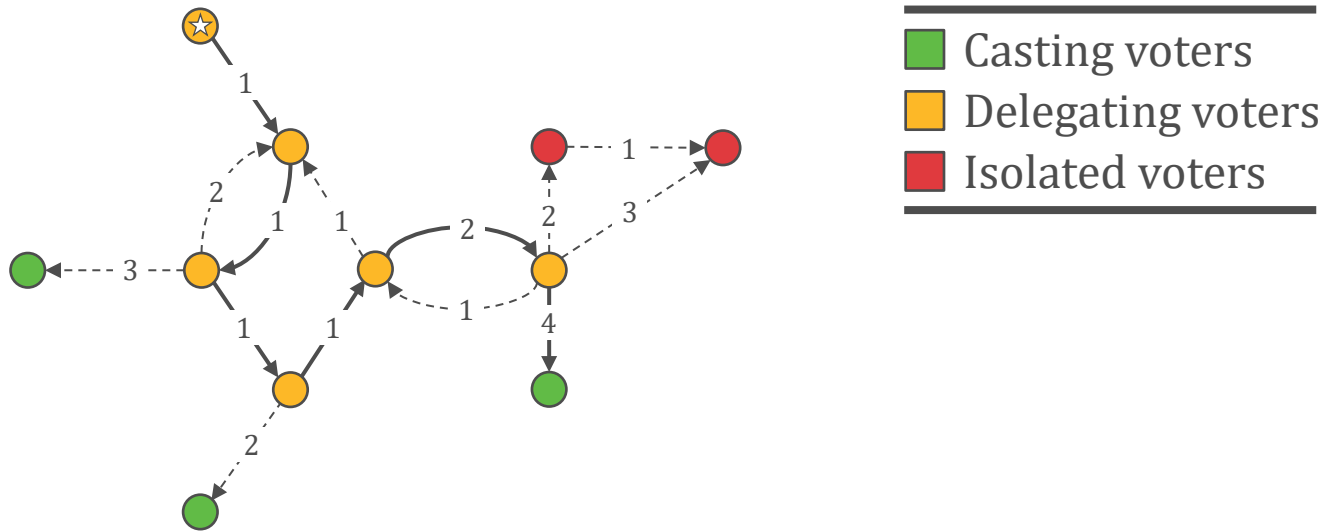
Liquid Democracy

Web Platform Makes Professor Most Powerful Pirate

RANKED DELEGATION: MODEL

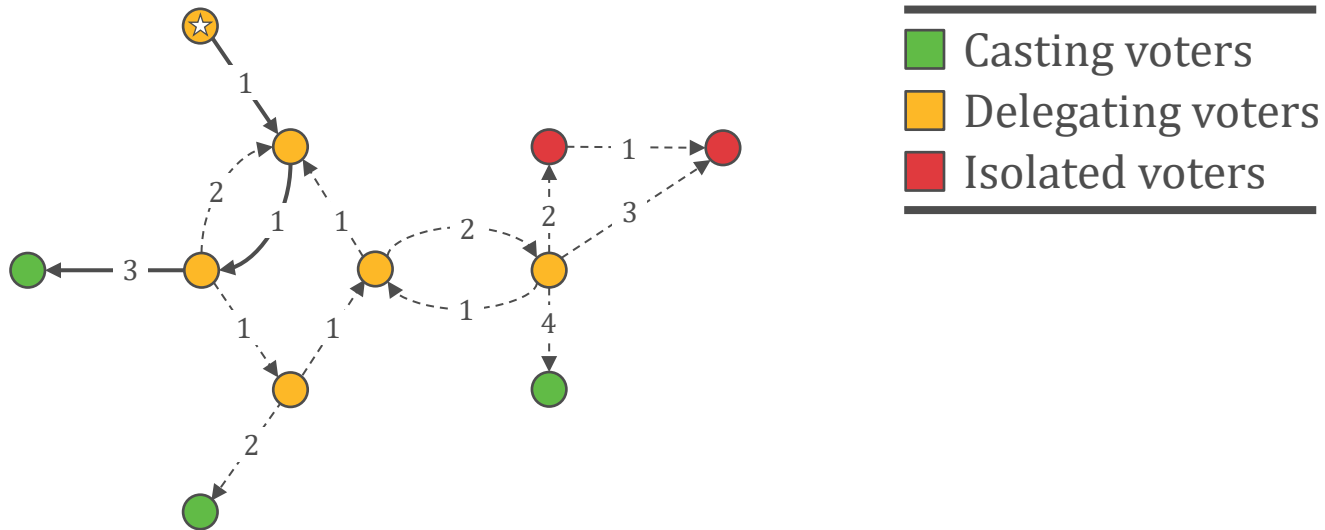
- Each voter ranks potential delegates
- Casting voters vote directly, delegating voters have a path to a casting voter, isolated voters do not
- A **ranked delegation mechanism** specifies, for each delegating voter, a path to a casting voter
- Every path corresponds to a sequence of ranks
- A **sequence mechanism** compares delegation paths using an order $\succ$ over rank sequences

SEQUENCE MECHANISMS



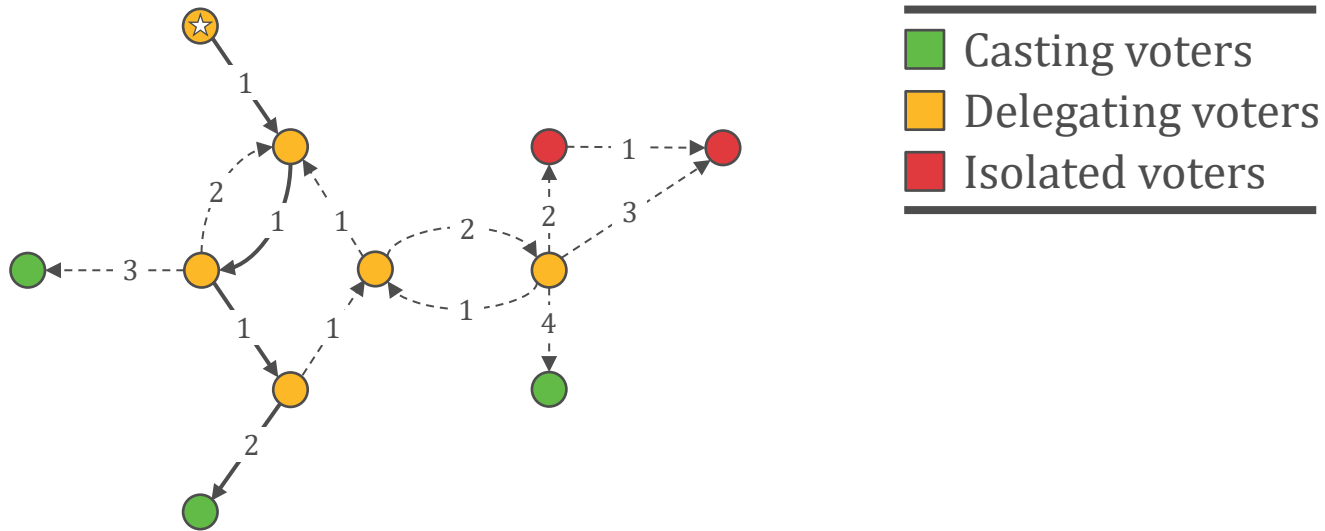
Depth-first delegation: rank sequences are compared lexicographically

SEQUENCE MECHANISMS



Breadth-first delegation: rank sequences are compared by their length (shorter is better), breaking ties lexicographically

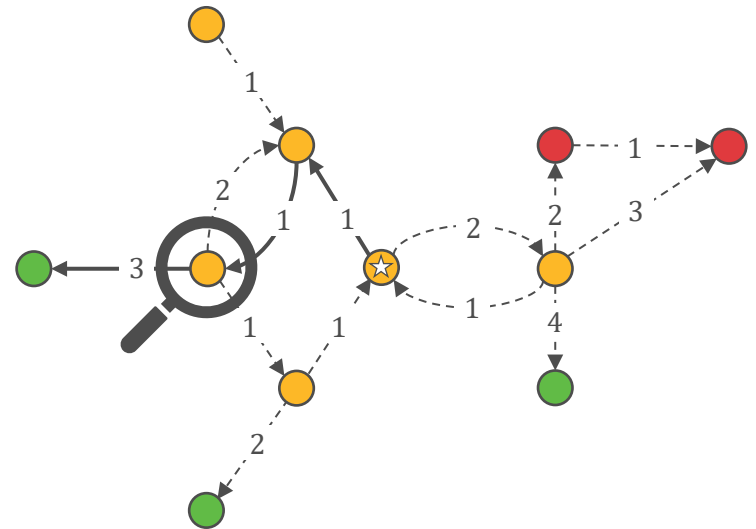
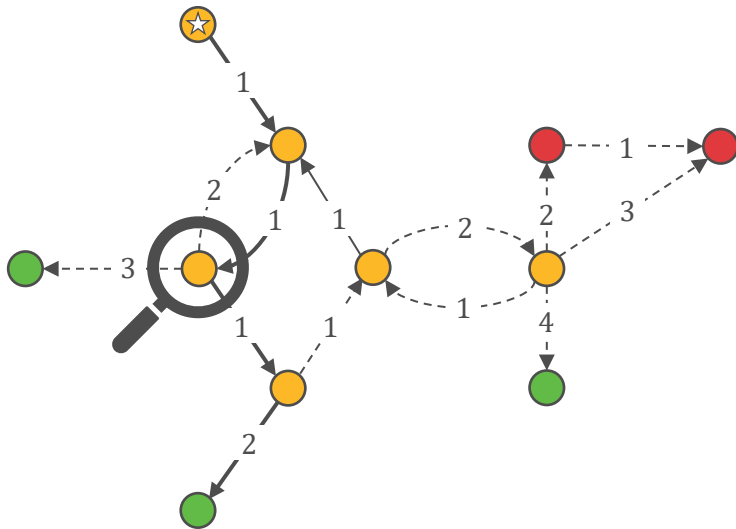
SEQUENCE MECHANISMS



Minimum sum: rank sequences are compared by their sum of ranks, breaking ties lexicographically

CONFLUENT DELEGATION

A ranked delegation mechanism is **confluent** if the delegation paths are consistent



CONFLUENT DELEGATION

A ranked delegation mechanism is **confluent** if the delegation paths are consistent

Poll 2

Which sequence mechanism is **not** confluent?

- Depth-first delegation
- Breadth-first delegation
- Minimum sum
- All are confluent



COPY ROBUSTNESS

- A ranked delegation mechanism is **copy-robust** if the following holds: if a delegating voter v has a delegation path of length 1 to a casting voter c , then the weight of c equals the total weight of c and v if v became a casting voter
- **Theorem:** Depth-first delegation is copy-robust
- **Theorem:** No sequence mechanism is both confluent and copy-robust

PROOF OF THEOREM



Delegation paths must be as above due to confluence



Delegation path maintained due to copy-robustness,
leading to $(2) \succ (1)$ and $(1) \succ (2)$

BIBLIOGRAPHY

A. Kahng, S. Mackenzie, and A. D. Procaccia.
Liquid democracy: An Algorithmic Perspective.
Journal of Artificial Intelligence Research, 2021.

M. Brill, T. Delemazure, A.-M. George,
M. Lackner, and U. Schmidt-Kraepelin.
Liquid Democracy with Ranked Delegations.
AAAI 2022.

