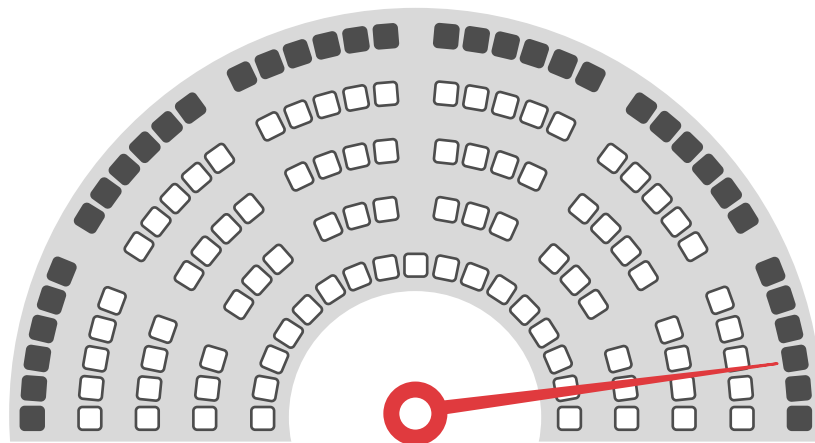




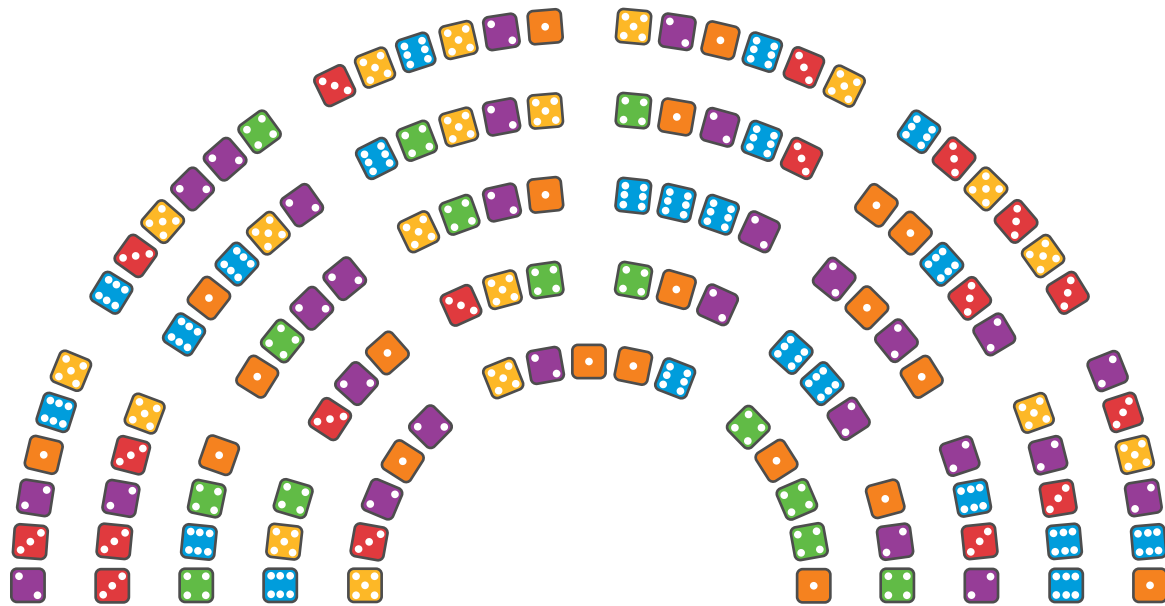
Optimized Democracy

Fall 2026 | Lecture 10

Sortition

Ariel Procaccia | Harvard University

HERE'S A RANDOM IDEA



Sortition—democracy built on
lotteries instead of elections

A BRIEF HISTORY OF SORTITION

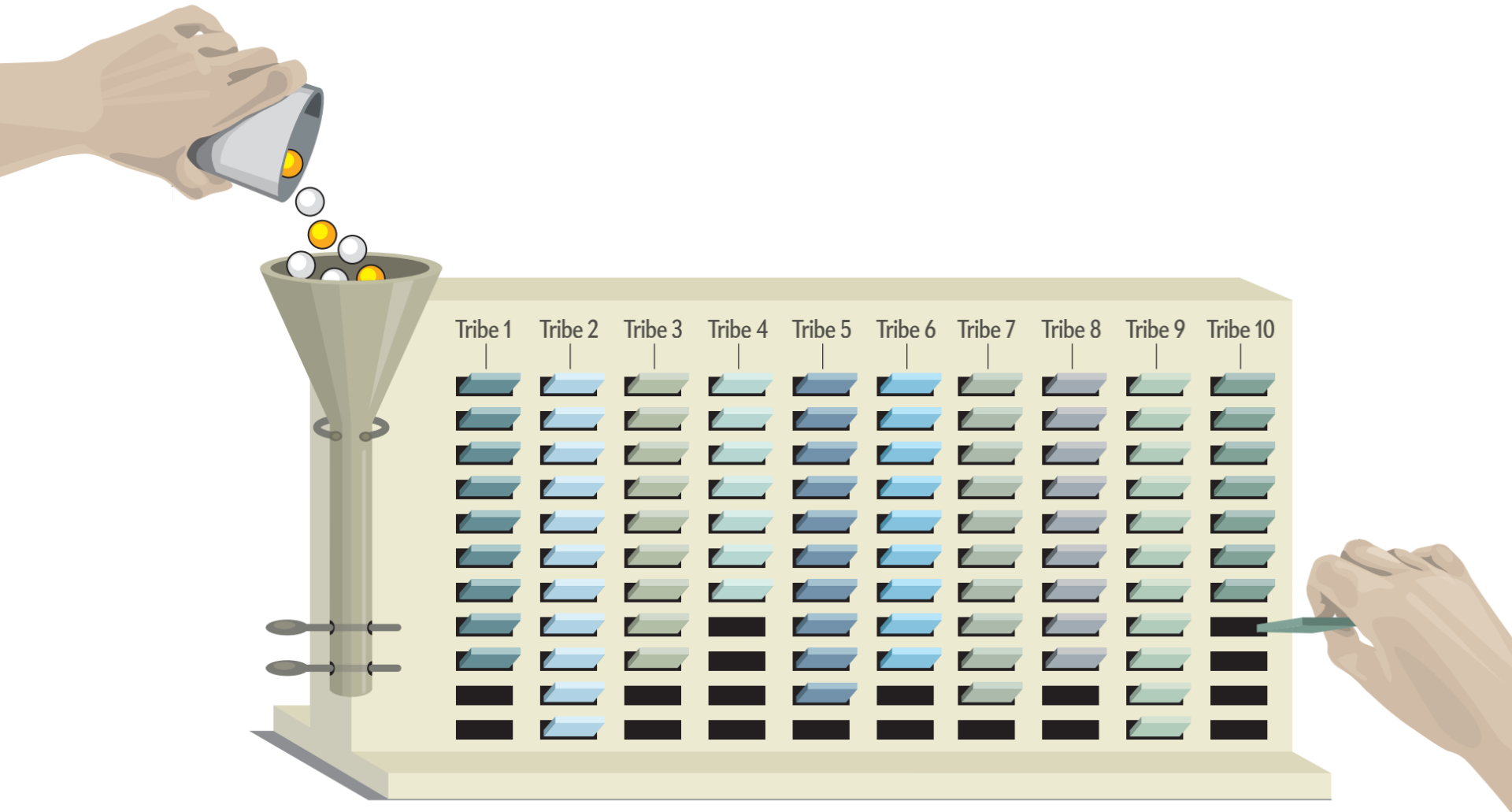
462-322 BC

Athens

Council of 500
and magistracies
chosen by lot



THE KLEROTERION



[Graphics by Jen Christiansen]

THE KLEROTERION

The diagram illustrates the Kleroterion, a mechanical device used for random selection in ancient Athens. It consists of a vertical column of 15 balls (7 yellow, 8 white) and a horizontal array of 10 tribes (Tribe 1 to Tribe 10). A hand is shown dropping a yellow ball into the column. As the ball falls, it passes through a series of horizontal slots, each corresponding to a tribe. The tribe whose slot the ball passes through is the selected tribe. The selected tribe is indicated by a yellow arrow pointing to the word "Selected".

[Graphics by Jen Christiansen]

THE KLEROTERION

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[Graphics by Jen Christiansen]

THE KLEROTERION



[Graphics by Jen Christiansen]

A BRIEF HISTORY OF SORTITION

462-322 BC

Athens

Council of 500
and magistracies
chosen by lot

1328-1530

Florence

The government
and legislative
council chosen by
lot

1776-present

USA

American and
French revolutions
make democracy
synonymous with
elections

21st Century

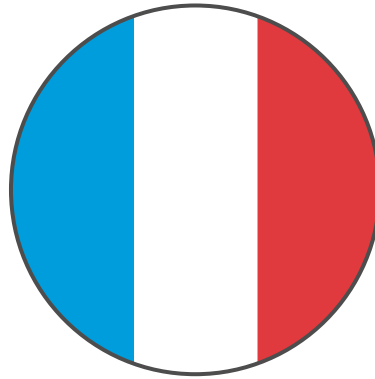
Worldwide

Citizens'
assemblies
organized by local
and national
governments

RANDOM ASSEMBLY REQUIRED



Ireland
2016
Constitution



France
2019-2020
Climate



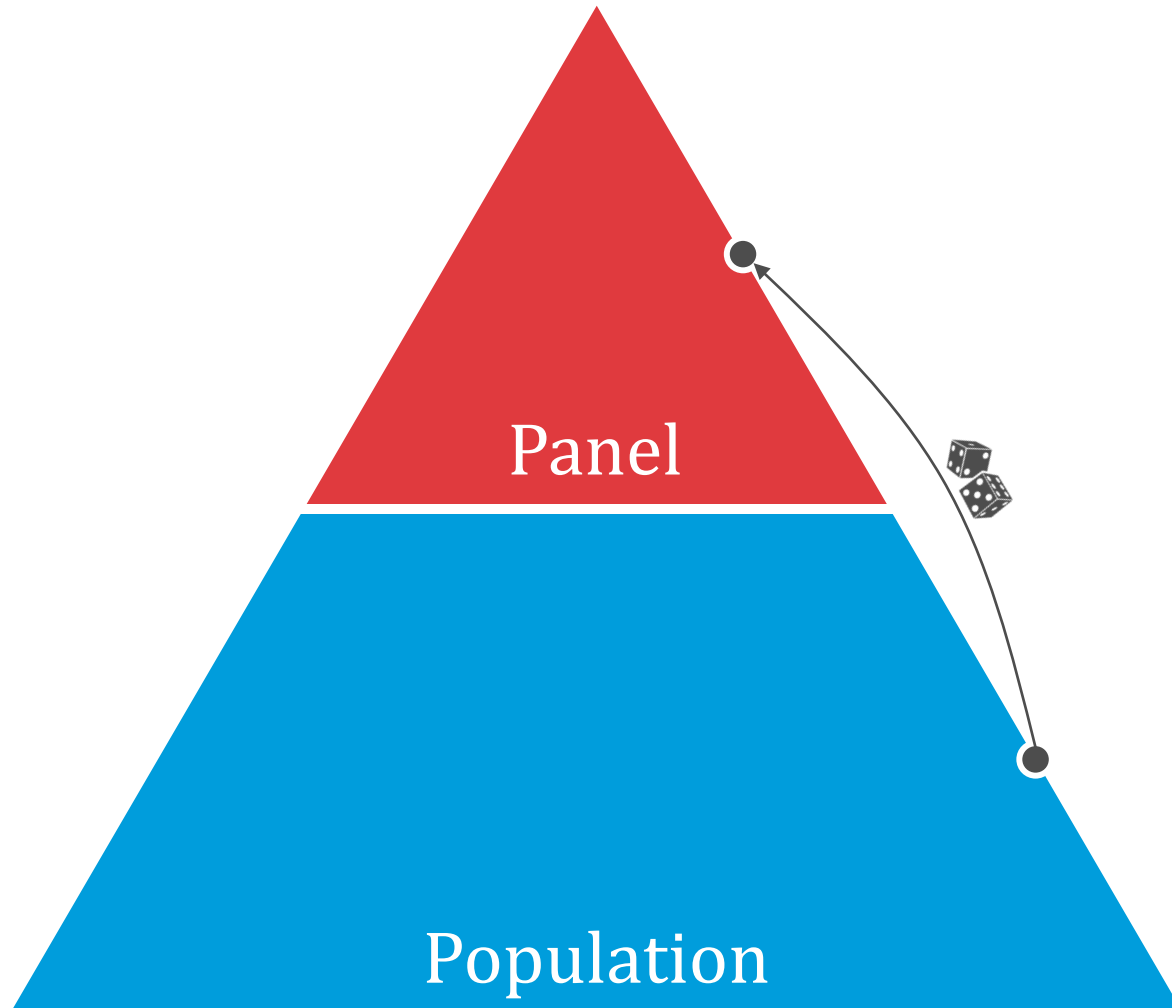
Belgium
Since 2019
Permanent



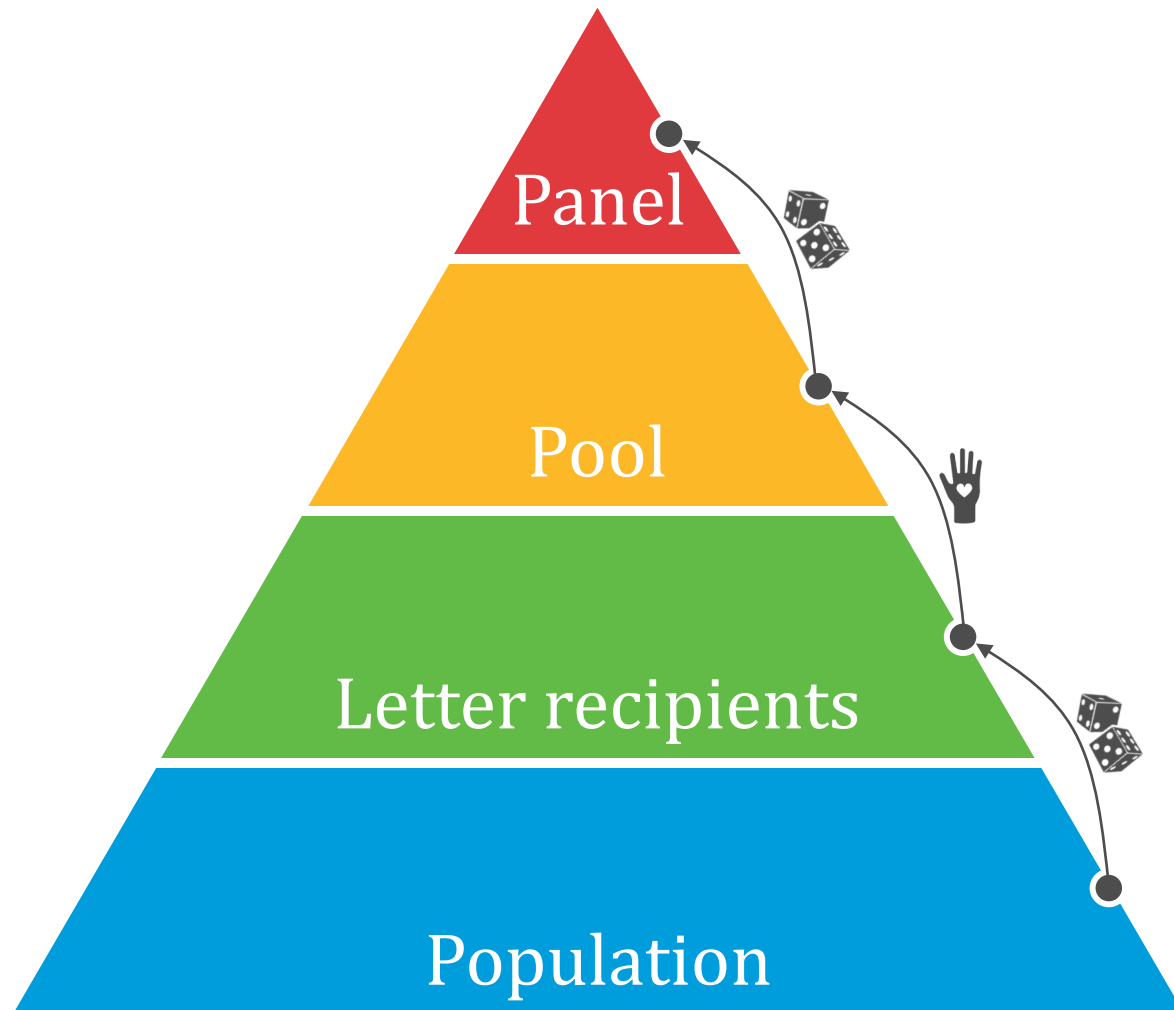


<https://www.youtube.com/watch?v=EDGp5eGnnxI>

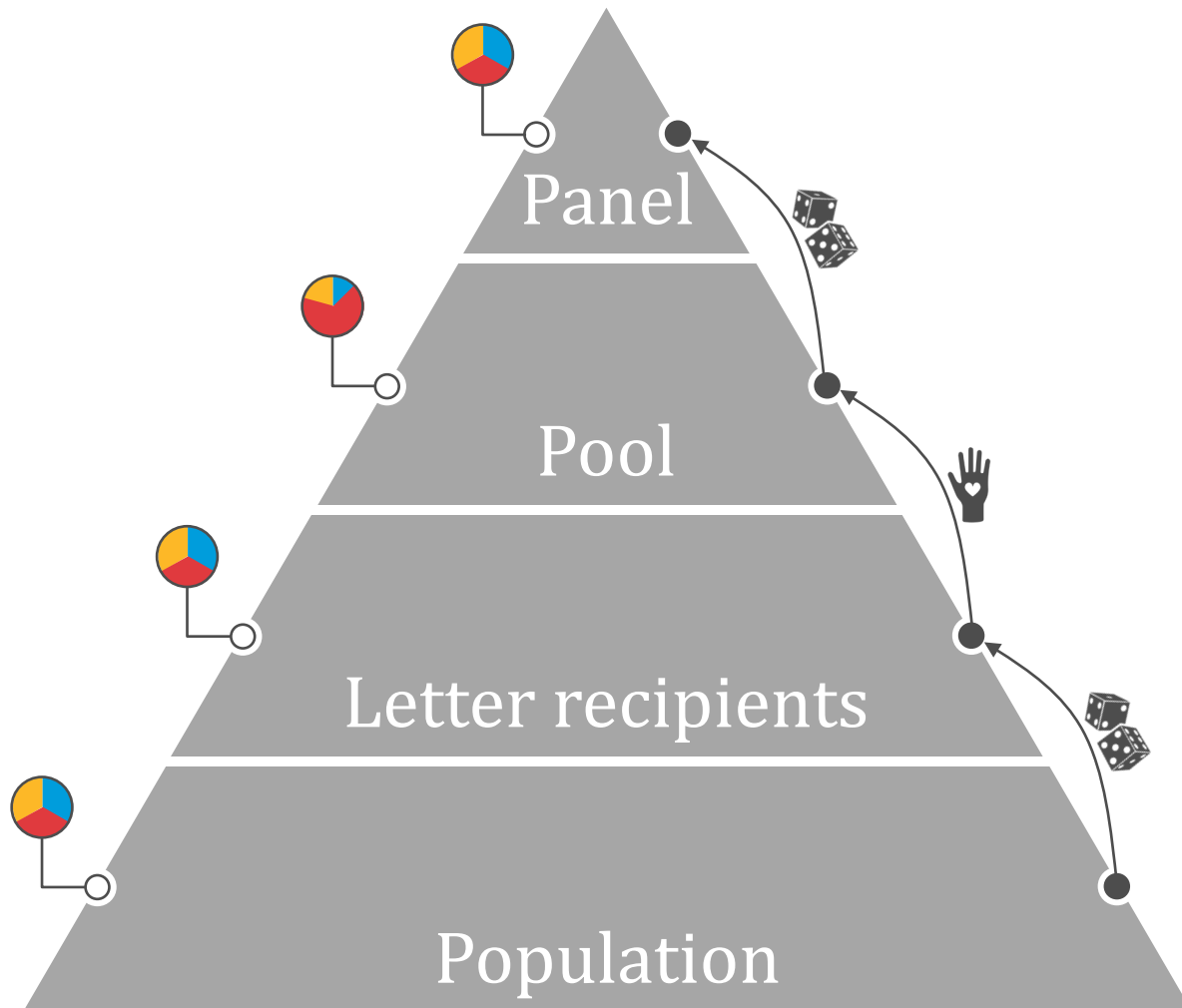
IDEAL SORTITION PIPELINE



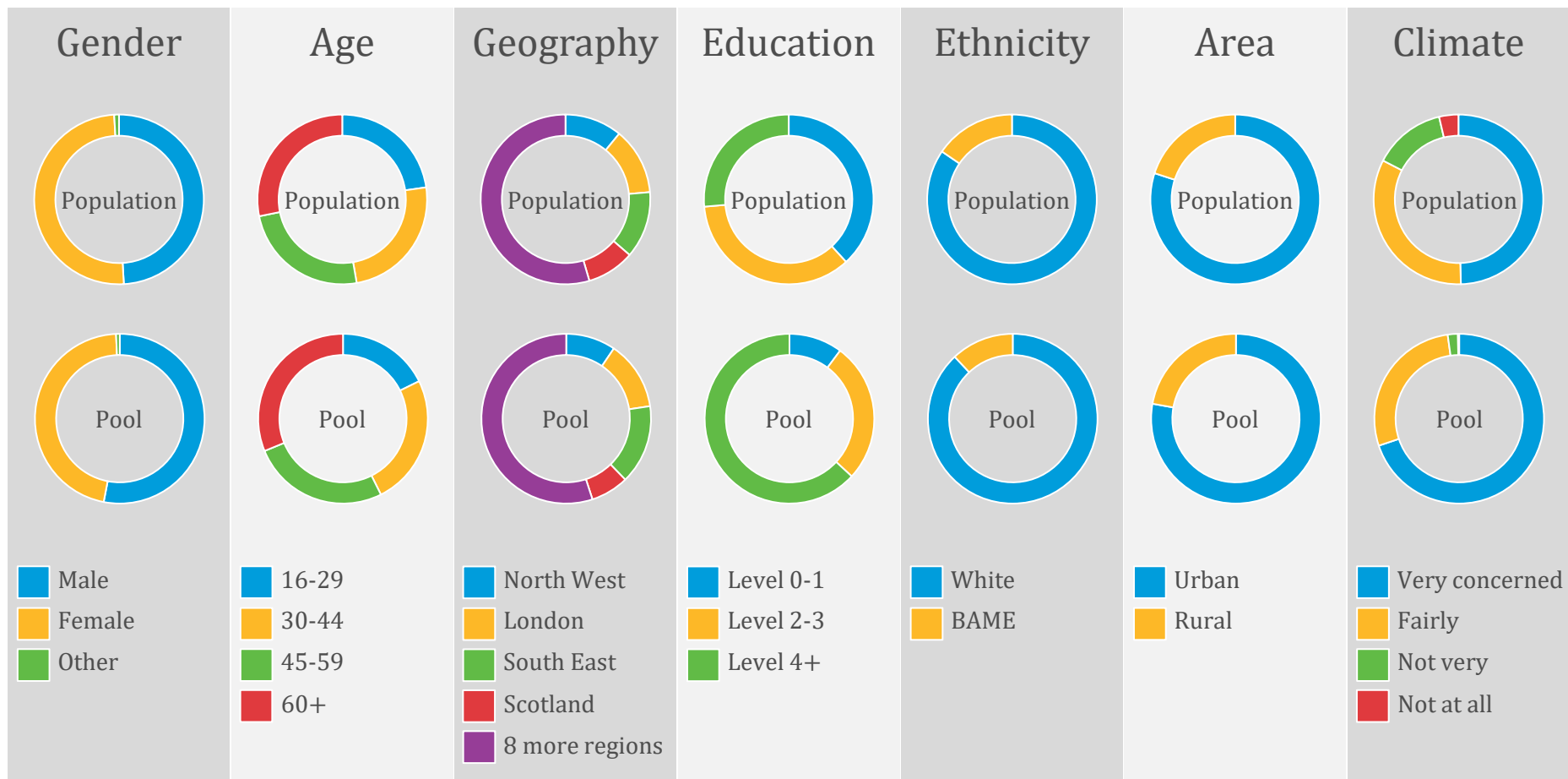
ACTUAL SORTITION PIPELINE



ACTUAL SORTITION PIPELINE

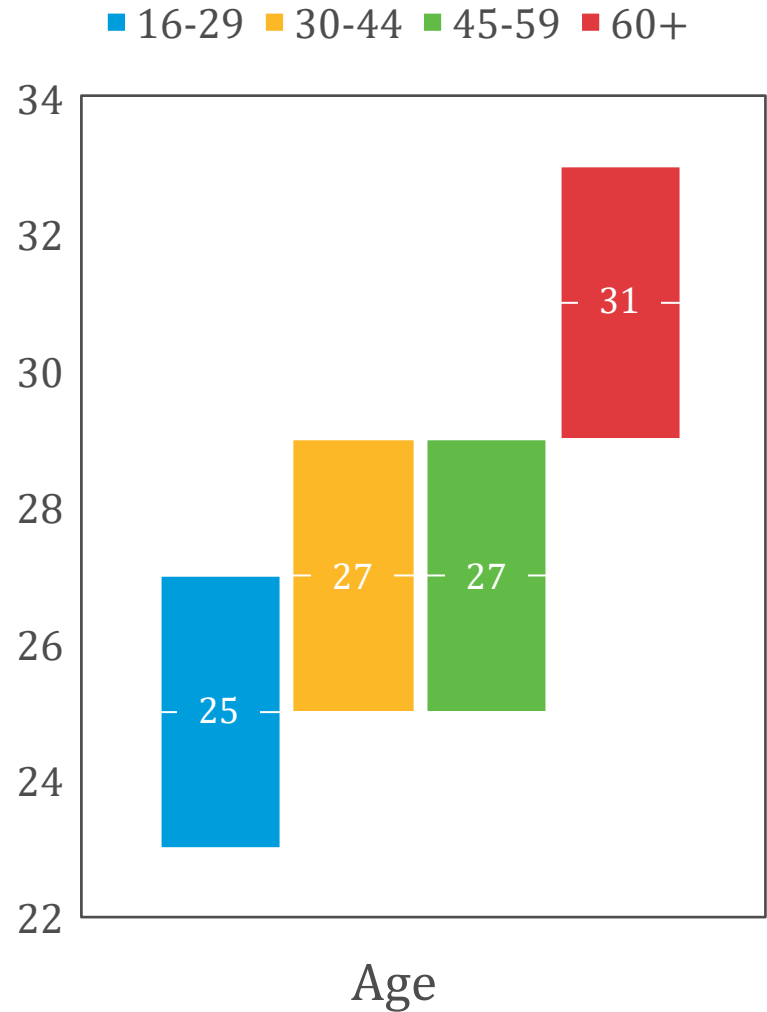
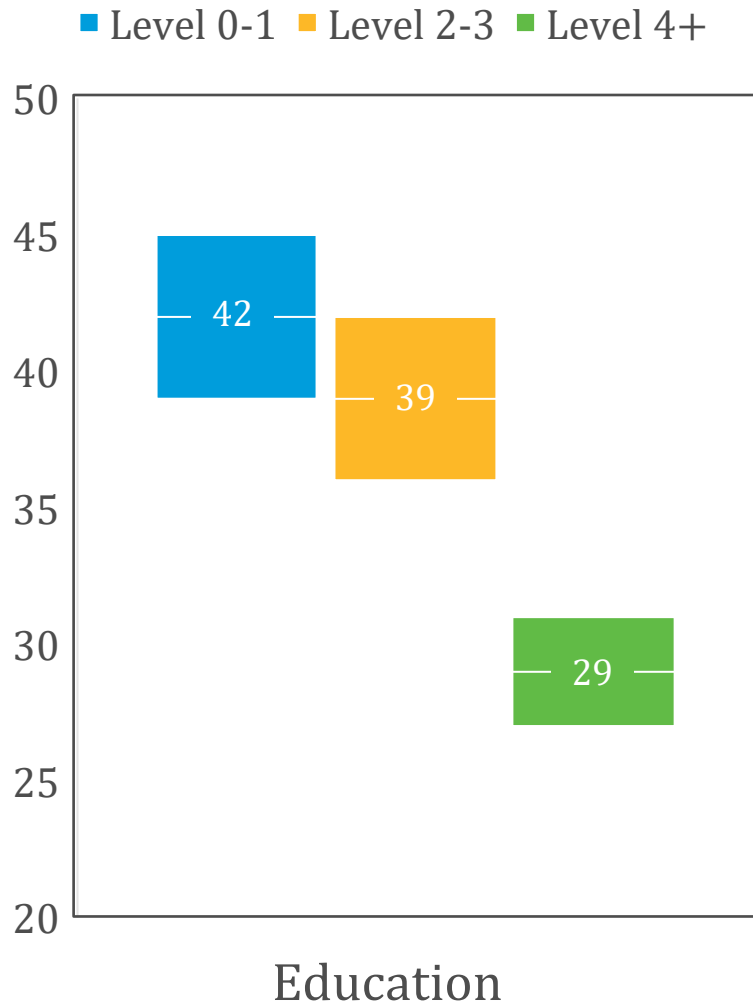


FEATURES



Climate Assembly UK (2020)
Pool size is $n = 1727$, panel size is $k = 110$

QUOTAS



THE SORTITION MODEL

- Set of **features** F , where each $f \in F$ has a set of **values** V_f
- Multiset of n **volunteers** N where each $\mathbf{x} \in N$ is a vector of feature values
- For each $f \in F$ and $v \in V_f$ there is an **upper quota** $u_{f,v}$ and a **lower quota** $\ell_{f,v}$
- The goal is to choose a **panel** P of k volunteers such that for all $f \in F, v \in V_f$,
$$\ell_{f,v} \leq |\{\mathbf{x} \in P: x_f = v\}| \leq u_{f,v}$$
- Finding a quota-feasible panel is NP-hard

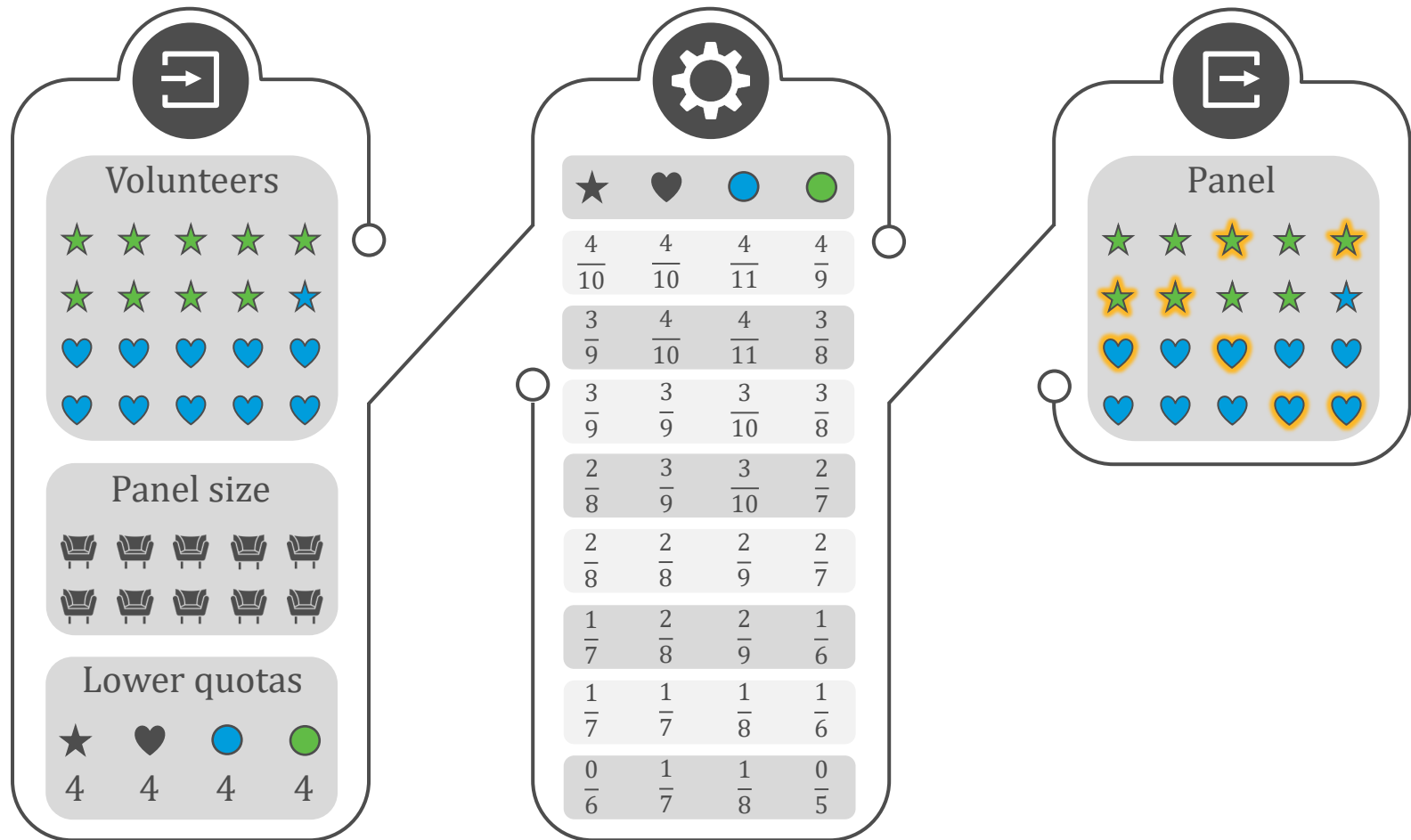
A GREEDY ALGORITHM

- At time t , a partial panel P_t has been selected ($P_0 = \emptyset$)
- For each $f \in F, v \in V_f$ define the score of v to be

$$\frac{\ell_{f,v} - |\{x \in P_t : x_f = v\}|}{|\{x \in N \setminus P_t : x_f = v\}|}$$

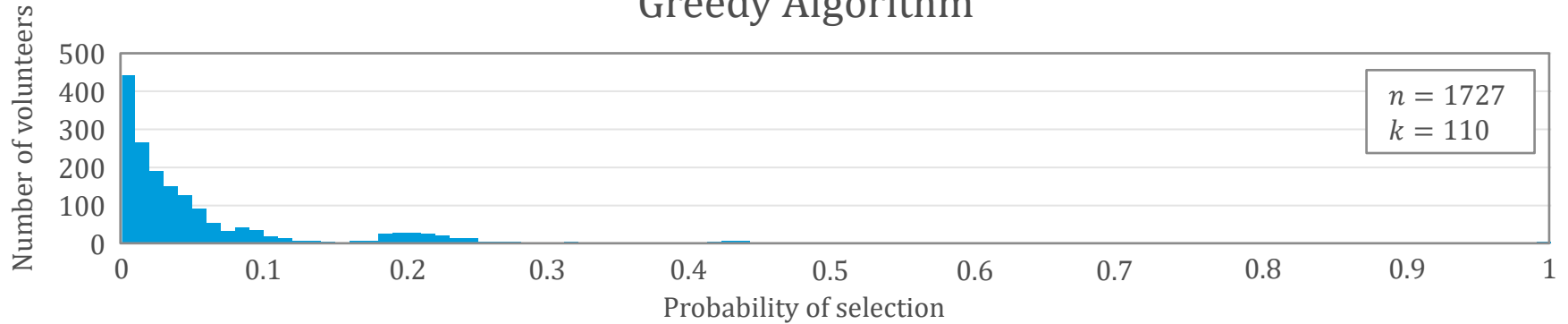
- For v with maximum score, select uniformly at random among $x \in N \setminus P_t$ such that $x_f = v$
- When all lower quotas have been filled, select uniformly at random among $N \setminus P_t$
- If any quotas cannot be satisfied, restart

A GREEDY ALGORITHM

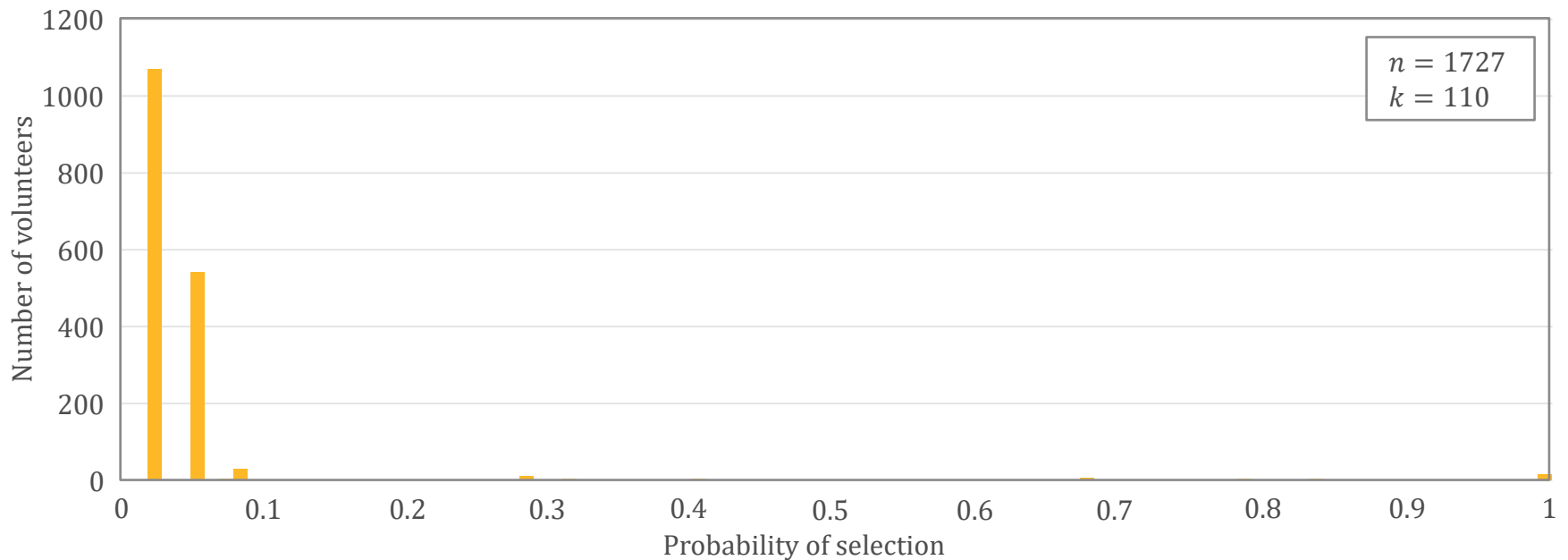


A GREEDY ALGORITHM

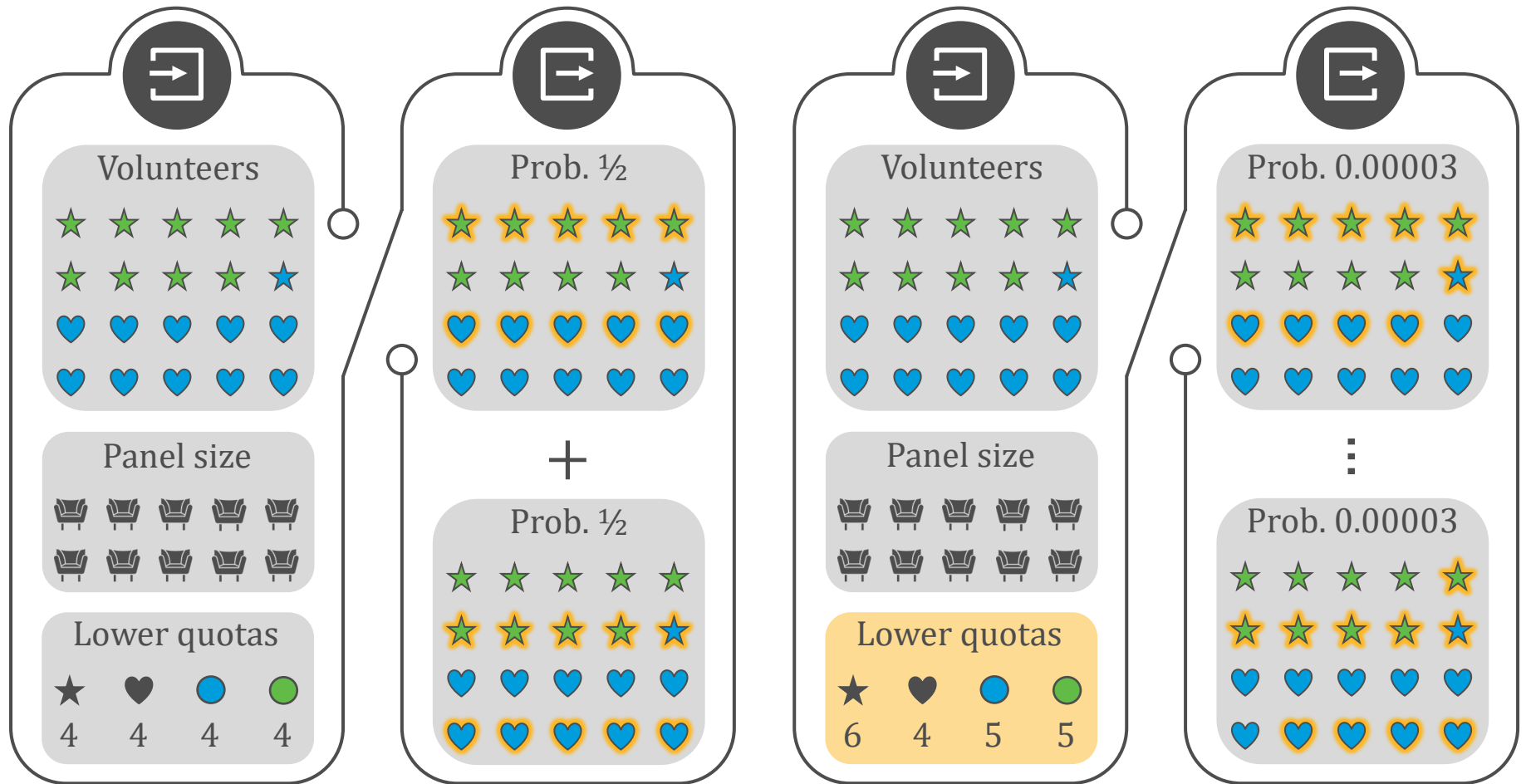
Greedy Algorithm



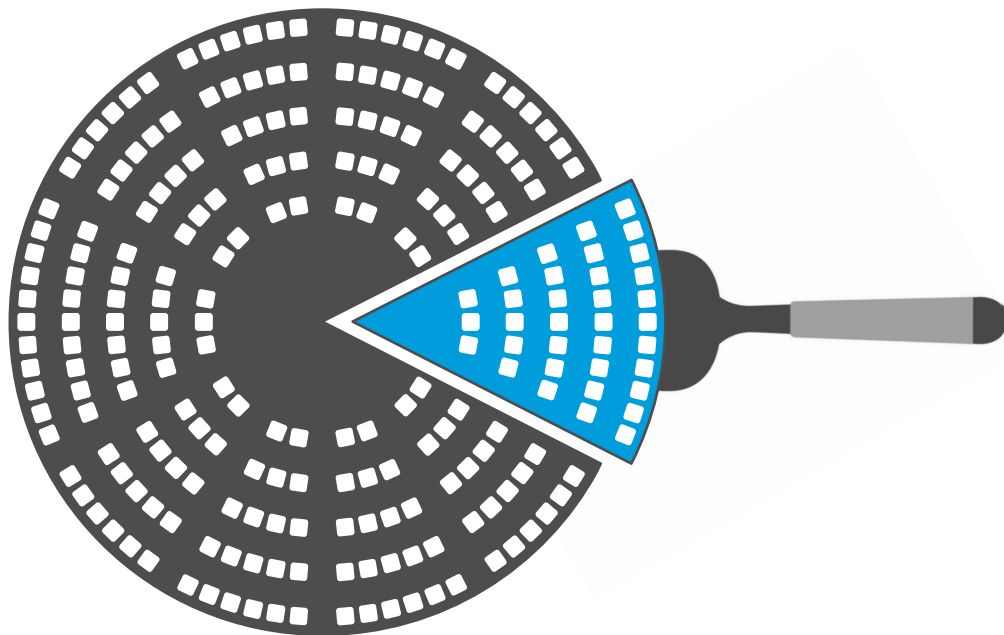
Mystery Challenger



LOADING THE DICE



FROM SORTITION TO FAIR DIVISION



A distribution over panels of size k divides overall selection probability of k between pool members

ALLOCATION RULES

- An **allocation rule** outputs a distribution $\mathcal{D}$ over quota-feasible panels of size k
- **Maximum Nash Welfare** maximizes the product $\prod_{x \in N} \Pr_{P \sim \mathcal{D}} [x \in P]$
- **Leximin** maximizes $\min_{x \in N} \Pr_{P \sim \mathcal{D}} [x \in P]$, subject to that max the second lowest probability, etc.

Poll

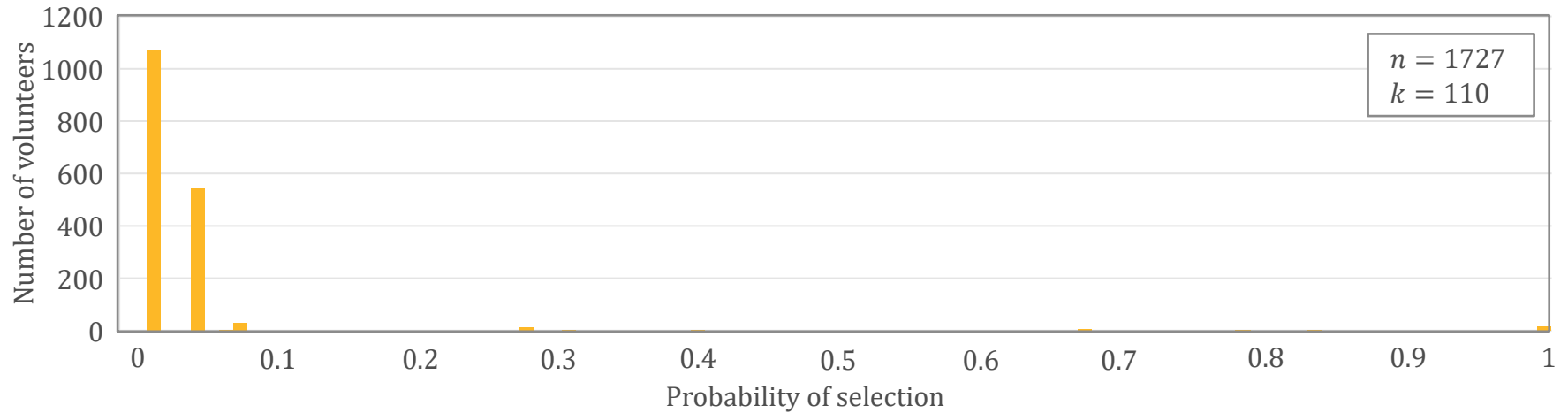
Which of the two rules equalizes volunteers' selection probabilities whenever the quotas make it feasible to do so?

- MNW • Leximin • Both rules • Neither one

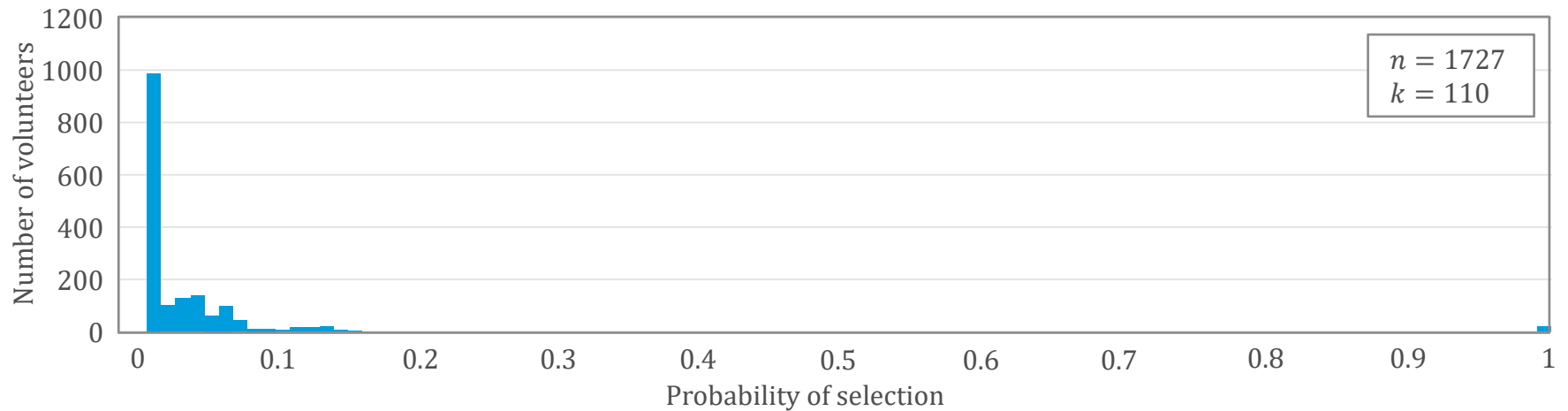


MYSTERY CHALLENGER UNMASKED

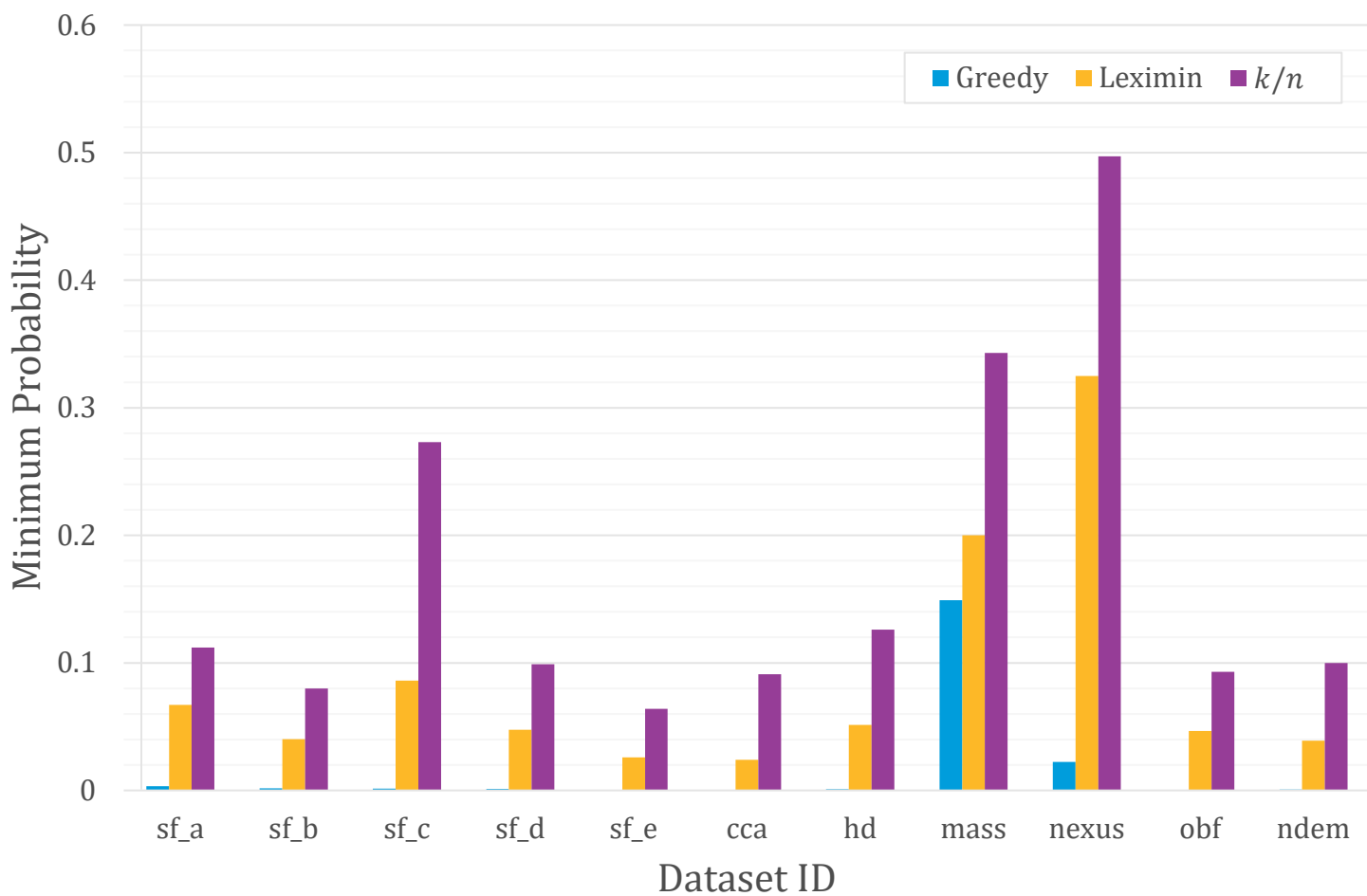
Leximin



Nash Welfare



EVERYONE DESERVES A FAIR CHANCE





Panelot

Online at panelot.org

VISUAL LOTTERIES



Michigan
2020
Covid-19 policy



Colorado
2023
Childcare



Germany
2023
Nutrition

CAMBRIDGE ASSEMBLY



<https://www.wcvb.com/article/cambridge-forms-civic-assembly-to-evaluate-safety-technology/74041338>

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