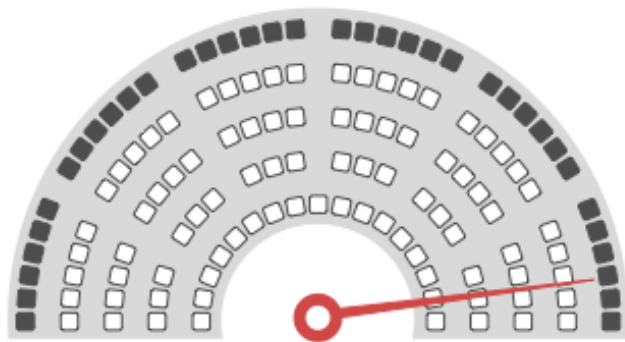




Optimized Democracy

Fall 2025

Preferences & Interpersonal Utility Comparisons

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PROVISIONAL REMARKS

Perhaps besides a calculator app, you shouldn't need any devices today.

(You'll likely enjoy it more!)

INTERPERSONAL UTILITY COMPARISONS

Computer scientists often assume it's possible to make **interpersonal utility comparisons**. Some examples from this semester:

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Distortion:

- Denote the (utilitarian) **social welfare** of $x \in A$ by $sw(x, \mathbf{u}) = \sum_{i \in N} u_i(x)$
- For a preference profile σ and $x \in A$, the **utilitarian distortion** of x at σ is

$$\text{dist}_u(x, \sigma) = \max_{y \in A} \max_{\mathbf{u} \succ \sigma} \frac{sw(y, \mathbf{u})}{sw(x, \mathbf{u})}$$

INTERPERSONAL UTILITY COMPARISONS

Computer scientists often assume it's possible to make **interpersonal utility comparisons**. Some examples from this semester:

Facility Location:

- Two objective functions
 - **Social cost:** $sc(y, x) = \sum_{i \in N} |y - x_i|$
 - **Maximum cost:** $mc(y, x) = \max_{i \in N} |y - x_i|$

WHAT IS “UTILITY”?

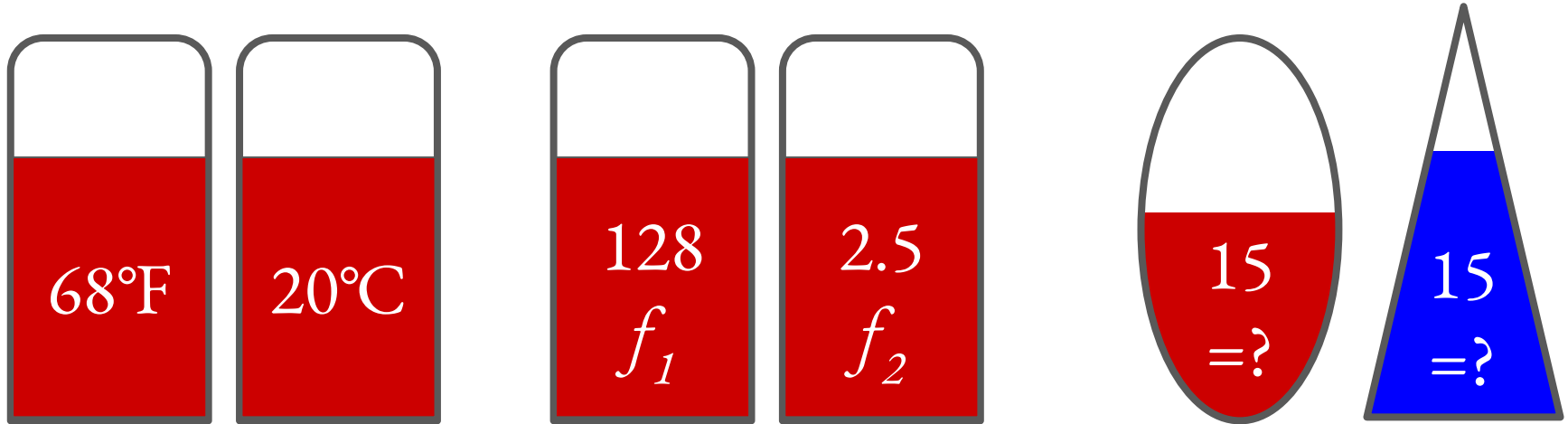
“Utility” in our context is NOT objective welfare, physical distance from some good, monetary value, happiness...

Think of a von Neumann-Morgenstern utility function that captures preferences over outcomes (and lotteries).

(i.e., a *subjective* utility function)

WHAT IS “UTILITY”?

If voter preferences differ, their utility functions will differ,
and do so in ways that defy simple “conversion”.



WHAT IS “UTILITY”?

Assuming you do have a subjective utility function that captures your preferences:

Where do you think it comes from?

PREFERENCES & WELFARE

Computer Scientists:

“... the existence of such a [interpersonally comparable] cardinal utility structure is rarely disputed...” (Anshelevich et al. 2021)

Economists (& most Philosophers):

“Interpersonal comparisons of utility (that is, of preferred positions on an individual’s preference scale) are known to be scientifically impossible in economics.” (Lemieux 2022)

POLL

Given what we discussed about where subjective utilities come from (assuming they exist):

Initial reaction: **who do you think is right?**

Instructional team note: distribute first sheet!

TODAY

- What really ARE preferences?
- Assuming we have subjective utility functions, are the values interpersonally comparable?
- What does a group need to know about each other to make them possible?

SETUP

Depending on the outcome of today's activity, you will receive one of the following snacks:

Bar (b)



Cheese* (c)



Jerky (j)



Snickers* (s)



THINK ABOUT EACH OPTION:

Which outcome would you prefer?

Any questions about any of them?

1. PERSONAL PREFERENCE GROUNDING

Individual task 1.1



	Prefer	Disprefer
<i>b</i>		
<i>c</i>		
<i>j</i>		
<i>s</i>		

1. PERSONAL PREFERENCE GROUNDING

Individual task 1.1

Matt's example

	Prefer	Disprefer
 <i>b</i>	<i>Environmentally friendly</i>	
 <i>c</i>		<i>Not great solo</i>
 <i>j</i>		<i>I'm a vegetarian</i>
 <i>s</i>	<i>Good solo</i>	<i>Try not to eat candy in AM</i>

2. PERSONAL ORDINAL RANK & UTILITIES

Individual task 2.1

	Ordinal Rank	Utility: $u(x)$
 <i>b</i>		
 <i>c</i>		
 <i>j</i>		
 <i>s</i>		

2. PERSONAL ORDINAL RANK & UTILITIES

Individual task 2.1

Matt's example

	Ordinal Rank	Utility: $u(x)$
 <i>b</i>	1	
 <i>c</i>	3	
 <i>j</i>	4	
 <i>s</i>	2	

BRIEF ASIDE

When you woke up today, did you have that preference ranking over those outcomes?

2. PERSONAL ORDINAL RANK & UTILITIES

Individual task 2.2

	Ordinal Rank	Utility: $u(x)$
 <i>b</i>	1	
 <i>c</i>	3	
 <i>j</i>	4	
 <i>s</i>	2	

RULES FOR UTILITIES

Values should be:

1. $0 \leq x \leq 1$, and must sum to 1
2. If you were indifferent between x and y , $u(x) = u(y)$
3. roughly consistent with obvious lotteries
[i.e., if $u(\text{Pr}(x)=0.5) = u(y)$ and $u(y) = 0.3$, then $u(x) = 0.6$]

2. PERSONAL ORDINAL RANK & UTILITIES

Individual task 2.2

Matt's example



b

Ordinal Rank

1

Utility

0.6



c

3

0.1



j

4

0



s

2

0.3

BRIEF ASIDE

Did anyone find that task difficult?

Instructional team note: distribute second sheet!

BRIEF ASIDE

Did anyone find that task difficult?

“... the existence of such a ~~[interpersonally comparable]~~
cardinal utility structure is rarely disputed...” (Anshelevich et
al. 2021)

IMPORTANT INSTRUCTION

Don't discuss the utility values you wrote down with your group until instructed to do so!

4. GROUP VOTE

Your group will be voting on one type of snack to receive at the end of the session based on your individual rankings.

All members of your group will receive the type of snack selected through your voting procedure.

No trading with, or donating to, others is allowed!

4. GROUP VOTE

	1:	2:	3:	4:	Votes
b					
c					
j					
s					

4. GROUP VOTE

	1: Ash	2: Bo	3: Cade	4: Matt	Votes
b				1	
c				3	
j				4	
s				2	

4. GROUP VOTE

	1: Ash	2: Bo	3: Cade	4: Matt	Votes
b	3	3	1	1	
c	1	2	2	3	
j	2	4	4	4	
s	4	1	3	2	

4. GROUP VOTE

	1: Ash	2: Bo	3: Cade	4: Matt	Votes
b	3	3	1	1	
c	1	2	2	3	
j	2	4	4	4	
s	4	1	3	2	

4. GROUP VOTE

	1: Ash	2: Bo	3: Cade	4: Matt	Votes
b	3	3	1	1	2
c	1	2	2	3	1
j	2	4	4	4	0
s	4	1	3	2	1

4. GROUP VOTE

	1: Ash	2: Bo	3: Cade	4: Matt	Votes
b	3	3	1	1	2
c	1	2	2	3	1
j	2	4	4	4	0
s	4	1	3	2	1

5. INFORMAL SOCIAL WELFARE ANALYSIS

Group task 5.1: Without looking at individual utilities, work through each possible outcome in order, and each member should share whether they would be happy with that being the outcome of the vote. Keep track of any particularly weighty considerations in the table below.

GROUP TASK 5.1

	Weighty Reason to Prefer	Weighty Reason to Disprefer
b		
c		
j		
s		

GROUP TASK 5.1

	Weighty Reason to Prefer	Weighty Reason to Disprefer
<i>b</i>	Cade is vegan	Ash is (kinda) keto
<i>c</i>		Cade is vegan
<i>j</i>		Cade is vegan, Matt is vego
<i>s</i>		Ash is (kinda) keto, Cade is vegan

5. INFORMAL SOCIAL WELFARE ANALYSIS

Group task 5.2: Consider the outcome of the vote, and decide whether you think that was the best for the social welfare of the group overall.

6. EXTENDED (EMPATHETIC) UTILITIES

- Say you know *what it would be like* for a group member to receive a certain snack, *given that person's preferences*.
- Then you should be able to compare how good that state is *for them* to how good various states are *for you*.
- If everyone in your group can do that,* and you all agree, then interpersonal utility comparisons are totally fine.

*(including over lotteries)

6. EXTENDED (EMPATHETIC) UTILITIES

One way to notate this mathematically:

$$u(x, i) = v(x, r_i)$$

Alternative

Voter

Voter features
(e.g. taste)



6. EXTENDED (EMPATHETIC) UTILITIES

	1: Ash	2: Bo	3: Cade	4: Matt
<i>b</i>	3	3	1	1
<i>c</i>	1	2	2	3
<i>j</i>	2	4	4	4
<i>s</i>	4	1	3	2

6. EXTENDED (EMPATHETIC) UTILITIES

	1: Ash	2: Bo	3: Cade	4: Matt
<i>b</i>	3	3	1	1
<i>c</i>	1	2	2	3
<i>j</i>	2	4	4	4
<i>s</i>	4	1	3	2

GROUP TASK 6.1

Without looking at individual utilities, as a group, try to come to a consensus over the extended preference ranking of each member's most preferred item as assigned during the individual preference ranking.

(In cases of genuine indifference, use the tie breaking convention below.)

GROUP TASK 6.1

	#1: Person / Item	#2: Person / Item	#3: Person / Item	#4: Person / Item
Extended Ranking	Cade / Bar	Ash / Cheese	Matt / Bar	Bo / Snickers

GROUP TASK 6.2:

Transcribe each individual's original utility values for that option in the row below each extended ranking.

FINAL DISCUSSION 1:

Did any groups find it easy to create the Extended Utility Function?

Did any groups find it difficult?

FINAL DISCUSSION 2:

Did your individual utility values line up with the Extended Ranking?

ANY FEEDBACK?

Survey link, with the option to
get free coffee and chat more:

