
Interpersonal Utility Comparisons

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D1

Abstract

Although we all make interpersonal utility comparisons, many economists and philosophers argue that our limited information about other people's minds renders them meaningless. If they are possible, interpersonal comparisons of utility *differences* must be distinguished from interpersonal comparisons of utility *levels*. Utilitarianism must assume the interpersonal comparability of utility differences to maximize a social welfare function, while Rawls's maximin principle requires interpersonal comparability of utility levels. Adopting an ordinalist or a cardinalist view of utility functions restricts the positions one can consistently take as to interpersonal comparability of utilities.

Keywords

Arrow, K.; Interpersonal utility comparisons; Maximin; Rawls, J.; Robbins, L.;

Suppose I am left with a ticket to a Mozart concert I am unable to attend and decide to give it to one of my closest friends. Which friend should I actually give it to? One thing I will surely consider in deciding this is which friend of mine would enjoy the concert *most*. More generally, when we decide as private individuals whom to help, or decide as voters or as public officials who are to receive government help, *one* natural criterion we use is who would derive the greatest benefit, that is, who would derive the *highest utility*, from this help. But to answer this last question we must make, or at least attempt to make, *interpersonal utility comparisons*.

At the common-sense level, all of us make such interpersonal comparisons. But philosophical reflection might make us uneasy about their meaning and validity. We have direct introspective access only to our *own* mental processes (such as our preferences and our feelings of satisfaction and dissatisfaction) defining our *own* utility function, but have only very indirect information about other people's mental processes. Many economists and philosophers take the view that our limited information about other people's

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minds renders it impossible for us to make meaningful interpersonal comparisons of utility.

Comparisons of Utility Levels vs. Comparisons of Utility Differences

In any case, if such comparisons are possible at all, then we must distinguish between interpersonal comparisons of utility levels and interpersonal comparisons of utility differences (i.e. utility increments or decrements).

It is one thing to compare the utility level $U_i(A)$ that individual i enjoys (or would enjoy) in situation A , with utility level $U_j(B)$ that another individual j enjoys (or would enjoy) in situation B (where A and B may not refer to the same situation). It is a very different thing to make interpersonal comparisons between utility differences, such as comparing the utility increment

$$\Delta U_i(A, A') = U_i(A') - U_i(A) \quad (1)$$

that individual i would enjoy in moving from situation A to situation A' , with the utility increment

$$\Delta U_j(B, B') = U_j(B') - U_j(B) \quad (2)$$

that individual j would enjoy in moving from B to B' . Either kind of interpersonal comparison might be possible without the other kind being possible (Sen 1970).

Some ethical theories would require one kind of interpersonal comparisons; others would require the other. Thus, *utilitarianism* must assume the interpersonal comparability of utility differences because it asks us to maximize a social utility function (social welfare function) defined as the *sum* of all individual utilities. (There are arguments for defining social utility as the *arithmetic mean*, rather than the *sum*, of individual utilities (Harsanyi 1955). But for most purposes – other than analysing population policies – the two definitions are equivalent because if the number of individuals can be taken for a *constant*, then

maximizing the sum of utilities is mathematically equivalent to maximizing their arithmetic mean.) Yet, we cannot add different people's utilities unless all of them are expressed in the same utility units; and in order to decide whether this is the case, we must engage in interpersonal comparisons of utility differences. (On the other hand, utilitarianism does not require comparisons of different people's utility levels because it does not matter whether their utilities are measured from comparable zero points or not.)

Likewise, the interpersonal utility comparisons we make in everyday life are most of the time comparisons of utility *differences*. For instance, the comparisons made in our example between the utilities that different people would derive from a concert obviously involve comparing utility differences.

In contrast, the utility-based version of Rawls's *Theory of Justice* (1971) does require interpersonal comparisons of utility *levels*, but does not require comparisons of utility *differences*. This is so because his theory uses the *maximin principle* (he calls it the *difference principle*) in evaluating the economic performance of each society, in the sense of using the well-being of the *worst-off* individual (or the worst-off social group) as its principal criterion. But to decide which individuals (or social groups) are worse off than others he must compare different people's utility levels. (In earlier publications, Rawls seemed to define the worst-off individual as one with the lowest utility level. But in later publications, he defined him as one with the smallest amount of 'primary goods'. For a critique of Rawls's theory, see Harsanyi 1975).

Ordinalism, Cardinalism and Interpersonal Comparisons

In studying comparisons between the utilities enjoyed by *one* particular individual i , we again have to distinguish between comparisons of utility *levels* and comparisons of utility *differences*. The former would involve comparing the utility levels $U_i(A)$ and $U_i(B)$ that i assigns to two different situations A and B . The latter would involve comparing the utility increment

$$\Delta U_i(A, A') = U_i(A') - U_i(A) \quad (3)$$

that i would enjoy in moving from situation A to situation A' , with the utility increment

$$\Delta U_i(B, B') = U_i(B') - U_i(B) \quad (4)$$

that he would enjoy in moving from B to B' .

If i has a well-defined utility function U_i at all, then he certainly must be able to compare the utility *levels* he assigns to various situations; and such comparisons will have a clear behavioural meaning because they will correspond to the preference and indifference relations expressed by his choice behaviour. In contrast, it is immediately less obvious whether comparing utility differences as defined under (3) and (4) has any economic meaning (but see below).

A utility function U_i permitting meaningful comparisons only between i 's utility levels, but not permitting such comparisons between his utility differences, is called ordinal; whereas a utility function permitting meaningful comparisons both between his utility levels and his utility differences is called cardinal.

As is well known, most branches of economic theory use only ordinal utilities. But, as von Neumann and Morgenstern (1947) have shown, cardinal utility functions can play a very useful role in the theory of risk taking. In fact, utility-difference comparisons based on von Neumann–Morgenstern utility functions turn out to have a direct behavioural meaning. For example, suppose that U_i is such a utility function, and let Δ_i^* and Δ_i^{**} be utility differences defined by (3) and by (4). Then, the inequality $\Delta_i^* > \Delta_i^{**}$ will be algebraically equivalent to the inequality

$$\begin{aligned} \frac{1}{2}U_i(A') + \frac{1}{2}U_i(B) &> \frac{1}{2}U_i(B') \\ &+ \frac{1}{2}U_i(A). \end{aligned} \quad (5)$$

This inequality in turn will have the behavioural interpretation that i *prefers* an equiprobability mixture of A' and of B to an equi-

probability mixture of B' and of A . Of course, once von Neumann–Morgenstern utility functions are used in the theory of risk taking, they become available for possible use also in other branches of economic theory, including welfare economics as well as in ethical investigations. (It has been argued that von Neumann–Morgenstern utility functions have no place in ethics (or in welfare economics) because they merely express people's attitudes toward *gambling*, which has no moral significance (Arrow 1951, p. 10; Rawls 1971, pp. 172 and 323). But see Harsanyi 1984.)

Note that by taking an ordinalist or a cardinalist position, one restricts the positions one can consistently take as to interpersonal comparability of utilities:

- (1) An *ordinalist* is logically free to *reject* both types of interpersonal comparisons. Or he may *admit* comparisons of different people's utility *levels*. But he *cannot* admit the interpersonal comparability of utility differences without becoming a cardinalist. (The reason is this. If the utility differences experienced by one individual i are comparable with those experienced by *another* individual j , this will make the utility differences experienced by *one* individual (say) i likewise indirectly comparable with one another, which will enable us to construct a *cardinal* utility function for each individual.)
- (2) A *cardinalist* is likewise logically free to *reject* both types of interpersonal comparisons. Or he may *admit* both. Or else he may admit interpersonal comparisons only for utility *differences*. (Though it is hard to see why anybody might want to reject interpersonal comparisons for utility levels if he admitted them for utility differences.) But he *cannot* consistently admit interpersonal comparisons for utility *levels* while rejecting them for utility *differences*. (This can be verified as follows. If utility levels are interpersonally comparable, then we can find four situations A , A' , B , and B' such that $U_i(A) = U_j(B)$ and $U_j(A') = U_i(B')$. But then we can conclude that

$$\Delta_i^* = U_i(A') - U_i(A) = \Delta_j^* = U_i(B') - U_i(B)$$

which means that at least the utility differences Δ_i^* and Δ_j^* are interpersonally comparable. But since U_i and U_j are *cardinal* utility functions, any utility difference Δ_i^{**} experienced by i is comparable with Δ_i^* , and any utility difference Δ_j^{**} experienced by j is comparable with Δ_j^* . Yet this means that *all* utility differences Δ_i^{**} experienced by i are comparable with *all* utility differences Δ_j^{**} experienced by j . Thus, cardinalism together with interpersonal comparability of utility levels *entails* that of utility differences.)

Extended Utility Functions

In what follows, I will use the symbols $A_i, B_i, \dots$ to denote the economic and non-economic resources available to individual i in situations $A, B, \dots$. Moreover, I will use the symbol A_j to denote an arrangement under which j has the same resources available to him as were available to individual i under arrangement A_i . These entities $A_i, B_i, \dots, A_j, B_j, \dots$ I will call positions.

Interpersonal utility comparisons would pose no problem if all individuals had the same utility function. For in this case, any individual j could assume that the utility level $U_i(A_i)$ that another individual i would derive from a given position A_i should be the *same* as he himself would derive from a similar position. Thus, j could write simply.

$$U_i(A_i) = U_j(A_j). \quad (6)$$

Of course, in actual fact, the utility of different people are rather *different* because people have different *tastes*, that is, they have different abilities to derive satisfactions from given resource endowments. I will use the symbols $R_i, R_j, \dots$ to denote the vectors listing the personal psychological characteristics of each individual $i, j, \dots$ that *explain* the differences among their utility functions $U_i, U_j, \dots$. Presumably, these vectors summarize the effects that the genetic make-up, the

education and the life experience of each individual have on his utility function. This means that any individual j can attempt to assess the utility level $U_i(A_i)$ that another individual j would enjoy in position A_i as

$$U_i(A) = V(A_i, R_i), \quad (7)$$

where the function V represents the psychological laws determining the utility functions $U_i, U_j, \dots$ of the various individuals $i, j, \dots$ in accordance with their psychological parameters specified by the vectors $R_i, R_j, \dots$. Since, by assumption, all differences among the various individuals' utility functions $U_i, U_j, \dots$ are fully explained by the vectors $R_i, R_j, \dots$, the function V itself will be the same for all individuals. We will call V an *extended utility function*. (See Arrow 1978; Harsanyi 1977, pp. 51–60; though the basic ideas are contained already in Arrow 1951, pp. 114–15.)

To be sure, we know very little about the psychological laws determining people's utility functions and, therefore, know very little about the true mathematical form of the extended utility function V . This means that, when we try to use Eq. (7), the best we can do is to use our – surely very imperfect – personal *estimate* of V , rather than V itself. As a result, in trying to make interpersonal utility comparisons, we must expect to make significant errors from time to time – in particular when we are trying to assess the utility functions of people with a very different cultural and social background from our own. But even if our judgements of interpersonal comparisons can easily be mistaken, this does not imply that they are meaningless.

Ordinalists will interpret both the functions U_i and the function V as ordinal utility functions and will interpret (7) merely as a warrant for interpersonal comparisons of utility levels (cf. Arrow 1978). In contrast, cardinalists will interpret all these as *cardinal* utility functions and will interpret (7) as a warrant for *both* kinds of interpersonal comparison (cf. Harsanyi 1977).

Limits to Interpersonal Comparisons

It seems to me that economists and philosophers influenced by *logical positivism* have greatly

exaggerated the difficulties we face in making interpersonal utility comparisons with respect to the utilities and the disutilities that people derive from ordinary commodities and, more generally, from the ordinary pleasures and calamities of human life. (A very influential opponent of the possibility of meaningful interpersonal utility comparisons has been Robbins 1932.) But when we face the problem of judging the utilities and the disutilities that other people derive from various *cultural* activities, we do seem to run into very real, and sometimes perhaps even unsurmountable, difficulties. For example, suppose I observe a group of people who claim to derive great aesthetic enjoyment from a very esoteric form of abstract art, which does not have the slightest appeal to me in spite of my best efforts to understand it. Then, there may be no way for me to decide whether the admirers of this art form *really* derive very great and genuine enjoyment from it, or merely *deceive themselves* by claiming that they do.

Maybe in such cases interpersonal comparisons of utility do reach unsurmountable obstacles. But, fortunately, very few of our personal moral decisions and of our public political decisions depend on such exceptionally difficult interpersonal comparisons of utility. (References additional to those listed below will be found in Hammond 1977 and in Suppes and Winet 1955).

See Also

- [Interdependent Preferences](#)
- [Interpersonal Utility Comparisons \(New Developments\)](#)

- [Pigou, Arthur Cecil](#)
- [Value Judgements](#)
- [Welfare Economics](#)

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