Optimized Democracy

Spring 2021, Lecture 9, 2021-02-24

Participatory Budgeting

Dominik Peters, Harvard University

Outline

- Aside on Single Transferable Vote
- More on approval-based committee elections: Phragmén's rule
- Participatory Budgeting

- *Repeating theme*: Can get proportionality by explicitly dividing "voting power" equally among voters. (Rather than magically proportional PAV.)

Single Transferable Vote

6	4	5	2
a	*b*	*c*	*b*
b	*c*	*a*	*a*
c	*a*	*b*	*c*

6	4	5	2
a	*b*	*c*	*a*
b	*c*	*a*	*b*
c	*a*	*b*	*c*

6	4	5	2
a	*b*		*b*
b		*a*	*a*
	a	*b*	

6	4	5	2
a		*c*	*a*
	c	*a*	
c	*a*		*c*

6	4	5	2
a			
		a	*a*
	a		

6	4	5	2
		c	
	c		
c			*c*

Single Transferable Vote for Committees

- STV can also be used to elect a k-committee.
- Initially, each voter gets a ‘budget’ of \$1.
- It costs $\$n/k$ to elect a candidate.
- As long as there is a candidate that is ranked first by voters who together have at least $\$n/k$, elect the candidate and charge those voters $\$n/k$.
- Otherwise, eliminate the candidate whose supporters are poorest, and repeat.
- Exercise: Show STV elects k candidates.

Proportionality for Solid Coalitions (PSC)

- Suppose a set $S \subseteq N$ with $|S| \geqslant \ell \frac{n}{k}$ has the same set $T \subseteq C$ of $|T| = t \geqslant \ell$ candidates they all rank top, so $T \succ C \setminus T$ for all $i \in S$ (not necessarily ranked in the same order).
- Then $|W \cap T| \geqslant \ell$.

- STV satisfies this! (no matter how spending is distributed)

STV satisfies PSC

- Let $S \subseteq N$ with $|S| \geqslant \ell \frac{n}{k}$ agree on $|T| = t \geqslant \ell$ candidates.
- Suppose PSC failed for S. Then there is a time when
  - $\ell - j$ candidates from T have been elected
  - j further candidates from T need to be elected for PSC
  - all but j candidates from T have been elected or eliminated.
- Group S has only paid at most $\$(\ell - j)\frac{n}{k}$ thus far, so has $\$j\frac{n}{k}$ left over. So at least one of the j candidates has $\$\frac{n}{k}$ support, and this will remain true until all j candidates have been elected.

Hare vs Droop Quota

- The value "$\frac{n}{k}$" is known as the Hare quota.
  - Intuition: electorate is split into equal-sized groups, each of which is assigned one seat.
- But we can also use $\frac{n}{k+1} + \varepsilon$, the Droop quota.
  - This works because there are at most k disjoint subsets of N of size $\frac{n}{k+1} + \varepsilon$.
  - Guarantees representation to smaller groups.
  - For $k = 1$, this says majority needs to be followed.
- Everything we've said works for Droop quota if we are more careful in the proofs.
  - PAV satisfies Droop EJR, Droop-STV satisfies Droop PSC

Open Problem

Does there exist a ranking-based committee rule that is monotonic and satisfies PSC?

Recap: Approval-based Committee Elections

- Proportional Approval Voting maximizes $\sum_i 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{|W \cap A_i|}$.
- PAV satisfies Extended Justified Representation: If $S \subseteq N$ with $|S| \geqslant \ell \frac{n}{k}$ agrees on ℓ candidates $T \subseteq \bigcap_{i \in S} A_i$, then $|W \cap A_i| \geqslant \ell$ for some $i \in S$.
- PAV is NP-complete to compute.
- Sequential PAV fails EJR even for $\ell = 1$.
- *Question:* Can we get something proportional in polynomial time?

Is PAV always right?

$k = 12$

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<tr><td>4</td><td>5</td><td>6</td><td>10</td><td>14</td><td>18</td></tr>
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EJR not strong enough to capture this!

Phragmén's Rule

EDVARD PHRAGMÉN

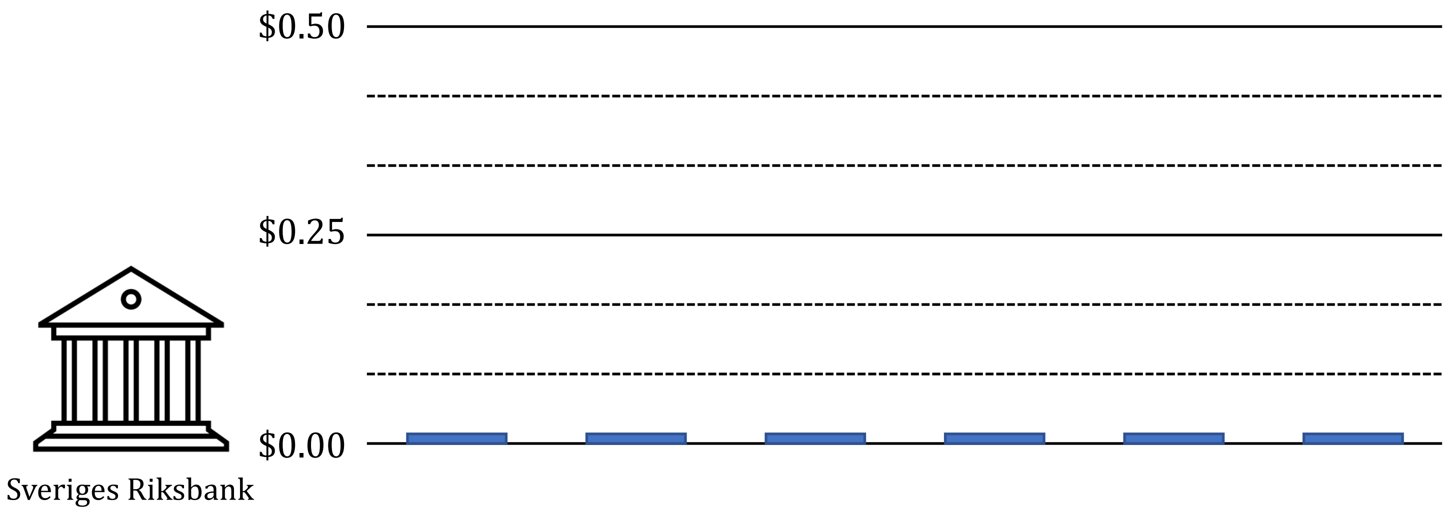
- Proposed in 1894. Thiele proposed PAV in 1895. Phragmén criticized it in 1899, for a reason similar to

4	5	6	10	14	18
	3		9	13	17
	2		8	12	16
	1		7	11	15

- Phragmén's proposal:
  - Each voter starts with a bank account with \$0.
  - Fill bank accounts at the same rate, until the approvers of some unelected candidate together hold $\$\frac{n}{k}$.
  - Elect the candidate and reset approvers' accounts to \$0.
  - Stop after k candidates are elected.

$k = 12$

Phragmén's Rule: Example



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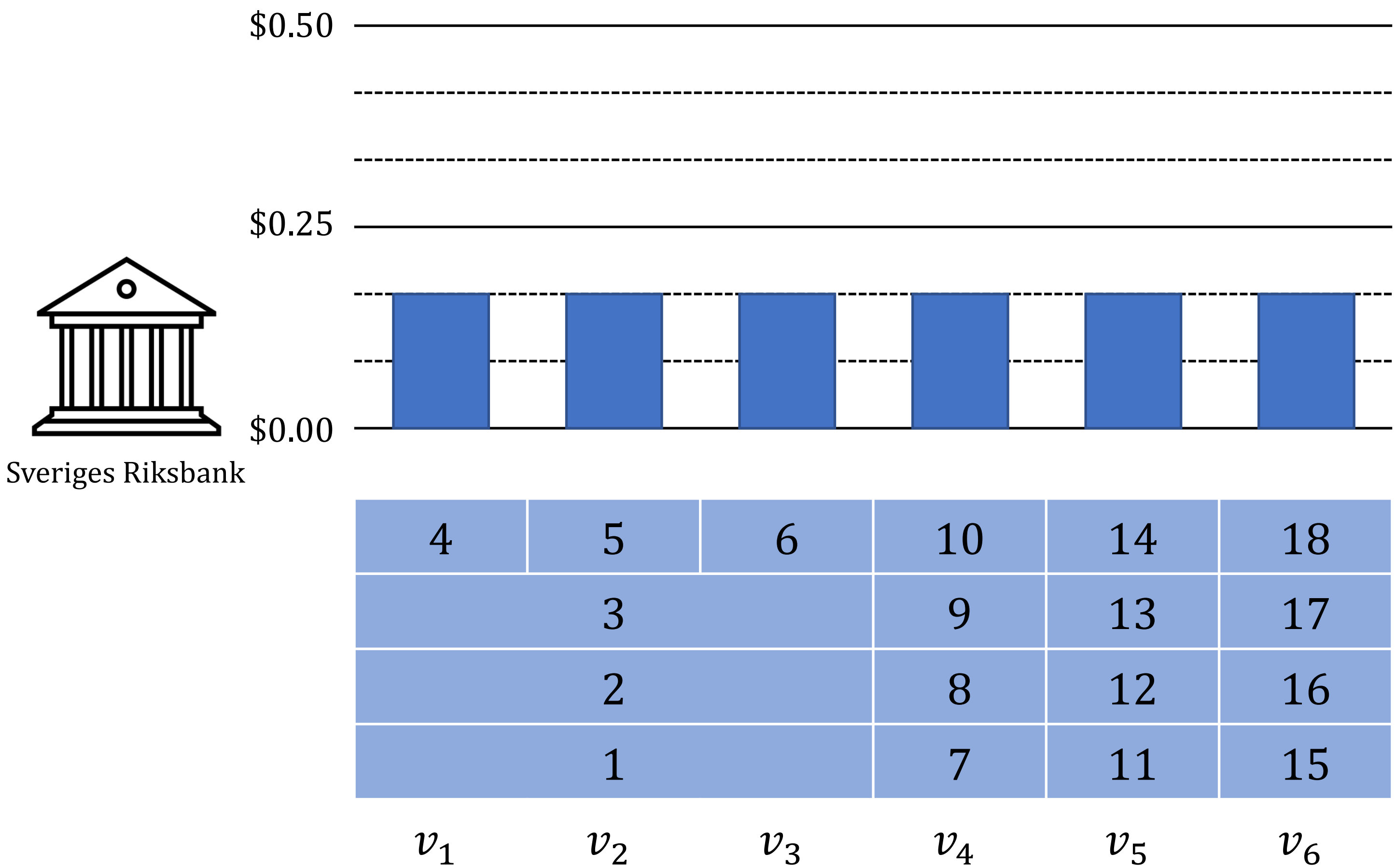

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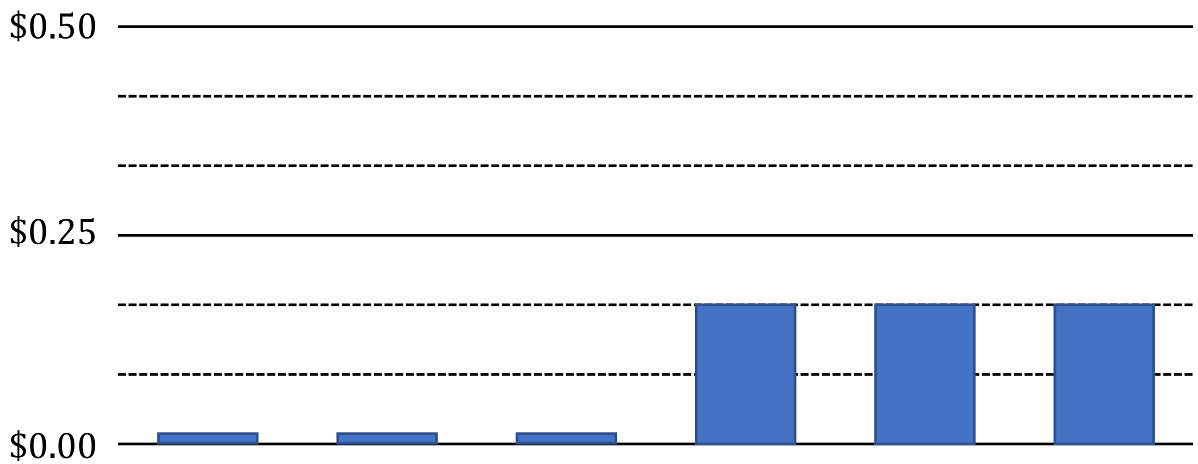

Sveriges Riksbank

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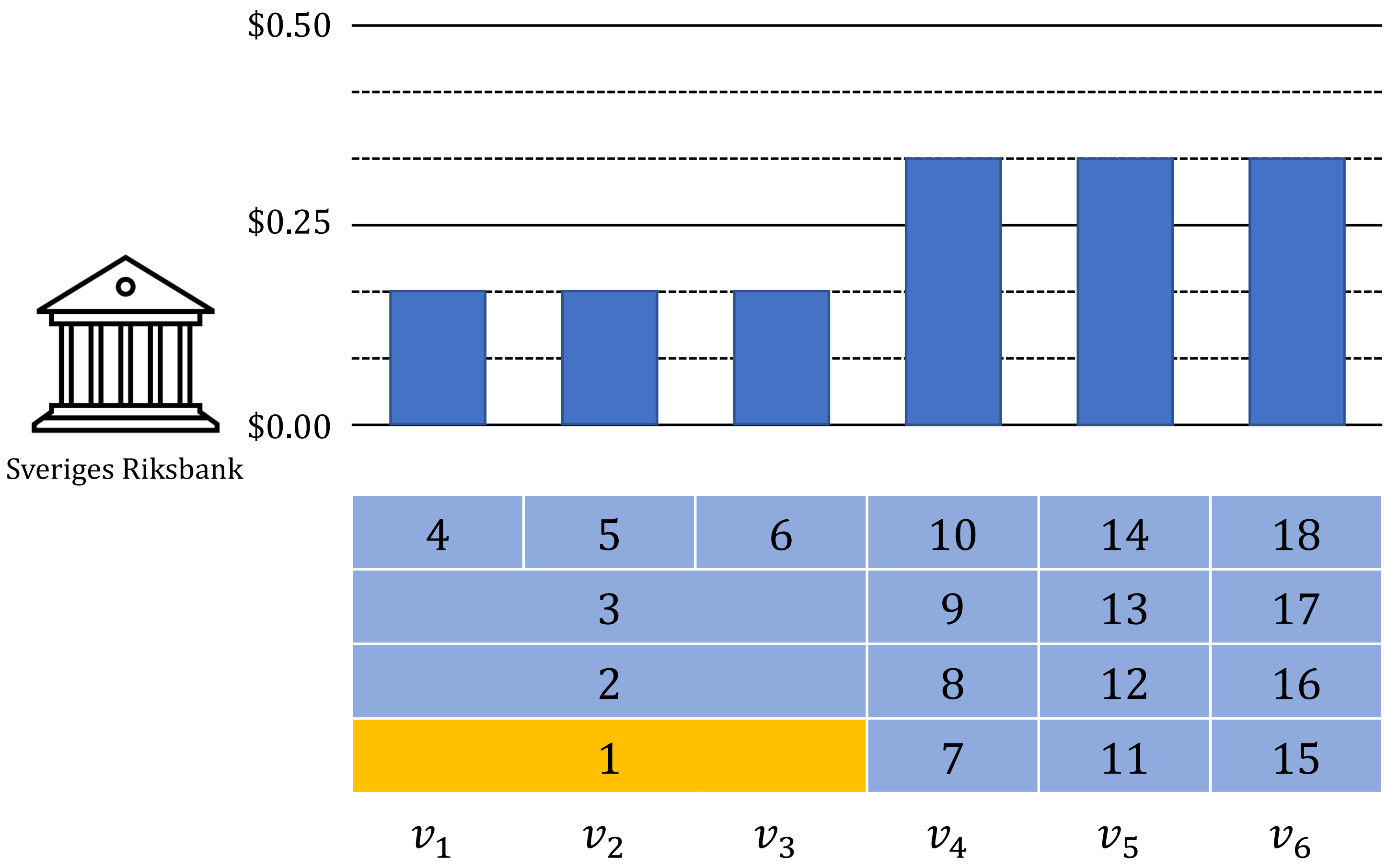

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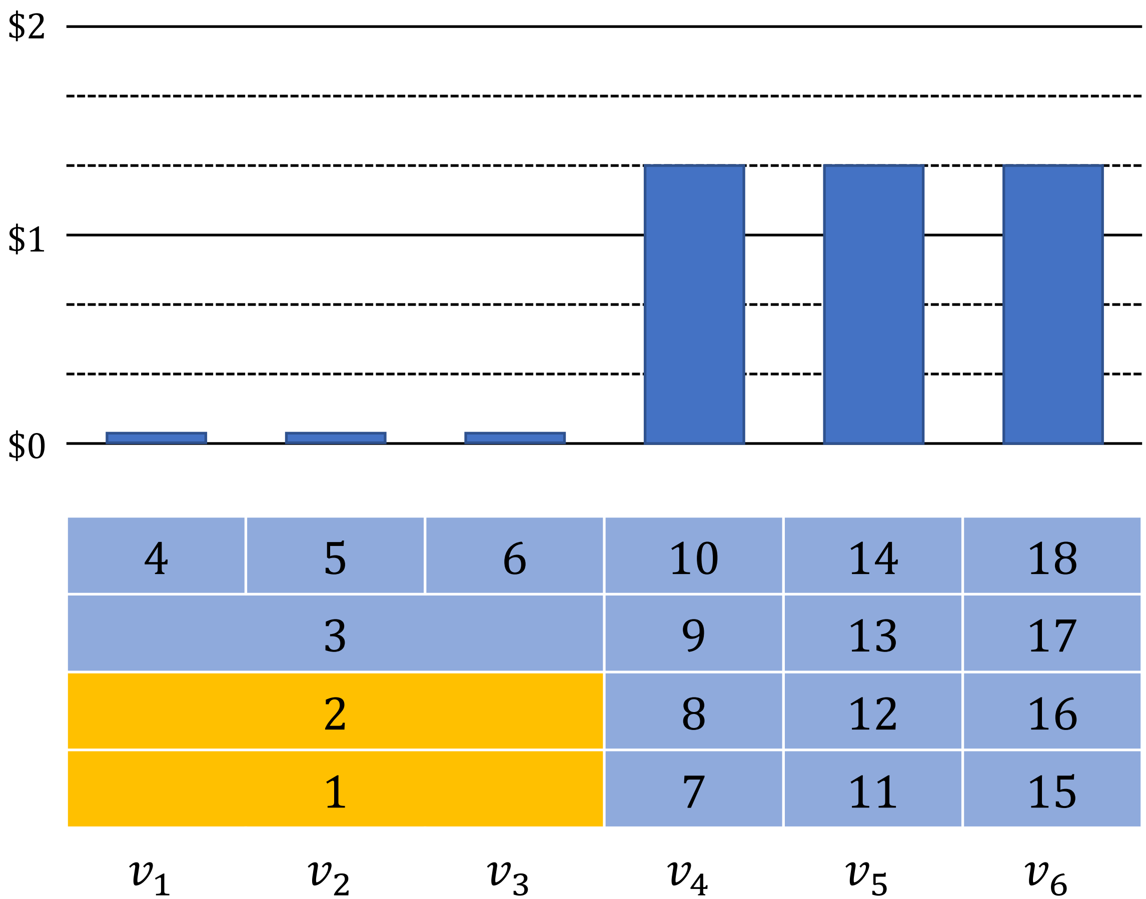

Phragmén’s Rule: Example

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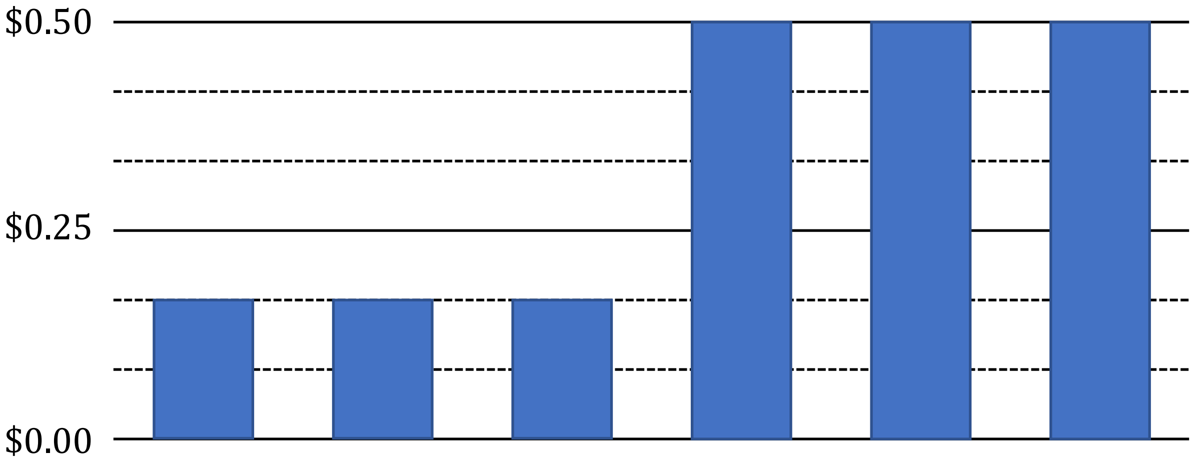

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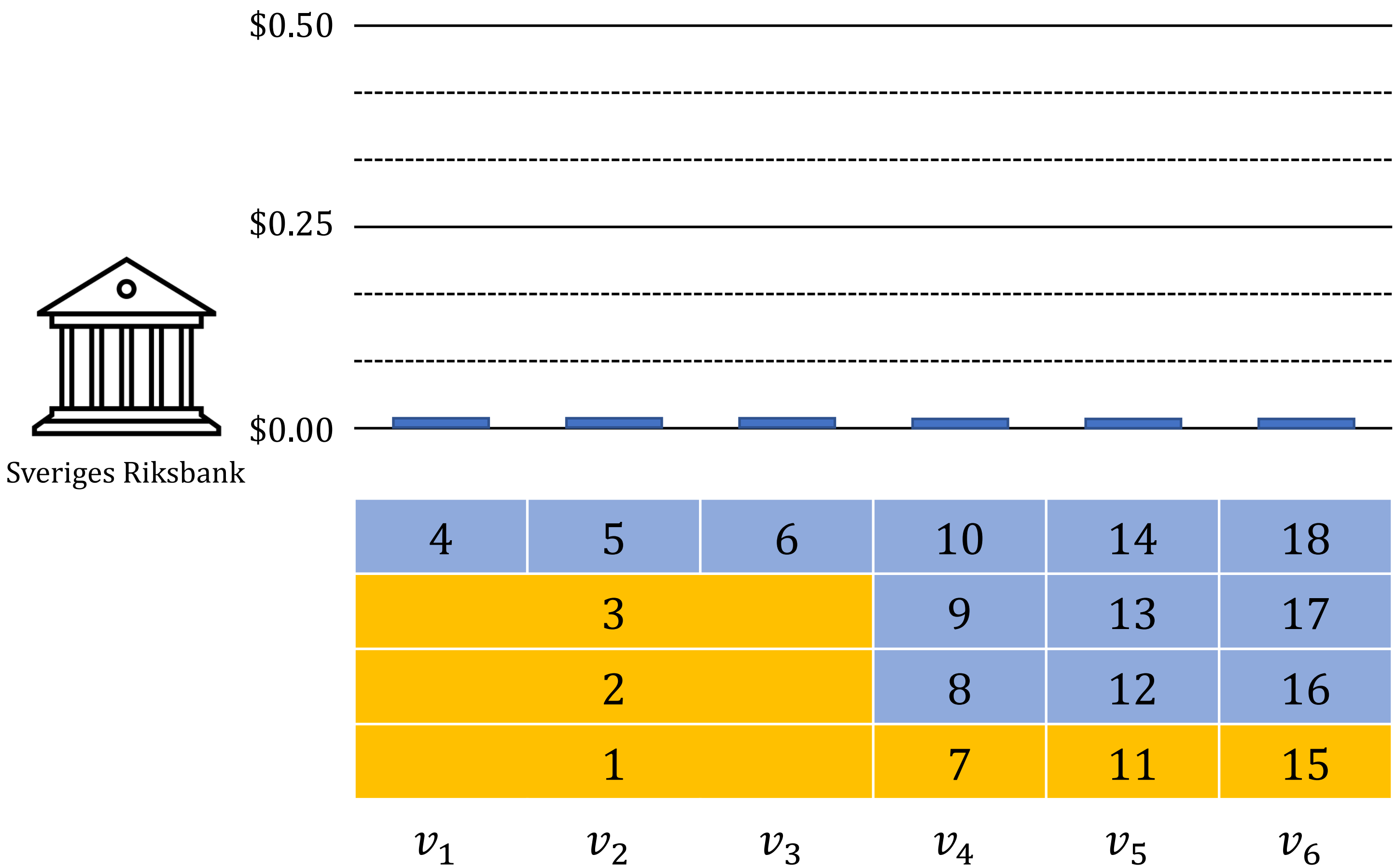

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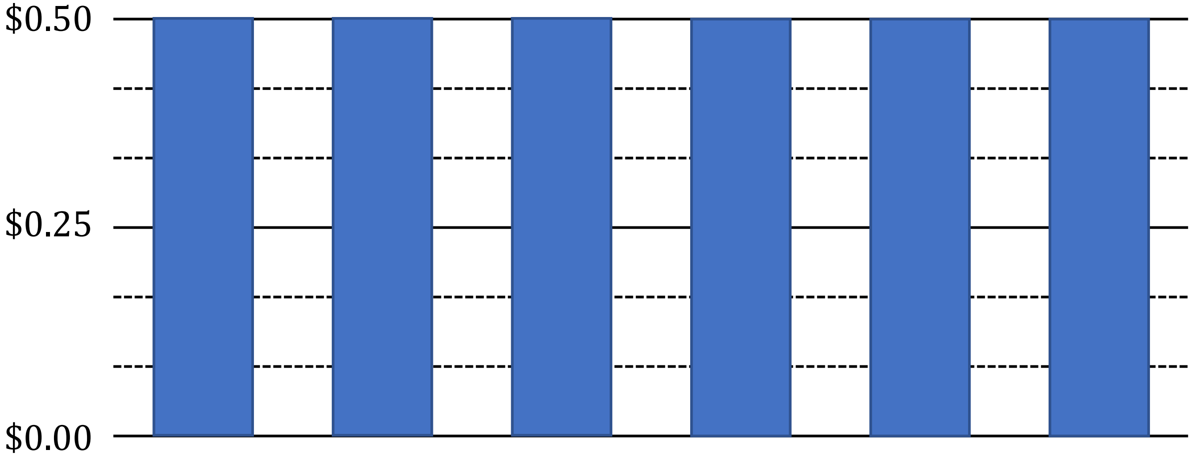

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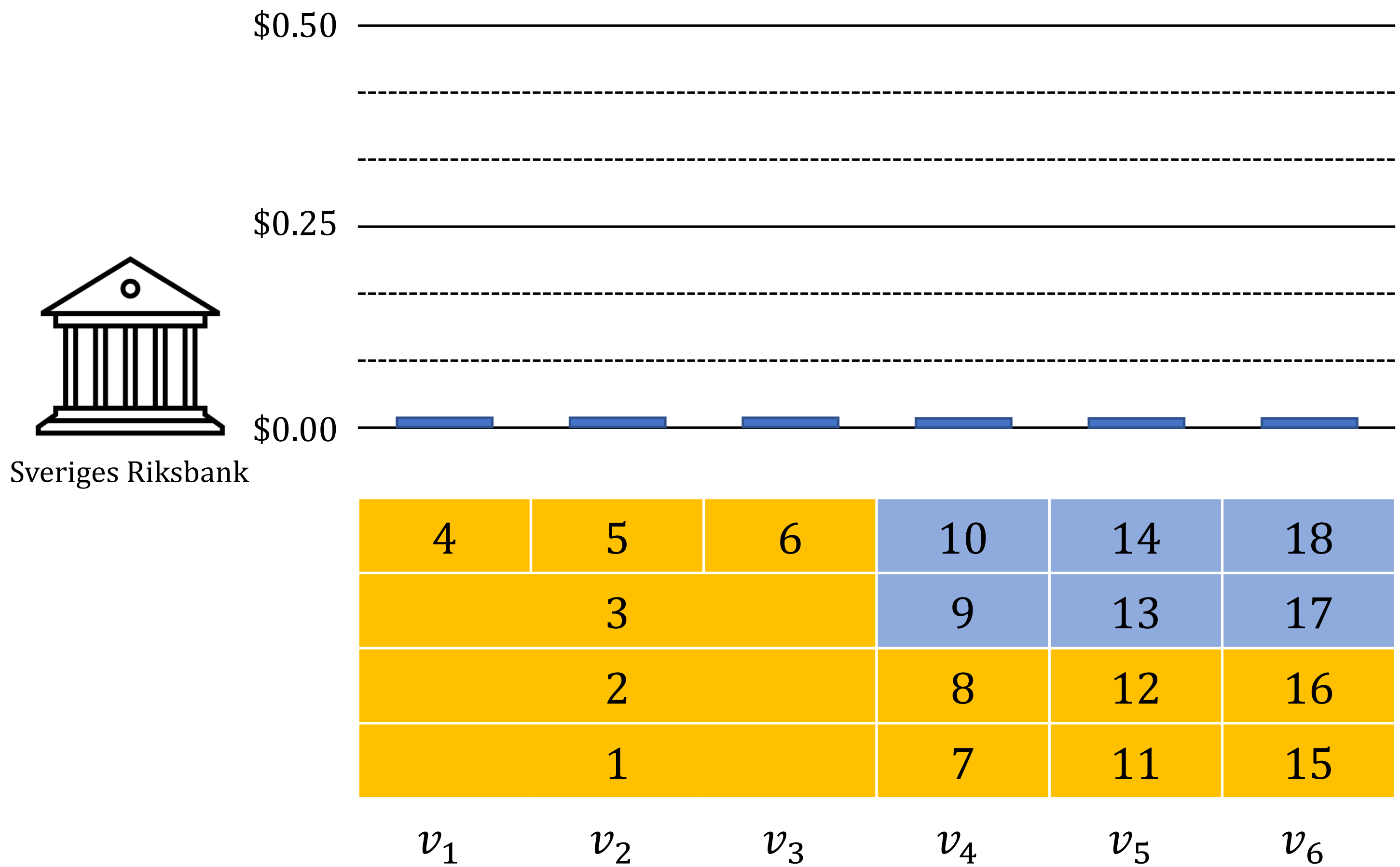
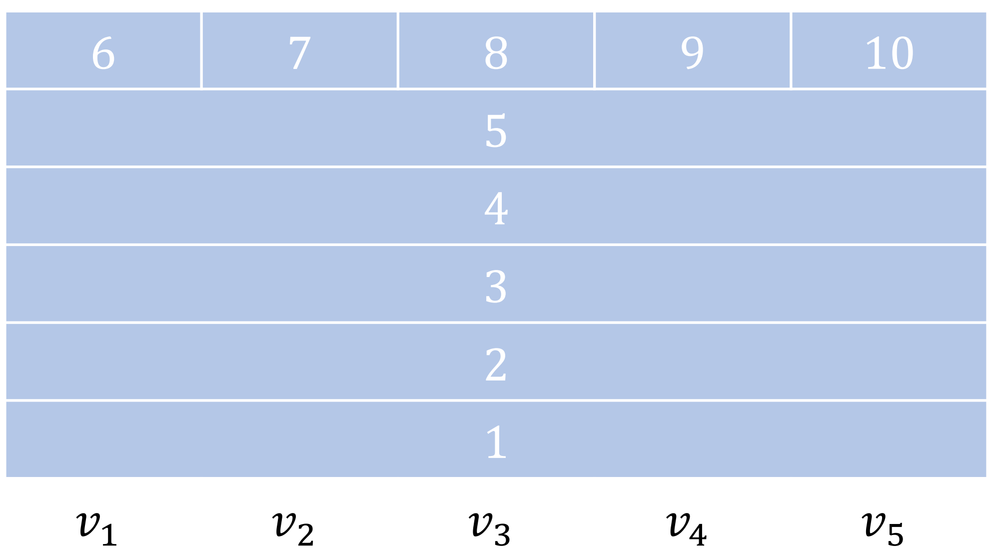


Phragmén’s Rule: Proportionality

- Phragmén’s rule violates EJR (largish example with 24 voters, 14 candidates, $k = 12$).
- But it satisfies a weaker version (“PJR”):
  If $S \subseteq N$ with $|S| \geqslant \ell \frac{n}{k}$ agrees on ℓ candidates $T \subseteq \bigcap_{i \in S} A_i$, then $|W \cap \bigcup_{i \in S} A_i| \geqslant \ell$.

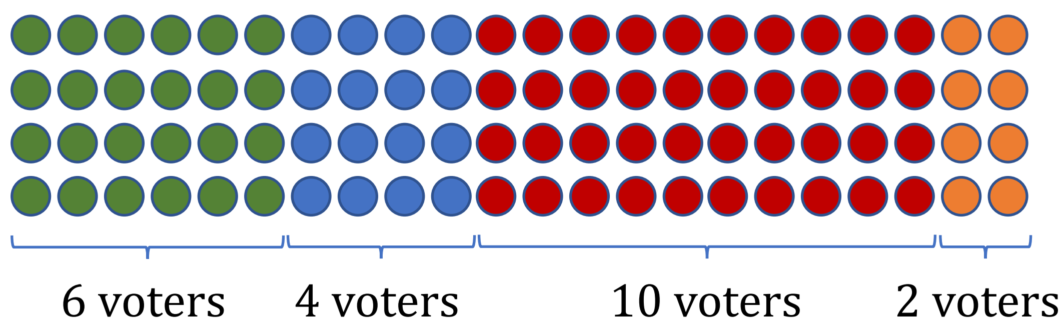


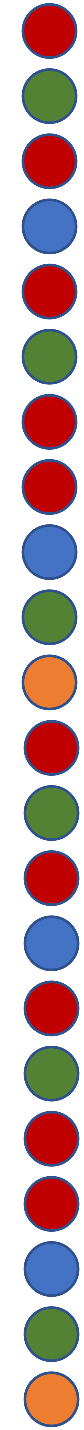
Phragmén's Rule: PJR

- If $S \subseteq N$ with $|S| \geqslant \ell \frac{n}{k}$ agrees on ℓ candidates $T \subseteq \bigcap_{i \in S} A_i$, then $|W \cap \bigcup_{i \in S} A_i| \geqslant \ell$.
- *Proof*: By the time Phragmén terminates, each voter has received at least \$1.
- If it terminates exactly at the \$1-point, then all money was spent. So S spent $\$\ell\frac{n}{k}$, and so they bought ℓ candidates from $\bigcup_{i \in S} A_i$.
- If it terminates strictly later, consider \$1-point. If then $|W \cap \bigcup_{i \in S} A_i| \leqslant \ell - 1$, then S now holds at least $\$\frac{n}{k}$, so can purchase a candidate from T.

Proportional Rankings

- Note: you don't have to stop Phragmén after it has elected *k* candidates (same for SeqPAV)
- This way, we get a *proportional ranking*.
- In particular, every prefix satisfies PJR. (Or think of party-list profiles.)
- Applications:
  - Ranking comments by upvotes
  - Displaying proposal variants in LiquidFeedback
- Open Problem: Do there exist EJR rankings?






PARIS
BUDGET PARTICIPATIF
CHOISISSEZ VOS PROJETS PRÉFÉRÉS
DU 6 AU 22 SEPTEMBRE 2019
budgetparticipatif.paris.fr
ÉTOILE

Youth Lead the Change: PB Boston

2015
$1,000,000

You have selected **1 / 4** projects.

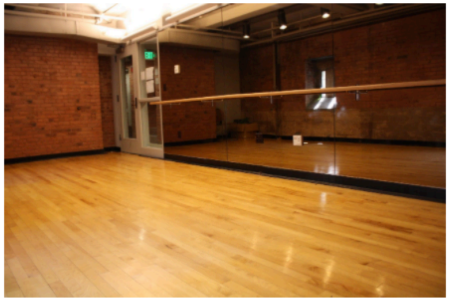
BCYF Hyde Park Dance Studio Renovation

A renovated dance studio at the Hyde Park Community Center for children of all ages.

Estimated Cost: $286,000

Location: BCYF Hyde Park Community Center, Hyde Park

Select



Click image for slide show

Bike Lane Installation

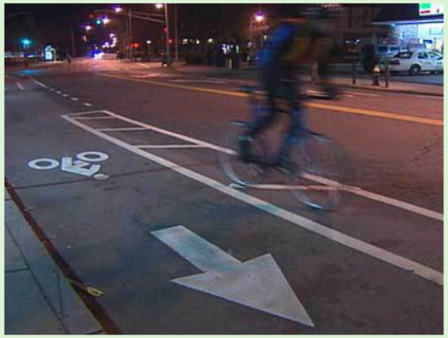
After a study, bike lanes will begin to be installed around Charlestown Navy Yard, Bunker Hill housing, and Charlestown High.

Estimated Cost: $200,000

Location: Charlestown

✔ Selected

Remove



Click image for slide

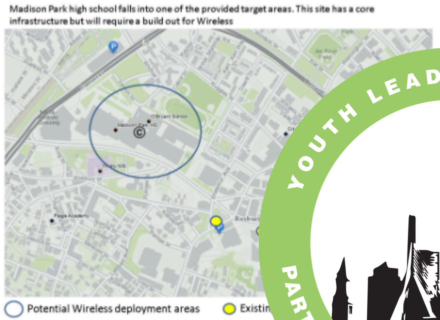
Wicked Free Wifi 2.0

Wicked free Wi-Fi 2.0 provides Wi-fi at locations with young people.

Estimated Cost: $119,000

Location: Various High Schools and Community Centers, Dorchester, Roxbury, East Boston, Charlestown

Select



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YOUTH LEAD THE CHANGE
PARTICIPATORY BUDGETING BOSTON

New York City District 8 Selected $650,000 of $2,000,000 total budget. You still have $1,350,000 left. Exit Help **Español** 中文

Education

Bathroom Renovations at M.S./H.S. 223

Renovation of girls' and boys' bathrooms including stalls, lighting, painting, and having walls re-glazed.

Estimated Cost: $150,000

Location: 360 E.145th St. (Bronx - Mott Haven)

✔ Selected Remove

Technology Upgrades

Technology upgrades for Park East High School and Central Park East High School.

Estimated Cost: $312,000

Location: 230 E.105th; 1573 Madison Ave. (El Barrio/East Harlem)

Select

Air Conditioning at Bronx Schools

Installation of 1 air conditioning system at 345 Brooke Avenue for schools X343, X224 & X334. Installation of 1 air conditioning system at PS 161x.

Estimated Cost: $500,000

Location: 628 Tinton Ave, 345 Brook Ave. (Bronx - Mott Haven & Longwood)

Air Conditioning: P.S.179, P.S.369, P352

Installation of two HVAC units at P.S. 179X, P.S. 369X and P.S. 352X.

Estimated Cost: $500,000

Location: 468 E. 140 St. (Bronx - Mott Haven)

✔ Selected Remove

4e arrondissement

Greedy: total utility **3 500**. Funds 5 projects, avg cost 293 000

Optimal: total utility **6 878**. Funds 14 projects, avg cost 98 928

Project Name	QPOP	Cost €	Votes	v / k€	Greedy	Optimal
Un mur végétalisé au croisement des rues Blancs Manteaux et Archiv		30 000	788	26	✓	✓
Un café solidaire dans le quartier de la tour Saint-Jacques		15 000	706	47	✓	✓
Une salle d'arts plastiques pour l'école Saint-Merri Renard		300 000	702	2	✓	✓
Rénovation énergétique exemplaire d'une école du 4e		1 000 000	655	1	✓	–
Végétalisation de la rue de l'Arsenal		120 000	649	5	✓	✓
Un collège Charlemagne accessible aux personnes à mobilité réduit		200 000	630	3	–	✓
Faire du hall d'accueil de la piscine Saint-Merri un lieu de convivia		20 000	528	26	–	✓
Des agrès sportifs place des Vosges		15 000	491	33	–	✓
Mise en valeur des pierres de la prison de la Bastille		20 000	473	24	–	✓
Un fauteuil roulant électrique pour rompre l'isolement		5 000	453	91	–	✓
Création d'un auvent sur une aire de jeux d'un square du 4e		150 000	410	3	–	✓
L'Ascenseur, un tiers-lieu pour l'égalité des chances ouvert sur le		350 000	315	1	–	✓
Baliser le passage du chemin de Compostelle dans le 4e		30 000	265	9	–	✓
Des arceaux vélos rue de la Reynie		10 000	240	24	–	✓
Faciliter l'accès au cellier d'Ourscamp		120 000	228	2	–	✓
				Total €	1 465 000	1 385 000

Participatory Budgeting: Model

- A set C of projects, each with a cost
- A budget limit B
- Outcome: set $W \subseteq C$ with $\sum_{c \in W} \text{cost}(c) \leq B$.
- A set N of n voters
- Each voter $i \in N$ approves a subset $A_i \subseteq C$.
- Mostly, we say that i's utility is $u_i(W) = |A_i \cap W|$ (this is a dichotomous preference assumption).
- Unit cost assumption: $\text{cost}(c) = 1$ for all c.

Three interpretations of "AV"

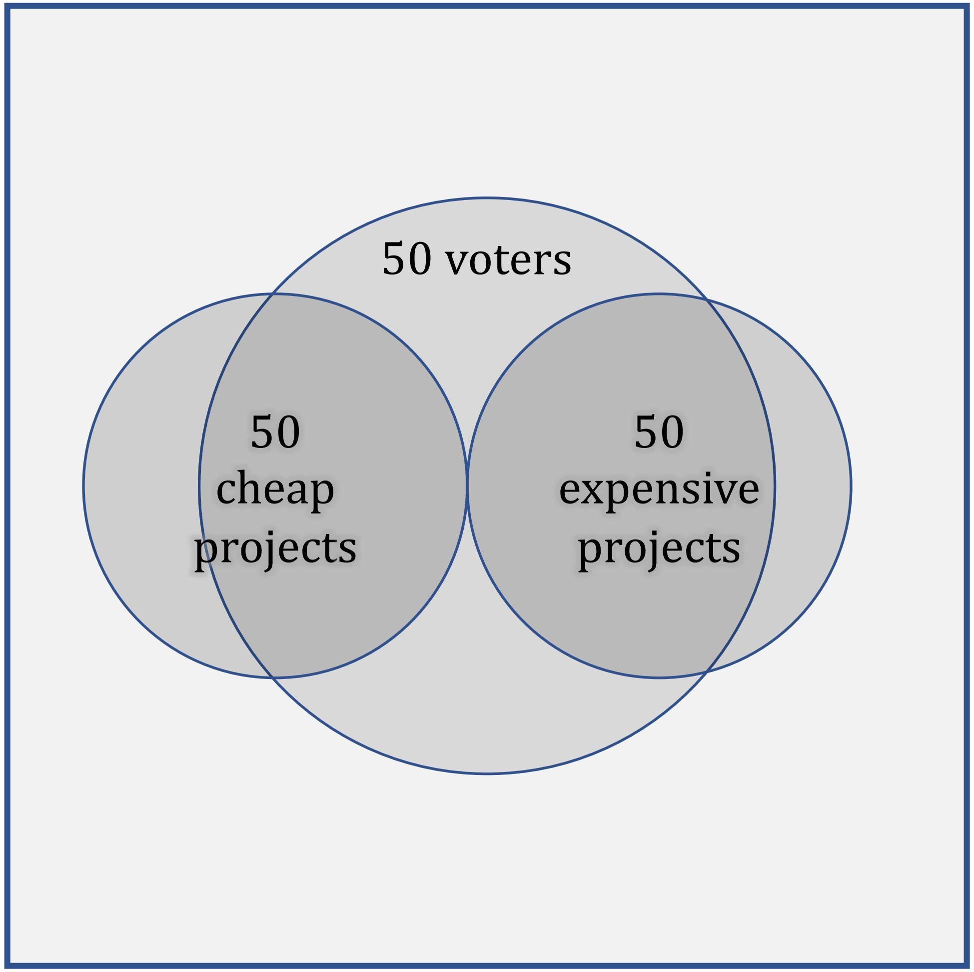
- Optimize $\sum_{i \in N} u_i(W) = \sum_{c \in W} \text{approval-score}(c)$.

- Greedy: add projects in order of approval score, skipping unaffordable projects.

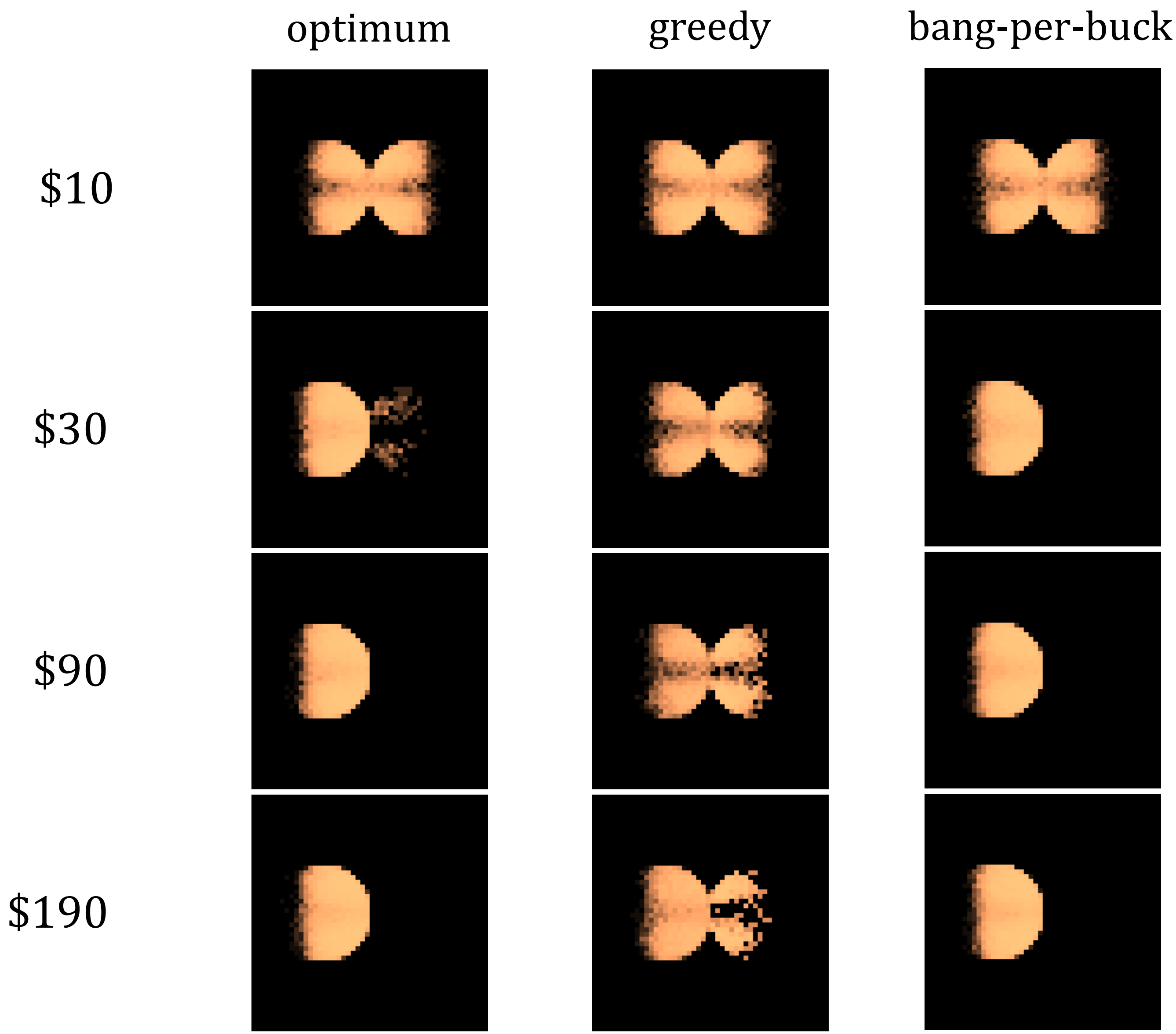
- Bang-per-buck greedy: add projects in order of approval score divided by cost.

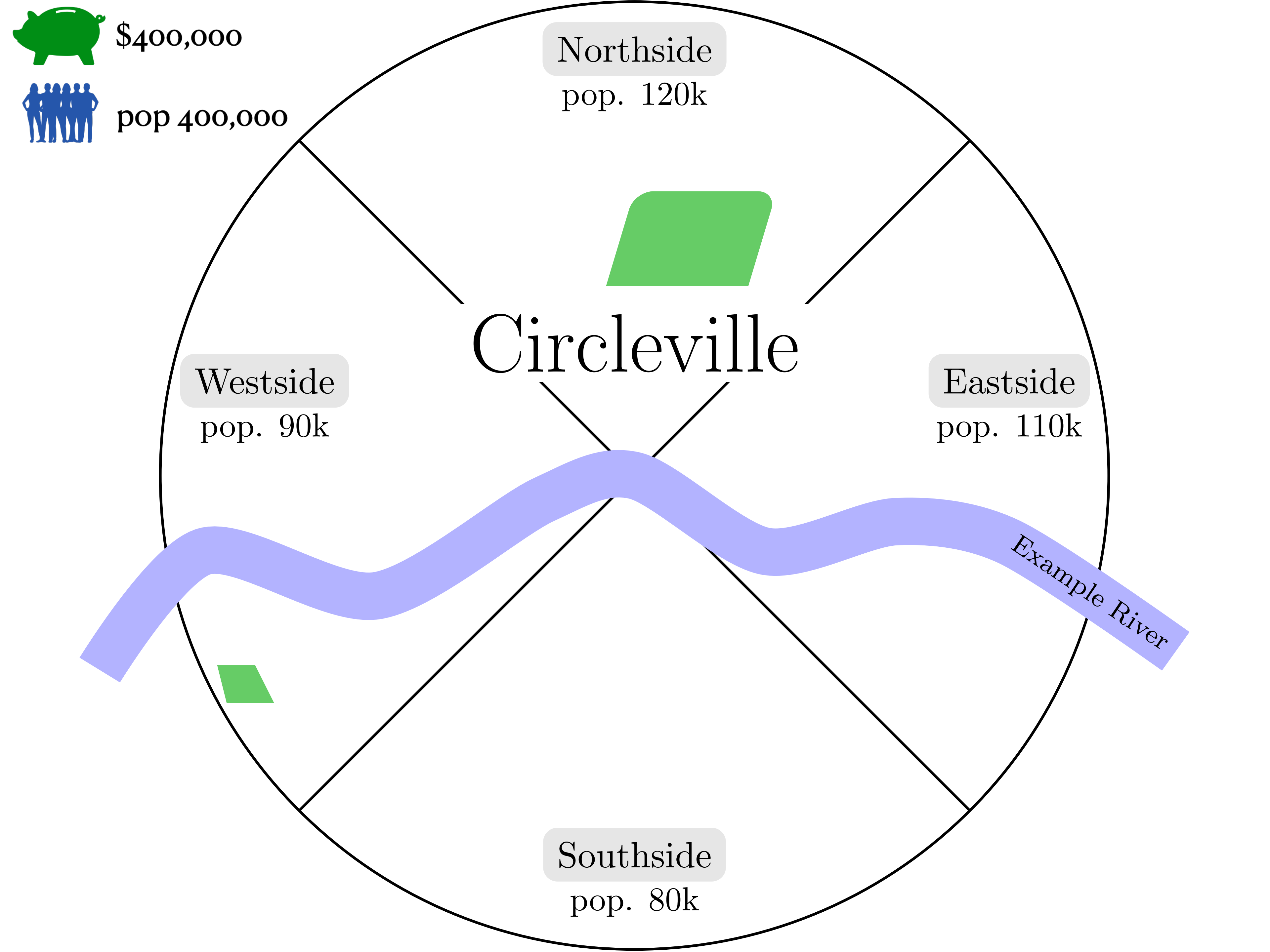
Experiments

Budget = \$1000. Cheap = \$10. Expensive = \$10, \$30, \$90, \$190.

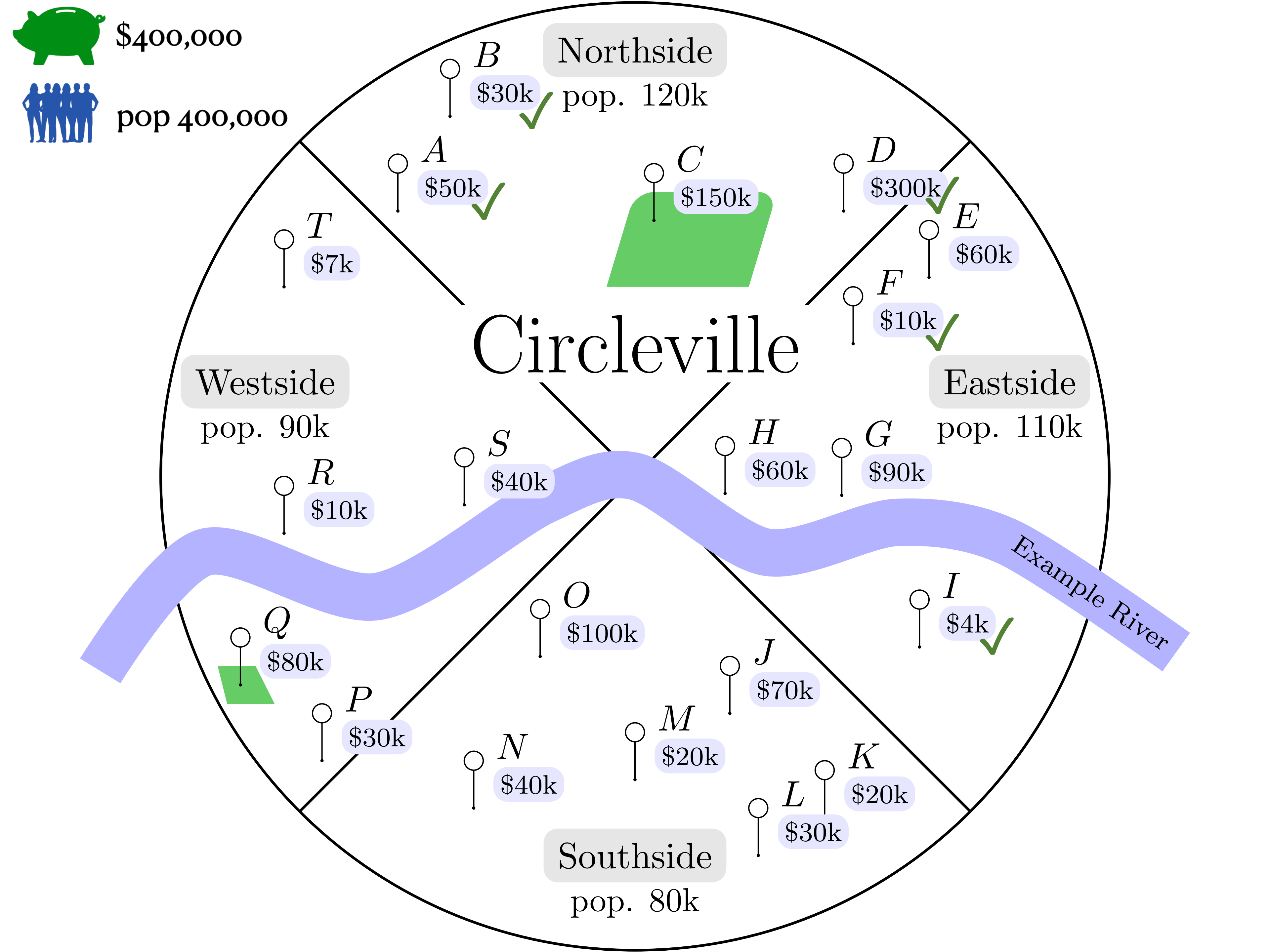


Experiments





$400,000
pop 400,000
Northside
pop. 120k
Circleville
Westside
pop. 90k
Eastside
pop. 110k
Example River
Southside
pop. 80k


$400,000
pop 400,000
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pop. 90k
Eastside
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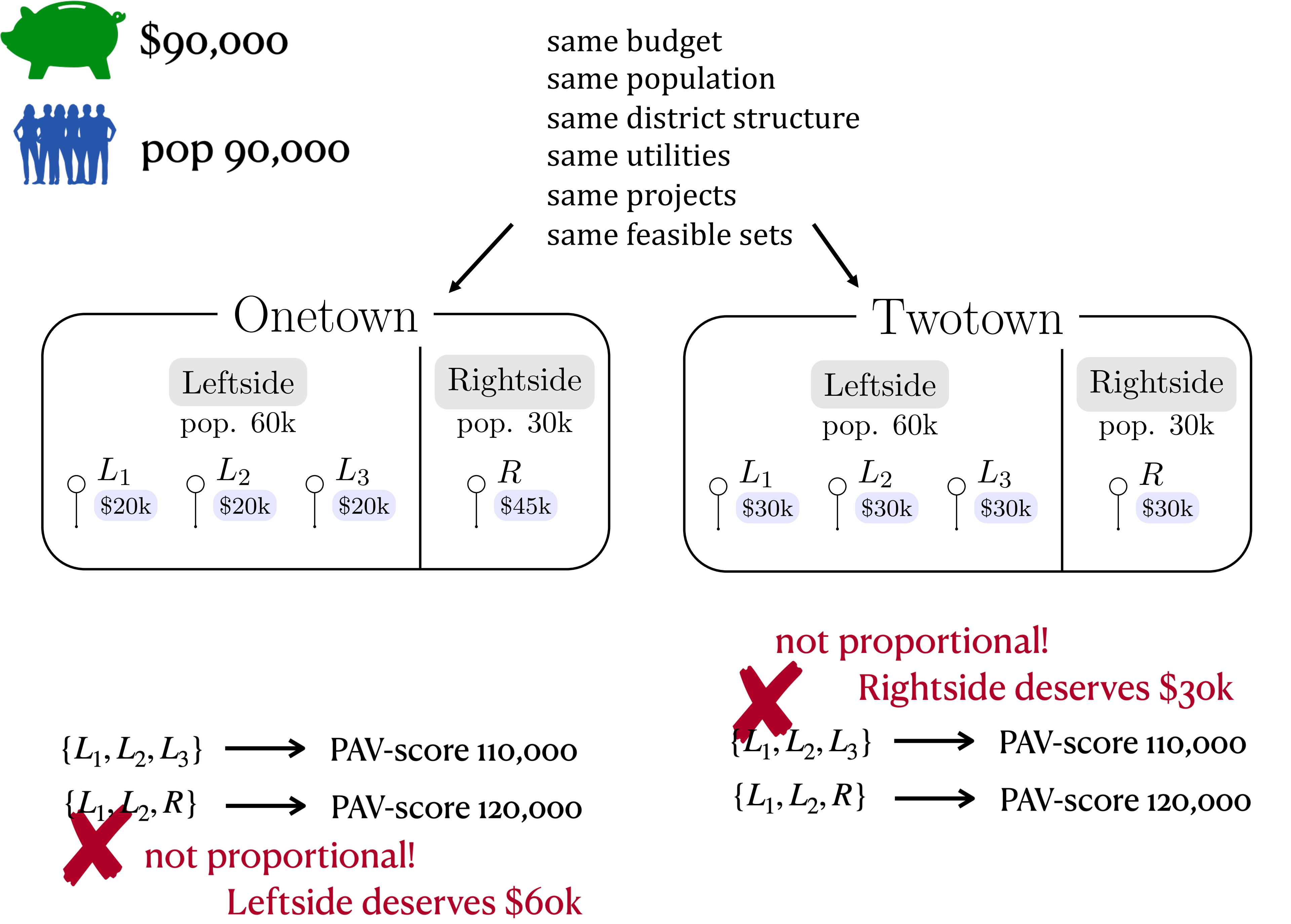
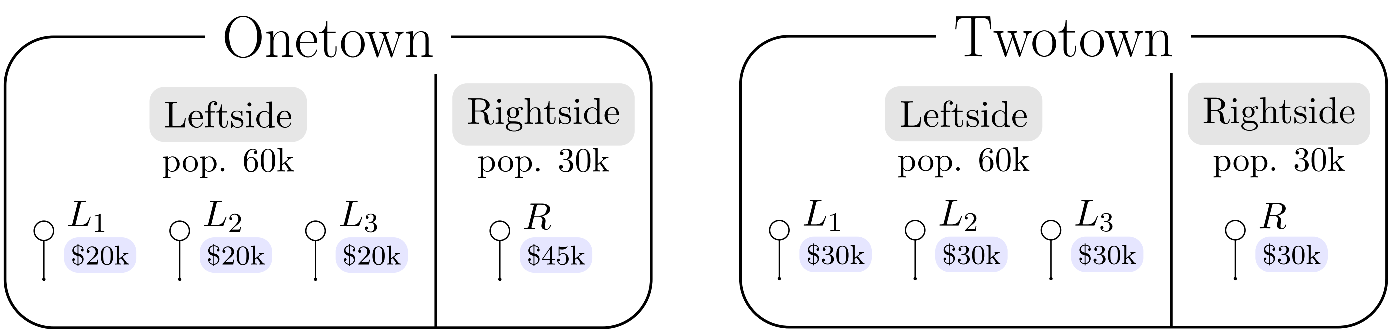
Southside
pop. 80k
Example River
B $30k
A $50k
C $150k
D $300k
E $60k
F $10k
T $7k
S $40k
R $10k
H $60k
G $90k
I $4k
O $100k
Q $80k
J $70k
P $30k
M $20k
N $40k
K $20k
L $30k



$90,000
pop 90,000
same budget
same population
same district structure
same utilities
same projects
same feasible sets
Onetown
Leftside
pop. 60k
L_1 $20k
L_2 $20k
L_3 $20k
Rightside
pop. 30k
R $45k
Twotown
Leftside
pop. 60k
L_1 $30k
L_2 $30k
L_3 $30k
Rightside
pop. 30k
R $30k
$\{L_1, L_2, L_3\} \longrightarrow$ PAV-score 110,000
$\{L_1, L_2, R\} \longrightarrow$ PAV-score 120,000
not proportional!
Leftside deserves $60k
not proportional!
Rightside deserves $30k
$\{L_1, L_2, L_3\} \longrightarrow$ PAV-score 110,000
$\{L_1, L_2, R\} \longrightarrow$ PAV-score 120,000

Phragmén for PB

- Phragmén's rule can easily be adapted:
  - Fill bank accounts
  - If the approvers of a project have enough money to finance its cost, implement the project
  - Stop when next project doesn't fit into the budget.
- Picks correct outcome in Onetown and Twotown.



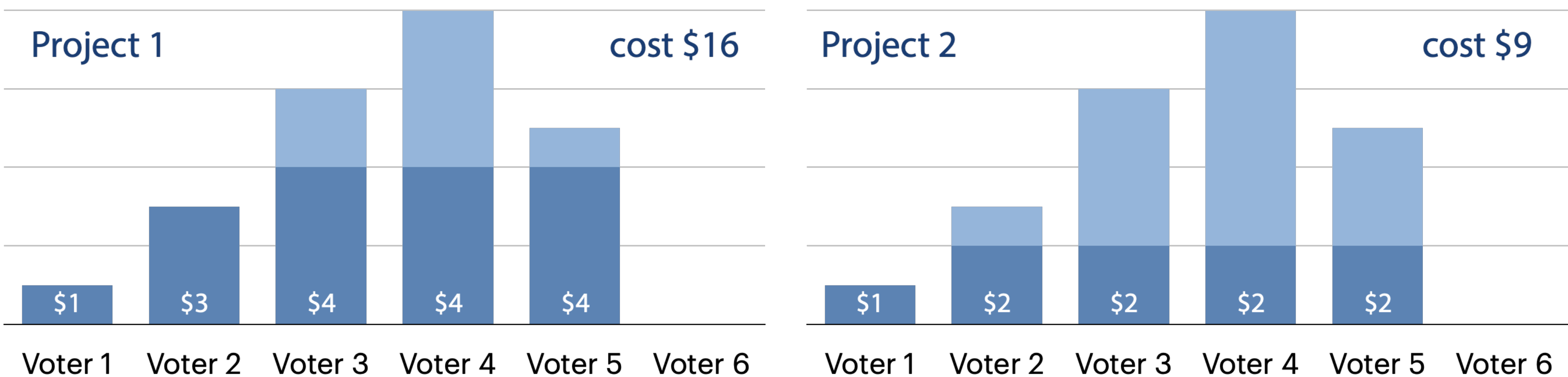
- Satisfies "PJR": If $\frac{|S|}{n} \geqslant \alpha$ and $\text{cost}(\bigcap_{i \in S} A_i) \geqslant \alpha \cdot B$, then $\text{cost}\big((W \cap \bigcup_{i \in S} A_i) \cup \{c\}\big) \geqslant \alpha \cdot B$ for some $c \in \bigcap_{i \in S} A_i$.

Rule X for PB

- Split the city budget evenly among residents.
- Put each resident's share $\frac{B}{n}$ in a bank account.
- Repeatedly, until the budget runs out:
    - identify a project whose supporters have enough money left to afford it
    - charge the cost to supporters

Rule X for PB

- Split the city budget evenly among residents.
- Put each resident's share $\frac{B}{n}$ in a bank account.
- Repeatedly, until the budget runs out:
  - always divide the cost of a project among supporters as evenly as possible
  - find an affordable project with the lowest max payment.



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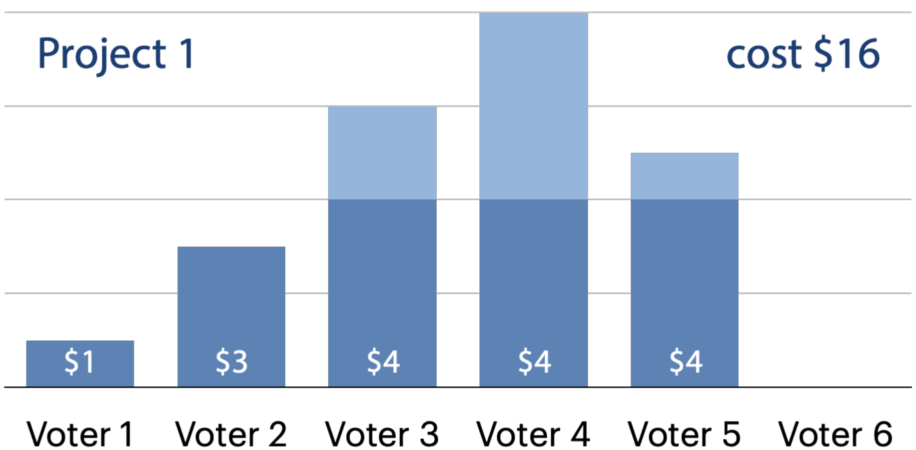

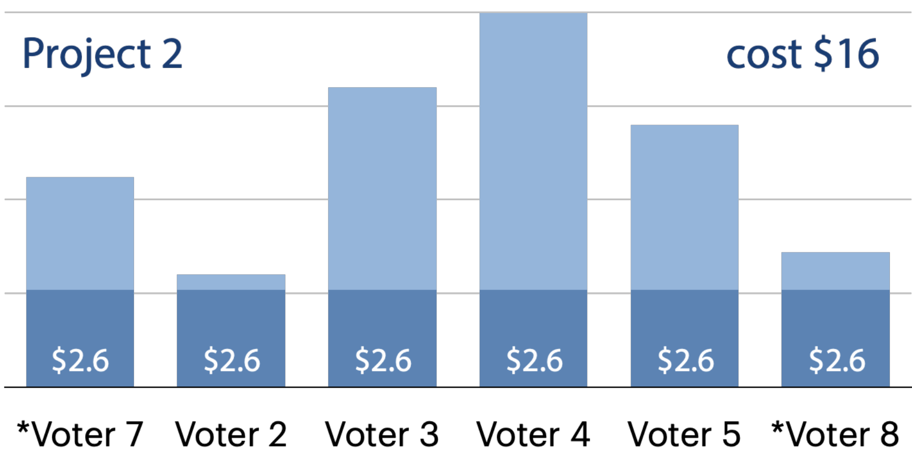

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- Repeatedly, until the budget runs out:
  - always divide the cost of a project among supporters as evenly as possible
  - find an affordable project with the lowest max payment.
- Rule X satisfies EJR!
  - Let $\frac{|S|}{n} \geqslant \alpha$, and take $T \subseteq \bigcap_{i \in S} A_i$ with $\text{cost}(T) \leqslant \alpha \cdot B$.
  - Then $u_i(W) \geq u_i(T)$ for some $i \in N$ (i.e. $|W \cap A_i| \geqslant |T|$.)

Additive Valuations

Motivating Example:
2019, Paris, 16th arrondissement
€560k: refurbish sports facility — 775 votes
€3k: materials for classroom project — 670 votes
— 1.15x as popular, 186x the cost!

- Utility of outcome: $u_i(W) = \sum_{c \in W} v_i(c)$.
- Phragmén: no obvious way of extending to additive utilities.
- Rule X: can extend using following idea: a voter's payment for a candidate should be proportional to the voter's utility for the candidate.
- Core may be empty!

Core for Additive Valuations

- A group $S \subseteq N$ with $\frac{|S|}{n} \geq \alpha$ *blocks* W if there is $T \subseteq C$ with $|T| \leq \alpha \cdot B$ such that $u_i(T) > u_i(W)$ for all $i \in S$.

	Voter 1	Voter 2	Voter 3	cost
$u_i(a)$	2	1	0	\$2
$u_i(b)$	0	2	1	\$2
$u_i(c)$	1	0	2	\$2
			Budget B	\$3

- An approximation exists if we put $|T| \leqslant \frac{\alpha}{32} \cdot B$. The factor of 32 might be improvable to 2, but not further.
- Existence open for approval utilities.

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