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Lecture 12: Social Choice

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1 Introduction

Today we're concluding our multi-agent section of the course with a lecture on social choice. Social choice theory is a theoretical framework to analyze the combination of opinions, preferences, interests, or welfares of individual agents to reach a collective decision. We'll cover one of the most prevalent and important applications of social choice theory – voting procedures.

1.1 Voting model

Before we get into individual voting rules, let's cover the general framework behind models in voting theory. The model includes:

Set of agents (voters)	$N = \{1, \ldots, n\}$
Set of alternatives (candidates)	$A = \{a_1, a_2, \ldots, a_m\}$
Agent preferences	σ_i denotes agent i's preferences
Voting rule	Function f outputs winner based on σ

1.2 Ballot types

There are several models in which agents may express their approval / disapproval for alternatives:

1. Rankings: each voter ranks all the alternatives in preferred order
2. Approvals: each voter approves up to k alternatives
3. Scores / stars: each voter rates each alternative

We'll focus on rankings as they are the main ballot type studied in voting theory.

2 Voting rules

As described above, each voting rule gives us a function for how to rank alternatives (candidates) based on agents' preferences.

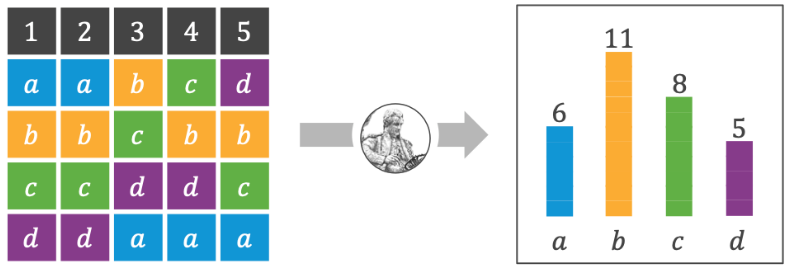
2.1 Plurality

This is a simple voting rule that is commonly used in elections.
Winner
Alternative with the most 1st-preference votes.
Example

In the above example, alternative a would be the winner since it received the most number of votes.
Application
Although simple, this is a problematic voting rule since it only looks at the top alternative for each voter. In the example above, it may be that voters 3, 4, and 5 all place candidate a in last place. France uses a version of this rule called **plurality with runoff**, where plurality is used to filter down to the top two alternatives, and then plurality is used again to a select winner.

2.2 Borda count

History
Proposed by Jean-Charles de Borda, a mid-1700s mathematician who is also credited for instigating the metric system.
Winner
For each agent, $m - k$ points are given to the alternative in the kth position (m is the number of alternatives). The alternative with most points wins.
Example



In the above example, alternative a gains 4-1=3 points from voter 1, 3 points from voter 2, and 4-4=0 points from voters 3,4, and 5 for a total of 6 points. Using the same method, b gains 11 points, c gains 8 points, and d gains 5 points. Note that in contrast to in plurality voting, b is the Borda count winner. This is because Borda voting is more sensitive to the full preferences of each voter.

Application

Slovenian elections use Borda.

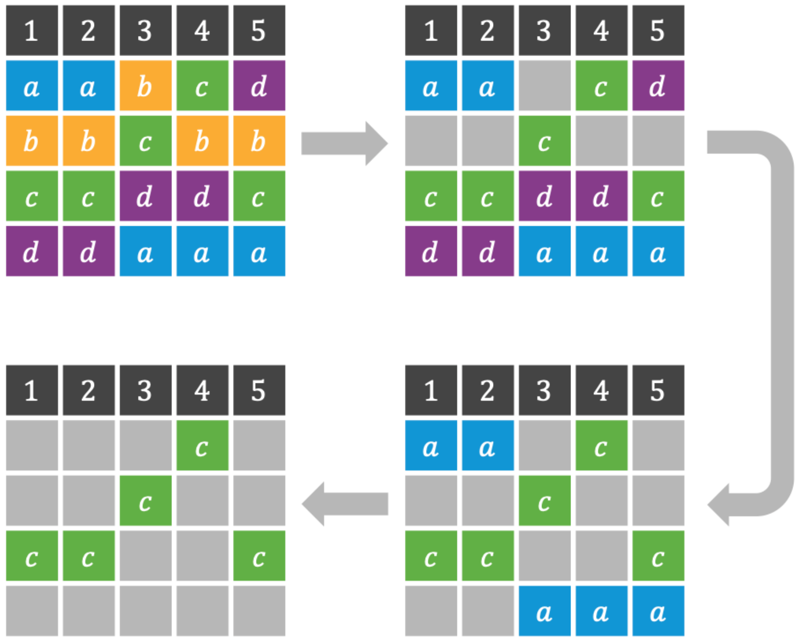
2.3 Single-Transferable Vote

Winner

STV successively eliminates alternatives that are ranked first by the smallest number of voters. Voter preferences are updated so that the second choice can take the place of the first choice for voters who selected that alternate first. Elimination and transferring of votes are repeated for $m-1$ rounds until a single alternative (which is our winner) is left.

Example

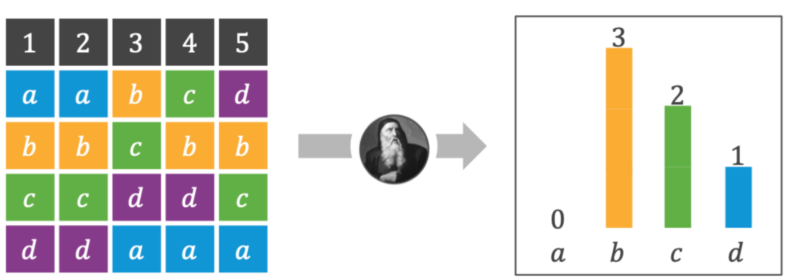
In the example below, alternative b and d receive the fewest first-ranked votes – we randomly pick b to be eliminated in the first round. Voter 3's vote is transferred to alternative c. In the second round, d is eliminated and voter 5's vote is transferred to c. Finally, a is eliminated and c is the STV winner.



Application
Also sometimes referred to as "alternative vote," "instant-runoff voting," and "ranked-choice voting." STV is used for elections in Ireland, parliamentary elections in Australia, and some statewide and city elections in the U.S. (one of the first cities it was used in was Cambridge, MA)!

2.4 Llull's Rule

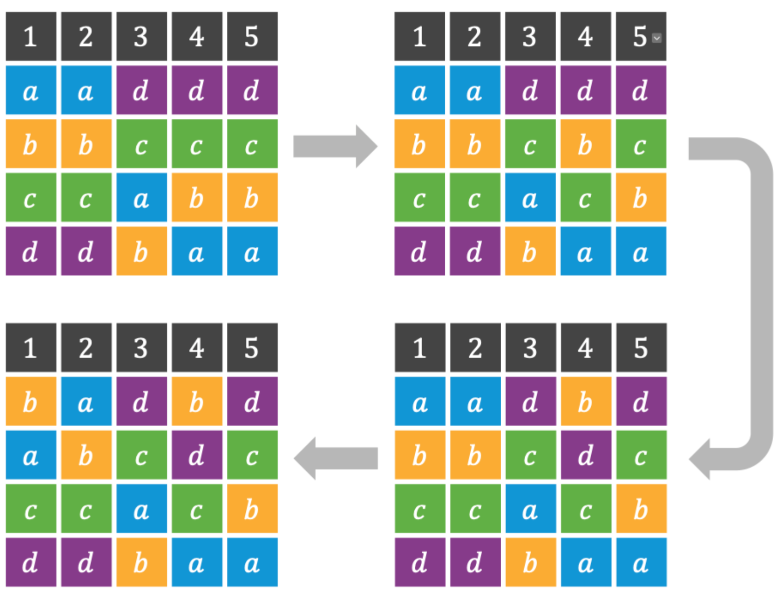
History
Proposed by Ramon Llull, a 13th-century philosopher and missionary. He proposed a rule that we will slightly tweak:
Winner
Each alternative receives a point for each head-to-head comparison it wins (including ties).
Example



In the above example, we calculate the winner by looking at head-to-head matchups. For example, alternative a loses 2-3 to each of the other alternatives in head-to-head matchups so gains 0 points. b is the winner as it wins head-to-head against each of the other alternatives.

2.5 Dodgson's Rule

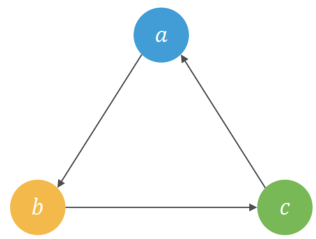
History
Proposed by Charles Lutwidge Dodgson, a mathematics professor at Oxford in the mid-1800s. Most people may know him under his pen name – Lewis Carroll. He was accused of plagiarizing Condorcet's work but evidence was shown that he most likely didn't read the relevant text.
Winner
The winner is the alternative a_k that needs the least number of swaps between adjacent alternatives for a_k to win all head-to-head against all other alternatives.
Example
Let's compute the Dogson score of b in the example below. We can make alternative b a Condorcet winner (win head-to-head against all other alternatives) with 3 swaps: twice for voter 4 (b swaps with c, then b swaps with d) and once for voter 1 (b swaps with a). Note that d is a Condorcet winner, and is also the Dodgson winner (0 swaps are needed to make d a Condorcet winner).



3 Condorcet Consistency

3.1 Condorcet Paradox

1	2	3
a	c	b
b	a	c
c	b	a



If we compare a to b, the majority of voters rank a above b. Therefore, we draw a directed arrow from a to b indicating that a "beats" b in a head-to-head competition. For the same reason, we draw arrows from b to c and arrows from c to a.

There's a cycle - there's no obvious winner. But in some situations, there will be an "obvious" winner - the person who defeats every other alternative in a head-to-head comparison.

3.2 Condorcet winner

An alternative is a Condorcet winner if it wins pairwise (head-to-head majority comparison) elections against all other alternatives.

3.3 Condorcet consistent

A voting rule is Condorcet consistent if the Condorcet winner, if exists, must be elected by the rule. Neither Plurality nor Borda Count are Condorcet consistent. Plurality only looks at the top preference for each voter, so it can choose a winner that loses the head-to-head matchup over all preferences. You can see an example of this in the example shown below in section 4.1, where 33 voters rank a first, but most other voters vote a behind all the other alternatives. Borda count is also not Condorcet consistent, since the points it gives to preferences sometimes inflate the points given to alternatives at the front. To illustrate this, consider the same example in section 4.1. Alternative c is a Condorcet winner since it wins the head-to-head matchup against all other alternatives, but b accrues more points in the weighted point method of Borda count and is the Borda count winner.

On the other hand, Llull's and Dodgson's rules are Condorcet consistent. For Llull's rule, a Condorcet winner has a score of $m-1$, whereas each other alternative has a score of at most $m-2$. For Dodgson's rule, a Condorcet winner requires zero swaps, whereas every other alternative requires at least one swap.

4 Independence of Clones

4.1 Definition of clones

A subset of alternatives K is a set of clones if no voter ranks any alternative outside of K between two alternatives of K.

1	2	3	4	5
a	a	b	c	d
b	b	a	b	c
c	c	d	a	b
d	d	c	d	a

In this voting profile, **a** and **b** are clones because for each of the voters, **a** and **b** are adjacent to one another. Under plurality, cloning a candidate can prevent them from winning.

1	2	3	4	5	6	7
a	a	a	a	c	c	c
b	b	c	c	a	a	a
c	c	b	b	b	b	b

1	2	3	4	5	6	7
a	a	d	d	c	c	c
d	d	a	a	d	a	a
b	b	c	c	a	d	d
c	c	b	b	b	b	b

For example, in the original voting profile, candidate **a** won the plurality vote. However, after cloning **a**, 2 of the candidates who'd previously voted **a** first vote for the clone of **a**, **d**, as first. Therefore, **c** ends up winning.

4.2 Independence of Clones

Like Condorcet consistency, independence of clones is another property of voting rules. A voting rule is independent of clones if and only if it satisfies the following two conditions:

1. An alternative that is a member of a set of clones wins if and only if some member of that set of clones wins after a member of the set is eliminated
2. An alternative that is not a member of a set of clones wins if and only if that same alternative wins after any clone is eliminated

For example, Lull's isn't independent of clones. To see this, imagine that we clone some alternative **a** many, many times (k for large enough k). Every alternative which was originally ranked above **a** now beats **a** and its k clones and therefore receives k additional points. Meanwhile, there isn't necessarily a "best" alternative amongst **a** and the k clones. Therefore, each of the clones might only be receiving $k/2$ points. In this case, if **a** was the original winner, some other alternative might now be the winner after receiving k additional points.

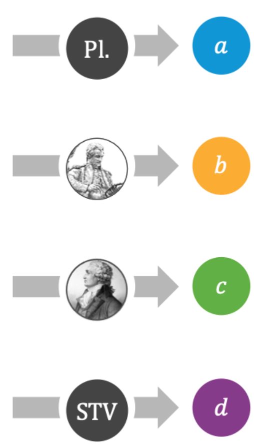
Meanwhile STV is independent of clones, because as clones are deleted, their votes get transferred to other clones. Therefore, all votes that belong to clones stay inside the set of clones, until eventually one of them remain standing.

5 Concluding thoughts

5.1 Is there one rule that is better than the others?

Above, we covered several voting rules out of tens if not hundreds of voting rules out there. However, not one of them is strictly better than the rest. In fact, there are situations where each voting rule may present a differing choice for the winner:

33 voters	16 voters	3 voters	8 voters	18 voters	22 voters
a	*b*	*c*	*c*	*d*	*e*
b	*d*	*d*	*e*	*e*	*c*
c	*c*	*b*	*b*	*c*	*b*
d	*e*	*a*	*d*	*b*	*d*
e	*a*	*e*	*a*	*a*	*a*



In the above situation, plurality would choose alternative a, borda count would choose alternative b, Llull's and Dodgson's rules would choose alternative c (it wins the most head-to-head matchups), and STV would choose alternative d. As such, depending on the application and context, one rule may fit the optimization objective better for a certain issue or environment.

5.2 Looking to the future

The advent of digital tools for voting makes it easy for organizations to choose any rule for voting. For example, in virtual democracy, AI can learn models of voters and be used to predict what they would want on unseen dilemmas. Prof. Procaccia has worked on multiple applications and research projects in computational social choice, including non-profit services such as Panelot. If you're interested in learning more, please reach out to Prof. Procaccia as well as check out CS238 (Optimized Democracy), usually offered in spring semesters.