CS182 Fall 2021
Midterm Exam

Name: ______________________________

Harvard ID: __________________________

Question 1: Search

1. **[10 pts]** Write down (using mathematical notation where appropriate) the definitions of admissible heuristics and consistent heuristics.

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2. **[15 pts]** Prove that A^* Tree Search with an admissible heuristic is optimal.

   Hint: This is precisely what we proved in class. Start by considering an optimal goal t^* and a suboptimal goal t. Let x be a node on the optimal path from s to t^* that was on the frontier when t was expanded. Argue that x should have been expanded before t.

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Question 2: Constraint Satisfaction Problems

1. **[10 pts]** Write down (using mathematical notation where appropriate) what it means for a variable X_i to be arc-consistent with respect to variable X_j in a CSP.

2. **[7 pts]** Consider an instance of the graph coloring problem with four colors, where the graph is a clique of four vertices. Formally, the vertices are $V = \{v_1, v_2, v_3, v_4\}$, their domains are $\{red, blue, green, yellow\}$, and for each $i \neq j$, there is a constraint that v_i and v_j must be assigned different colors.

   Would running the AC-3 algorithm (which enforces arc consistency) on this instance find a solution, or would additional search be required? Explain your reasoning.

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3. **[8 pts]** Now suppose that the previous instance is revised as follows: the domain of v_1 is restricted to $\{red\}$, the domain of v_2 is restricted to $\{blue\}$, and the domain of v_3 is restricted to $\{green\}$; the domain of v_4 is still $\{red, blue, green, yellow\}$.

   Would running the AC-3 algorithm (which enforces arc consistency) on this instance find a solution, or would additional search be required? Explain your reasoning.

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Question 3: Optimization

1. **[10 pts]** Write down (using mathematical notation where appropriate) the definitions of convex functions, convex sets, and convex optimization problems.

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2. **[20 pts]** Let $N = \{1, \ldots, n\}$ be a set of players, let M be a set of items, and let $v_{i\alpha}$ denote the value of player $i \in N$ for item $\alpha \in M$. Recall that an allocation $A_1, \ldots, A_n$ is a partition of the items, where A_i is the bundle allocated to player i. We say that an allocation is *equitable* if for all $i, k \in N$,

$$\sum_{\alpha \in A_i} v_{i\alpha} = \sum_{\alpha \in A_k} v_{k\alpha}.$$

In words, any two players i and k assign equal values to their own bundles.

Write down an integer program that finds an equitable allocation, if one exists.

Hint: Use binary variables $x_{i\alpha} \in \{0, 1\}$ that are 1 if and only if item α is assigned to player i. There are three constraints: equitability, each item is allocated to precisely one player, and the variable domains. Note that the indices used here for players and items are different from those used in class, to improve the readability of handwritten solutions.

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Question 4: Game Theory

1. **[10 pts]** Say in words what a Nash equilibrium is.

2. **[10 pts]** The following game tree describes a zero-sum game between a max player (triangles pointing up) and a min player (triangles pointing down). Assign payoffs (for the max player) to the leaves in a way that the alpha-beta pruning algorithm would have to examine all eight leaves, i.e., no pruning would take place. Assume that leaves are examined from left to right.