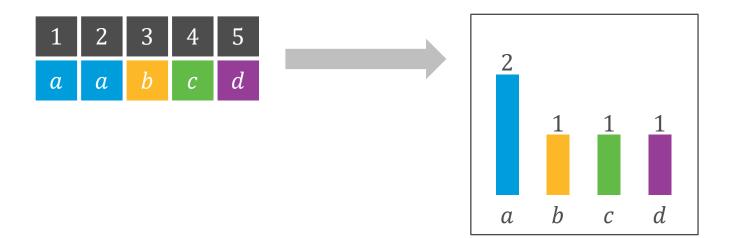


# Fall 2022 | Lecture 12 Social Choice Ariel Procaccia | Harvard University

#### **PLURALITY**

- Each person votes for a single alternative, and the alternative with most points wins
- A highly problematic voting rule!



# SOME BALLOT TYPES







Rankings

Approvals

Scores/stars

We will focus on rankings!



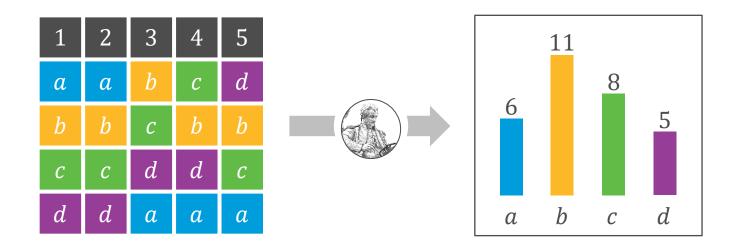
# Jean-Charles de Borda

1733-1799

Mathematician, engineer, and naval officer. Also remembered as an instigator of the metric system.

## **BORDA COUNT**

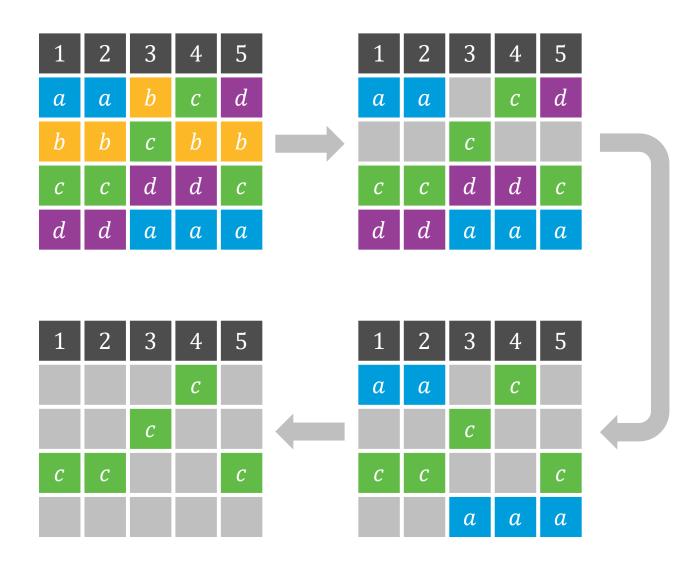
• Each voter awards m - k points to the alternative placed in the k'th position, where m is the number of alternatives



### SINGLE TRANSFERABLE VOTE

- Also known as "alternative vote," "instantrunoff voting" and (misleadingly) "rankedchoice voting"
- Votes are tabulated in rounds, where in each round the alternative with the lowest plurality score is eliminated; last alternative left standing is the winner

## SINGLE-TRANSFERABLE VOTE



### STV AROUND THE WORLD



■ Ireland

Used for all public elections

Canada

Used in Ontario for municipal elections

Australia

Used for parliamentary elections

USA

Used for statewide elections in ME and AK, and in cities like Cambridge, MA

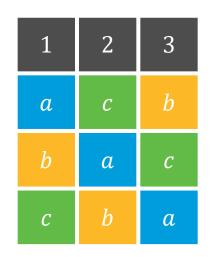


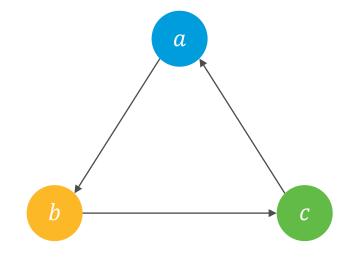
# Marquis de Condorcet

1743-1794

Philosopher, mathematician, enlightened nobleman. Also known for dying mysteriously in prison.

### THE CONDORCET PARADOX





The preferences of the majority may be cyclical!

# CONDORCET CONSISTENCY

- A Condorcet winner is an alternative that defeats every other alternative in a head-tohead majority comparison
- A rule is Condorcet consistent if it always selects a Condorcet winner whenever it is presented with a profile that contains one
- Poll 1: Which rule is Condorcet consistent?
  - Plurality
  - Borda count
  - Both rules
  - Neither one



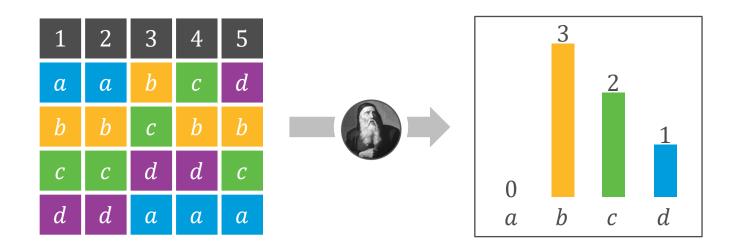
#### Ramon Llull

*c.* 1232–1315

Monk, missionary, and philosopher; one of the most influential intellectuals of his time. Also remembered for publishing a medieval parenting guide.

# LLULL'S RULE

 Each alternative receives one point for each head-to-head comparison it wins (as well as for tied comparisons)



• Llull's rule is Condorcet consistent



# Charles Lutwidge Dodgson

1832-1898

Professor of mathematics at Oxford, pioneer photographer, and beloved author. Also known for not plagiarizing Condorcet's work.

### DODGSON'S RULE

- The Dodgson score of an alternative x is the minimum number of swaps between adjacent alternatives needed to make x a Condorcet winner; select an alternative with minimum score
- Dodgson's rule is Condorcet consistent
- Dodgson's rule is NP-hard to compute!

# DODGSON'S RULE

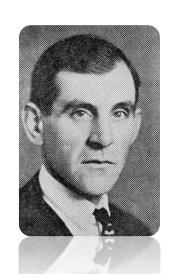
What is the Dodgson score of *b*?

1	2	3	4	5	1	2	3	4	5	
a	a	d	d	d	a	a	d	d	d	
b	b	С	С	С	b	b	С	b	С	
С	С	a	b	b	С	С	a	С	b	
d	d	b	a	а	d	d	b	a	а	- 1
										-
1	2	3	4	5	1	2	3	4	5	
b	a	d	b	d	a	a	d	b	d	
a	b	С	d	С	b	b	С	d	С	
С	С	а	С	b	С	С	a	С	b	
d	d	b	а	а	d	d	b	a	а	

# 1934 ELECTION: OREGON GOVERNOR



Charles H. Martin Dem. 116,667 votes



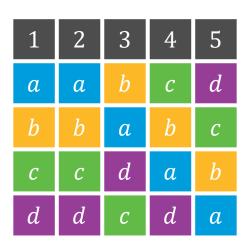
Peter Zimmerman Ind. (Rep.) 95,519 votes



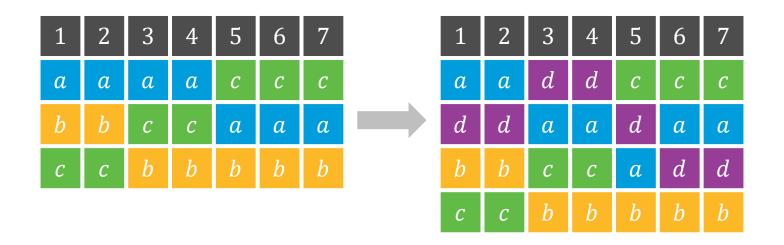
Joe E. Dunne Rep. 86,923 votes

### INDEPENDENCE OF CLONES

- A subset of alternatives *K* is a set of clones if no voter ranks any alternative outside of *K* between two alternatives of *K*
- In the following profile, *a* and *b* are clones:



### INDEPENDENCE OF CLONES



Under plurality, cloning a candidate may prevent them from winning

# INDEPENDENCE OF CLONES

- A voting rule is independent of clones if and only if it satisfies the following two conditions:
  - 1. An alternative that is a member of a set of clones wins if and only if some member of that set of clones wins after a member of the set is eliminated
  - 2. An alternative that is not a member of a set of clones wins if and only if that same alternative wins after any clone is eliminated
- Poll 2: Which rule is independent of clones?
  - Single-transferable vote
  - Borda
  - Llull
  - None of the above

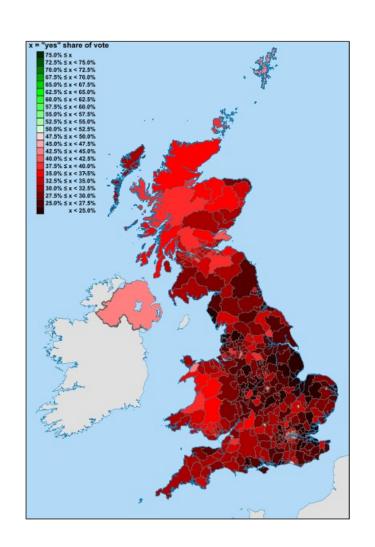


# AWESOME EXAMPLE

33 voters	16 voters	3 voters	8 voters	18 voters	22 voters	Pl. a
а	b	С	С	d	е	
b	d	d	e	e	С	b
С	С	b	b	С	b	
d	е	а	d	b	d	
e	а	е	а	а	а	STV

# IS SOCIAL CHOICE PRACTICAL?

- UK referendum (2011): Choose between plurality and STV as a method for electing MPs
- Academics agreed STV is better
- But STV was seen as beneficial to a particular politician



## COMPUTATIONAL SOCIAL CHOICE

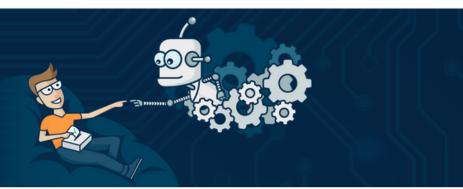
However, in emerging paradigms of democracy and tools for group decision making, the designer is free to choose any voting rule!





#### **AI-Driven Decisions**

RoboVote is a free service that helps users combine their preferences or opinions into optimal decisions. To do so, RoboVote employs state-of-the-art voting methods developed in artificial intelligence research. Learn More



#### Poll Types

RoboVote offers two types of polls, which are tailored to different scenarios; it is up to users to indicate to RoboVote which scenario best fits the problem at hand.



#### Objective Opinions

In this scenario, some alternatives are objectively better than others, and the opinion of a participant reflects an attempt to estimate the correct order. RoboVote's proposed outcome is guaranteed to be as close as possible — based on the available information — to the best outcome. Examples include deciding which product prototype to develop, or which company to invest in, based on a metric such as projected revenue or market share. Try the demo.



#### Subjective Preferences

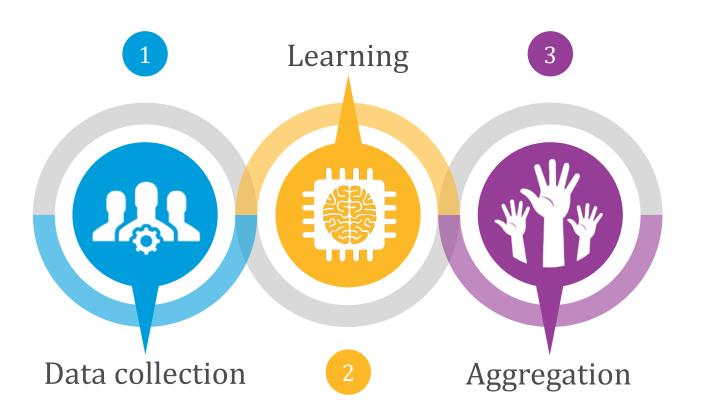
In this scenario participants' preferences reflect their subjective taste; RoboVote proposes an outcome that mathematically makes participants as happy as possible overall. Common examples include deciding which restaurant or movie to go to as a group, which destination to choose for a family vacation, or whom to elect as class president. Try the demo.

Ready to get started?

CREATE A POLL



# VIRTUAL DEMOCRACY



# VIRTUAL DEMOCRACY

