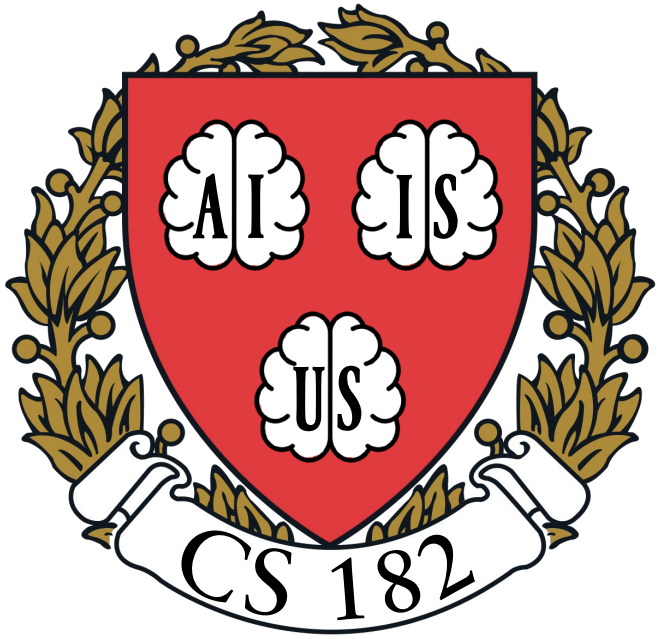


Fall 2021 | Lecture 3

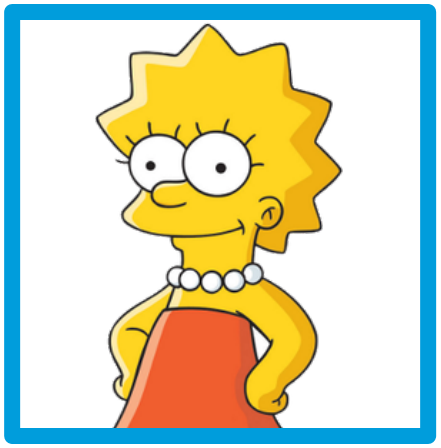
Informed Search

Ariel Procaccia | Harvard University

UNINFORMED VS. INFORMED

Uninformed

Can only generate successors and distinguish goals from non-goals



Informed

Strategies that know whether one non-goal is more promising than another

REMINDER: TREE SEARCH

function TREE-SEARCH($problem, strategy$)

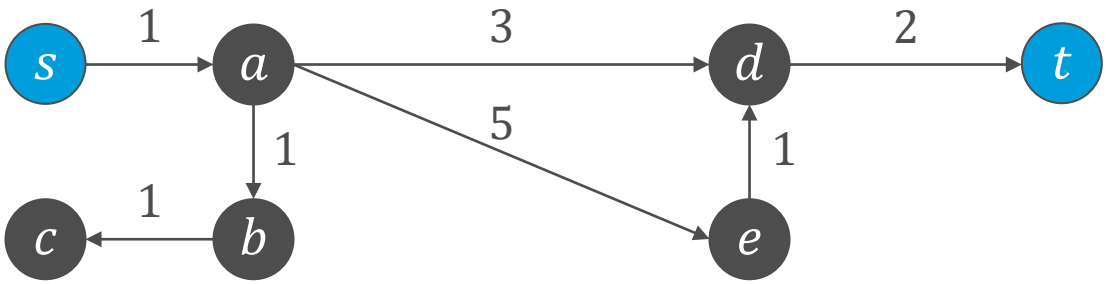
set of frontier nodes contains the start state of $problem$

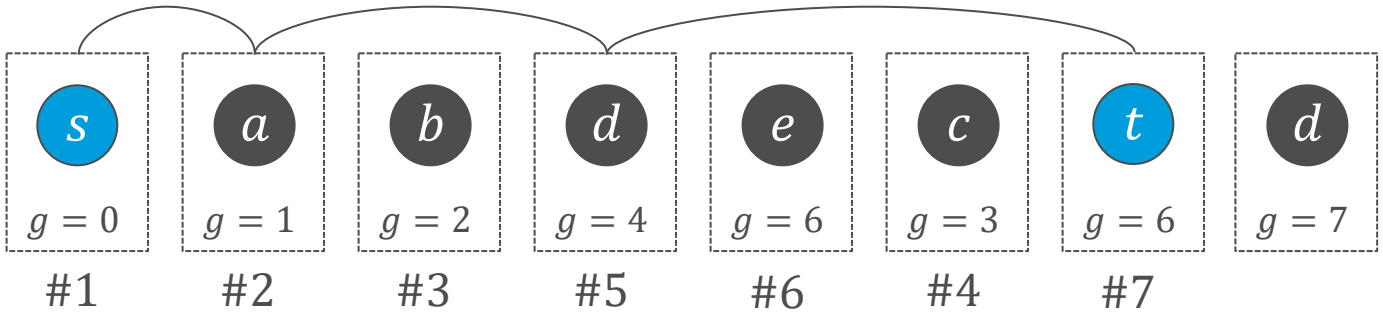
loop

- if there are no frontier nodes then return failure
- choose a frontier node for expansion using $strategy$
- if the node contains a goal then return the corresponding solution
- else expand the node and add the resulting nodes to the set of frontier nodes

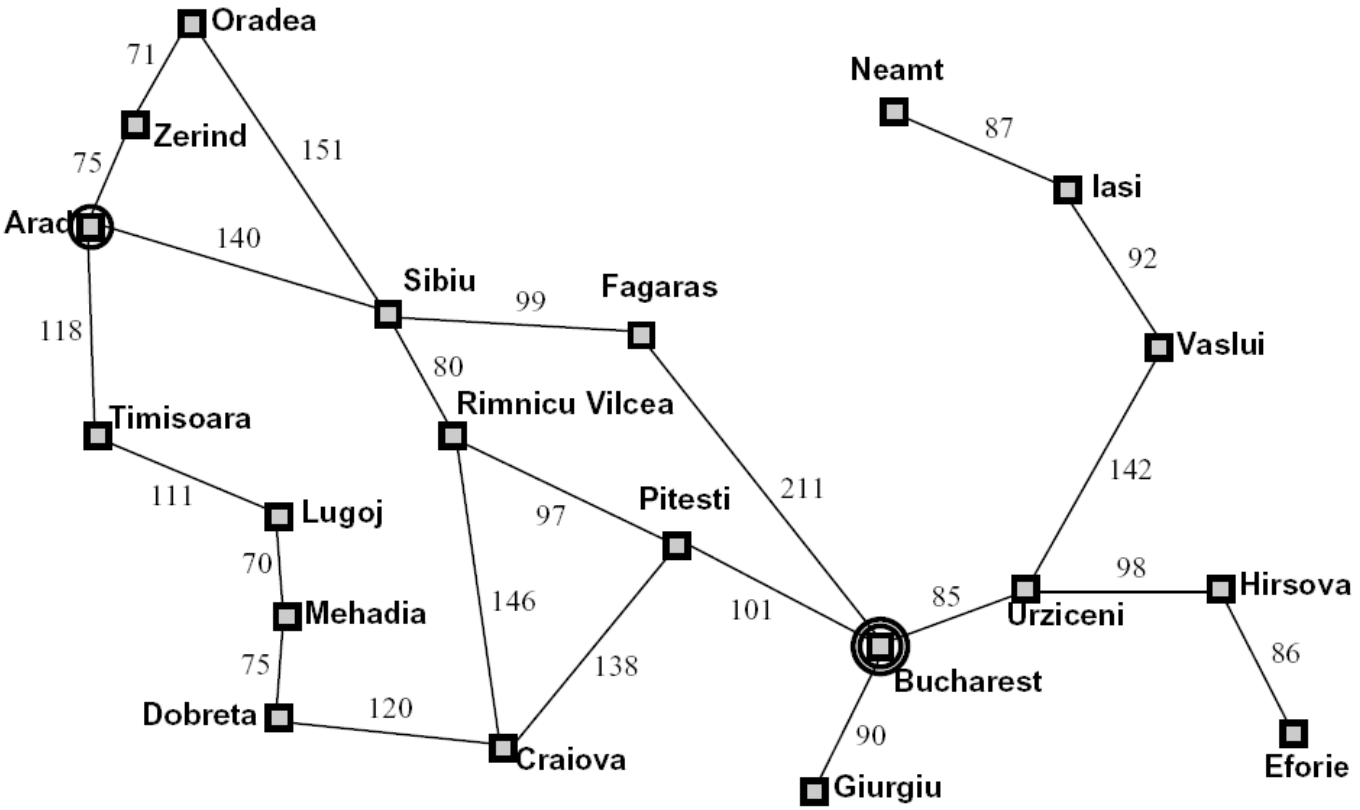
UNIFORM COST SEARCH

- Strategy: Expand by $g(x) =$ work done so far





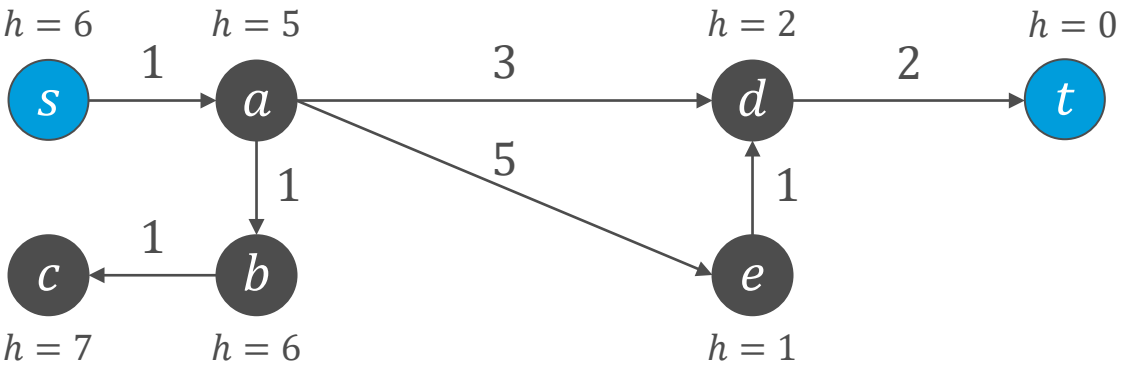
EXAMPLE: HEURISTIC

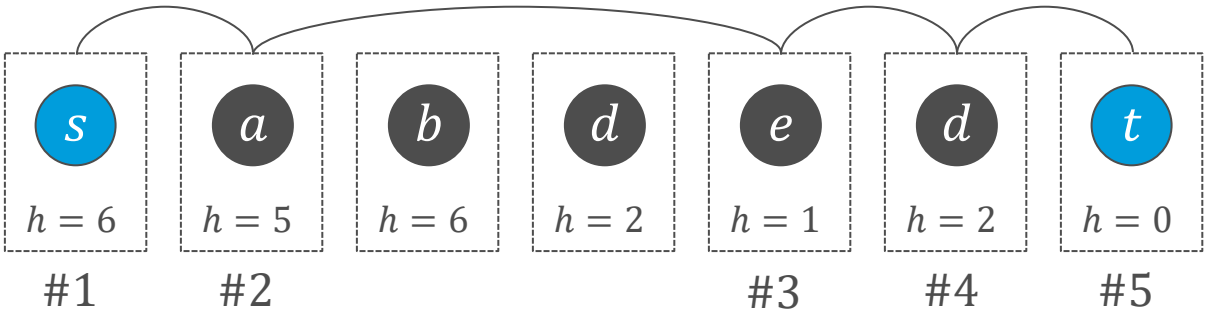


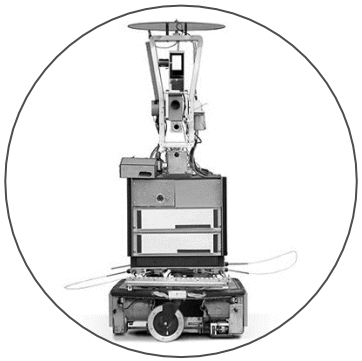
City	Arad	Sibiu	RV	Fagaras	Pitesi
Aerial distance from Bucharest	366	253	193	176	100

GREEDY SEARCH

- Strategy: Expand by $h(x)$ = heuristic evaluation of cost from x to goal







Shakey the Robot

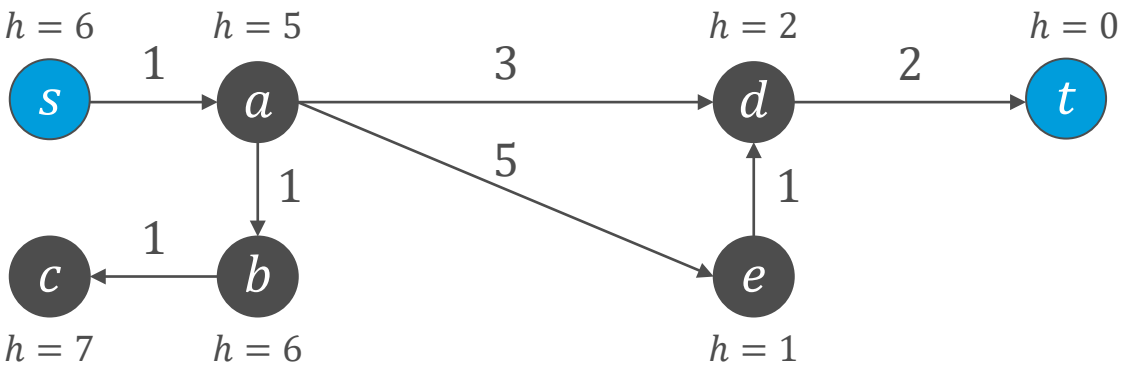
1966-1972

First mobile robot equipped with automated planning capabilities. Its pathfinding algorithm was A*.

A* SEARCH

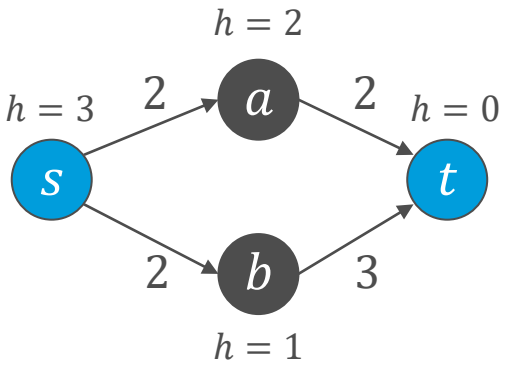
- Strategy: Expand by $f(x) = h(x) + g(x)$
- Poll 1: Which node is expanded fourth?

1. d
2. e
3. t ✓
4. c



A* SEARCH

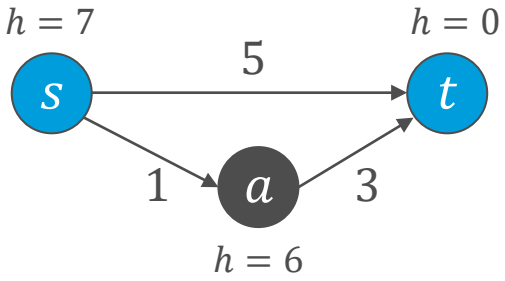
- Should we stop when we discover a goal?



- No: Only stop when we expand a goal

A* SEARCH

- Is A* optimal?



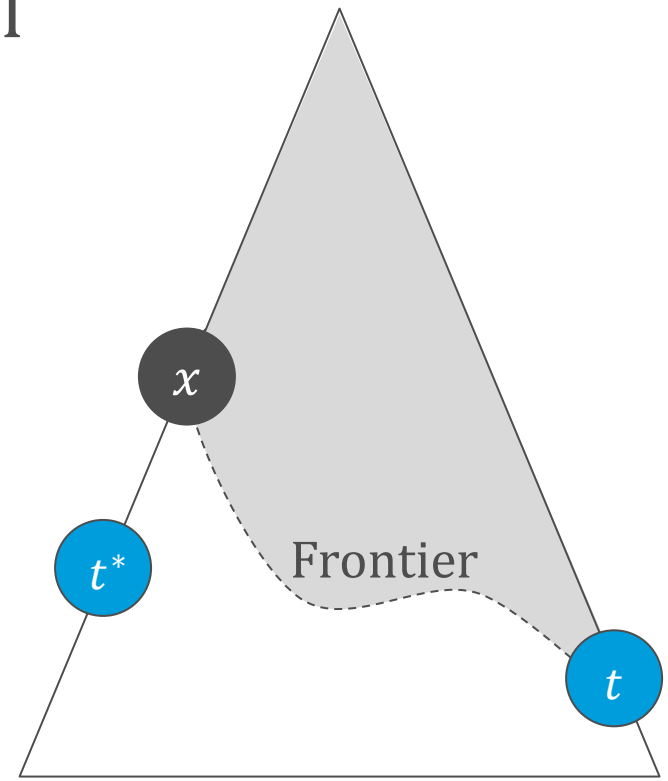
- Good path has pessimistic estimate
- Circumvent this issue by being optimistic!

ADMISSIBLE HEURISTICS

- h is admissible if for all nodes x,
$$h(x) \leq h^*(x),$$
where h^* is the cost of the optimal path to a goal
- Example: Aerial distance in the pathfinding example
- Example: $h \equiv 0$
- Theorem: A* tree search with an admissible heuristic returns an optimal solution

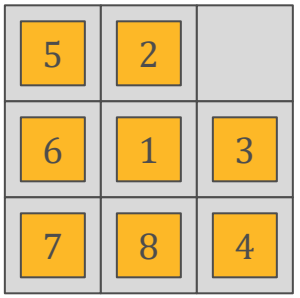
PROOF OF THEOREM

- Assume suboptimal goal t is expanded before optimal goal t^*
- There is a node x on the optimal path to t^* that has been discovered but not expanded
- $$\begin{aligned} f(x) &= g(x) + h(x) \\ &\leq g(x) + h^*(x) \\ &= g(t^*) < g(t) \\ &= f(t) \end{aligned}$$
- x should have been expanded before t! ■

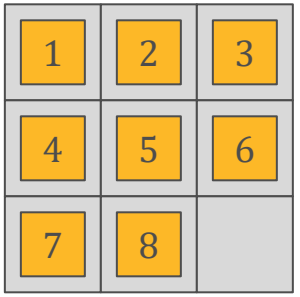


8-PUZZLE HEURISTICS

- h_1: #tiles in wrong position
- h_2: sum of Manhattan distances of tiles from goal
- Poll 2: Which heuristic is admissible?
  1. Only h_1
  2. Only h_2
  3. Both h_1 and h_2 ✓
  4. Neither one



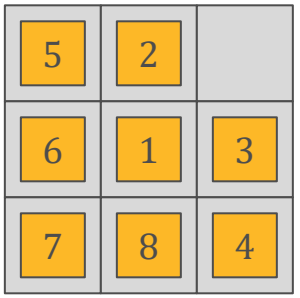
Example state



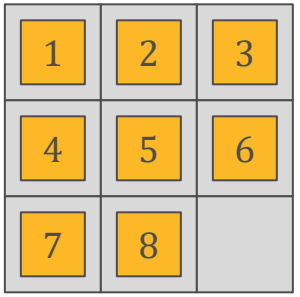
Goal state

8-PUZZLE HEURISTICS

- h_1: #tiles in wrong position
- h_2: sum of Manhattan distances of tiles from goal
- h dominates h' iff $\forall x, h(x) \geq h'(x)$
- Poll 3: What is the dominance relation between h_1 and h_2?
  1. h_1 dominates h_2
  2. h_2 dominates h_1 ✓
  3. h_1 and h_2 are incomparable



Example state



Goal state

8-PUZZLE HEURISTICS

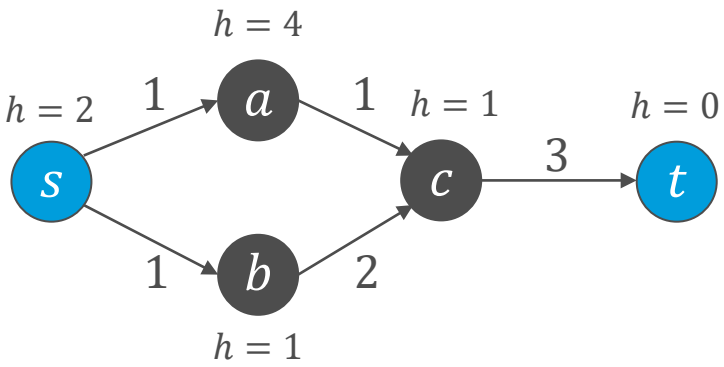
- The following table gives the number of nodes expanded by BFS and A* with the two heuristics, averaged over random 8-puzzles, for various solution lengths

Length	BFS	$A^*(h_1)$	$A^*(h_2)$
16	17270	1683	364
18	41558	4102	751
20	91493	9905	1318
22	175921	22955	2548
24	290082	53039	5733

- Moral: Good heuristics are crucial!

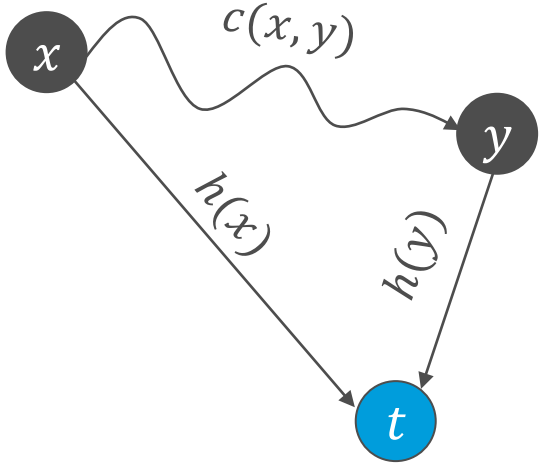
A* GRAPH SEARCH

- Recall: Graph search is the same as tree search, but never expand a node twice
- Is optimality of A* under admissible heuristics preserved? No!



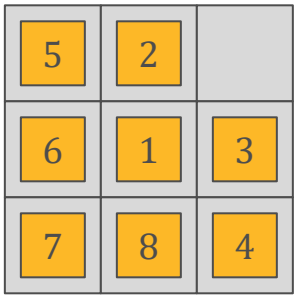
CONSISTENT HEURISTICS

- $c(x, y)$ = cost of cheapest path between x and y
- h is consistent if for every two nodes x, y, $h(x) \leq c(x, y) + h(y)$
- Assume $h(t) = 0$ for each goal t, then consistency implies admissibility because $h(x) \leq \min_t \big(c(x, t) + h(t)\big) = \min_t c(x, t) = h^*(x)$
- Theorem: A* graph search with a consistent heuristic returns an optimal solution

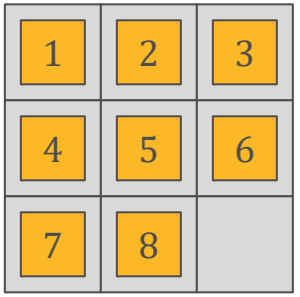


8-PUZZLE HEURISTICS, REVISITED

- h_1: #tiles in wrong position
- h_2: sum of Manhattan distances of tiles from goal
- Poll 4: Which heuristic is consistent?
  1. Only h_1
  2. Only h_2
  3. Both h_1 and h_2 ✓
  4. Neither one



Example state



Goal state