Fall 2021 | Lecture 2

Uninformed Search

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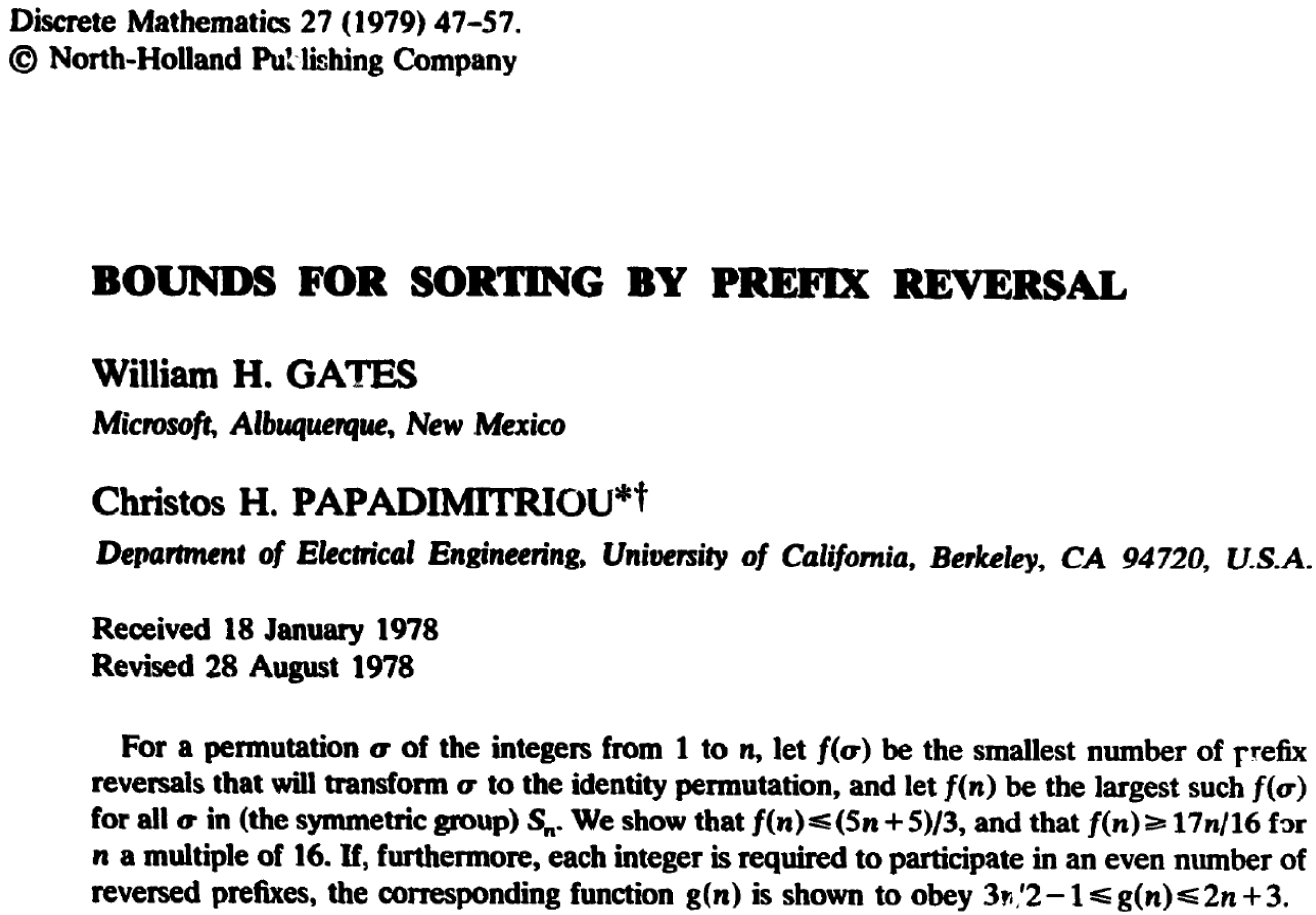
SEARCH PROBLEMS

- A search problem consists of
  - States (configurations)
  - Start state and goal states
  - Successor function: maps states to (action,state,cost) triples
- This is a powerful and flexible representation that captures a wide variety of concrete problems

EXAMPLE: PANCAKES



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BOUNDS FOR SORTING BY PREFIX REVERSAL

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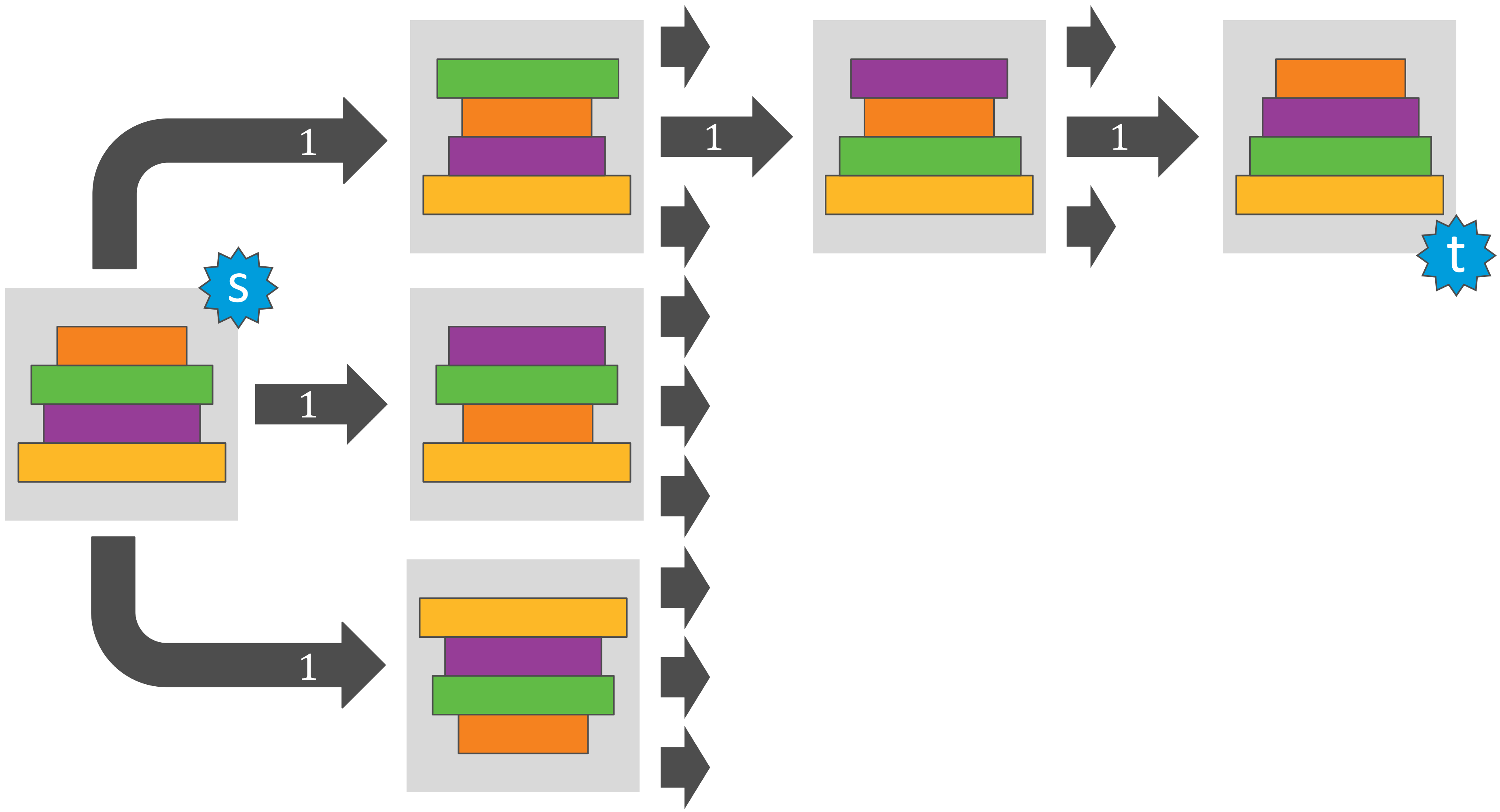
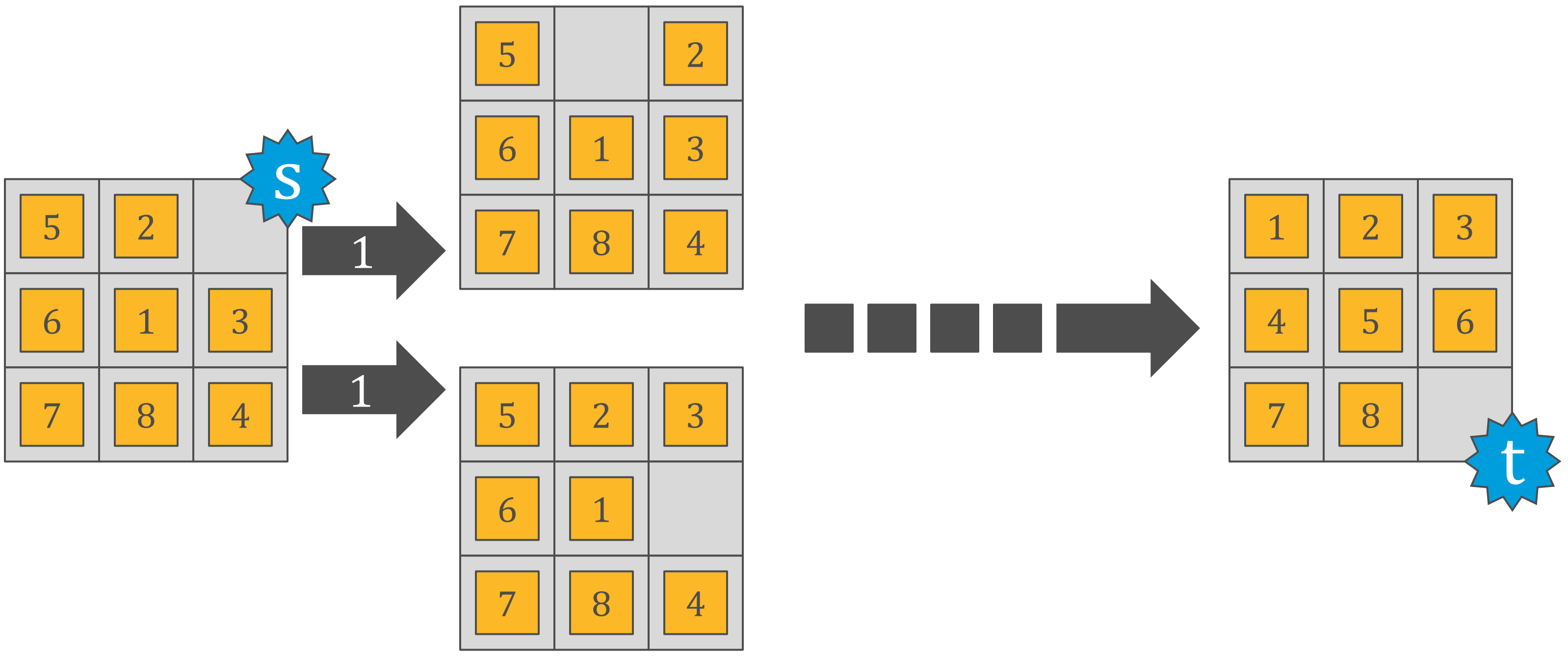
For a permutation σ of the integers from 1 to n, let $f(\sigma)$ be the smallest number of prefix reversals that will transform σ to the identity permutation, and let $f(n)$ be the largest such $f(\sigma)$ for all σ in (the symmetric group) S_n. We show that $f(n) \leq (5n+5)/3$, and that $f(n) \geq 17n/16$ for n a multiple of 16. If, furthermore, each integer is required to participate in an even number of reversed prefixes, the corresponding function $g(n)$ is shown to obey $3n/2 - 1 \leq g(n) \leq 2n+3$.

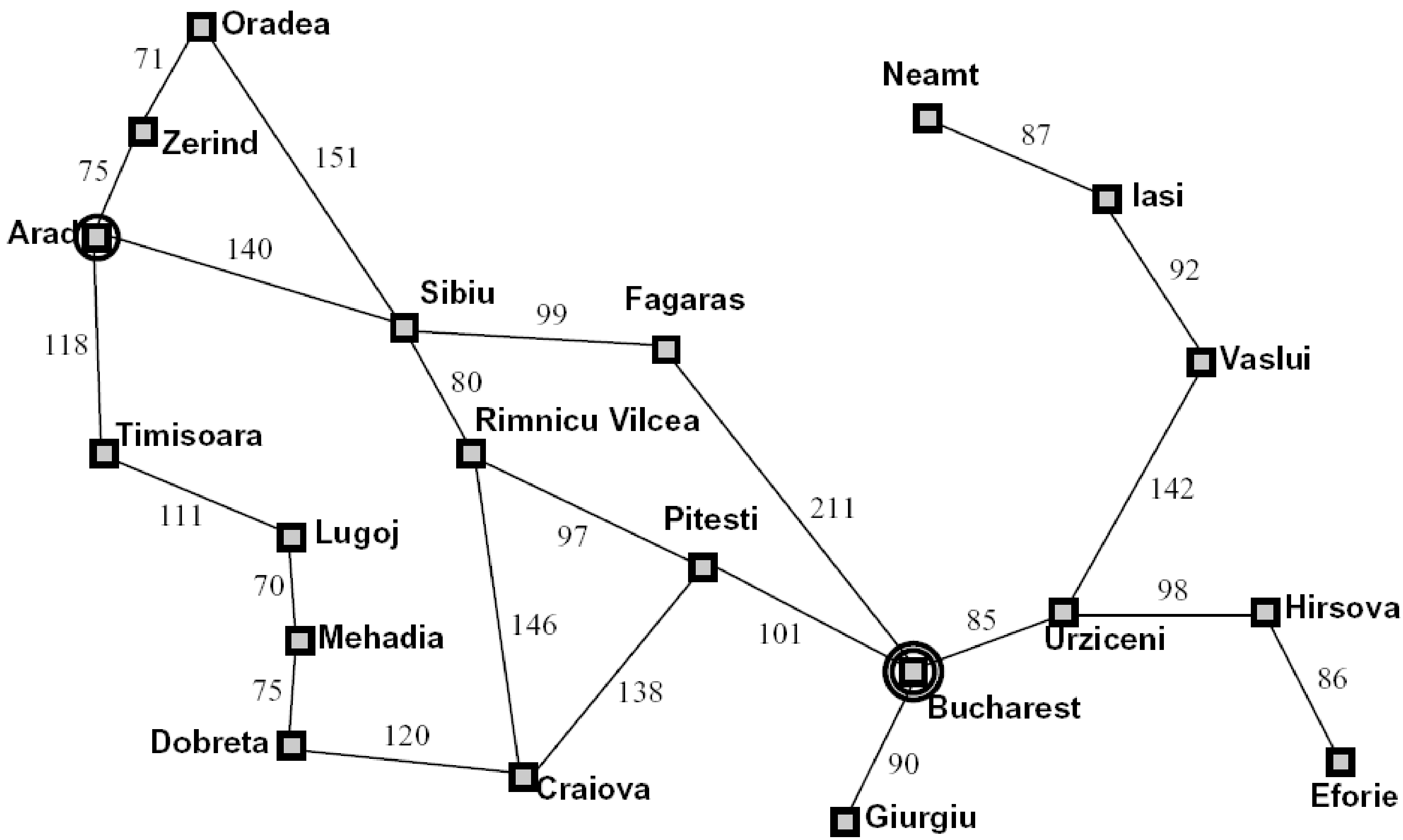
EXAMPLE: PANCAKES



EXAMPLE: 8-PUZZLE



EXAMPLE: PATHFINDING



TREE SEARCH

function TREE-SEARCH(*problem*, *strategy*)

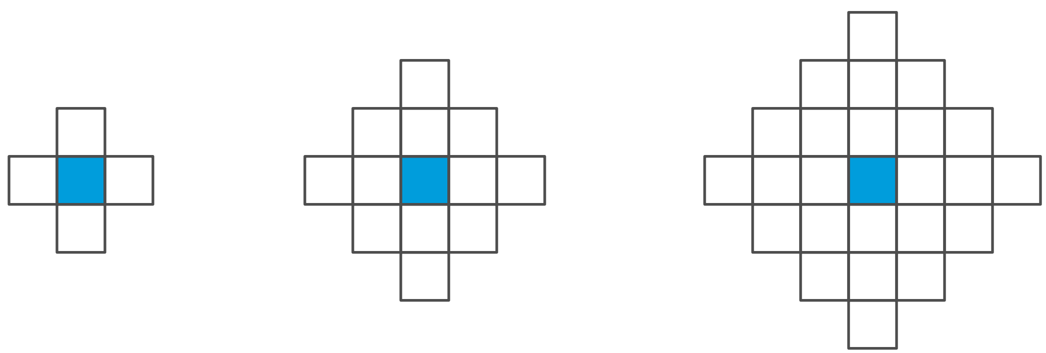
set of frontier nodes contains the start state of *problem*

loop

- **if** there are no frontier nodes **then return** failure
- choose a frontier node for expansion using *strategy*
- **if** the node contains a goal **then return** the corresponding solution
- **else** expand the node and add the resulting nodes to the set of frontier nodes

TREE SEARCH

Algorithms that forget their history are doomed to repeat it!



In a rectangular grid, search tree of depth d has 4^d leaves, but there are only $4d$ states within d steps of any given state

GRAPH SEARCH

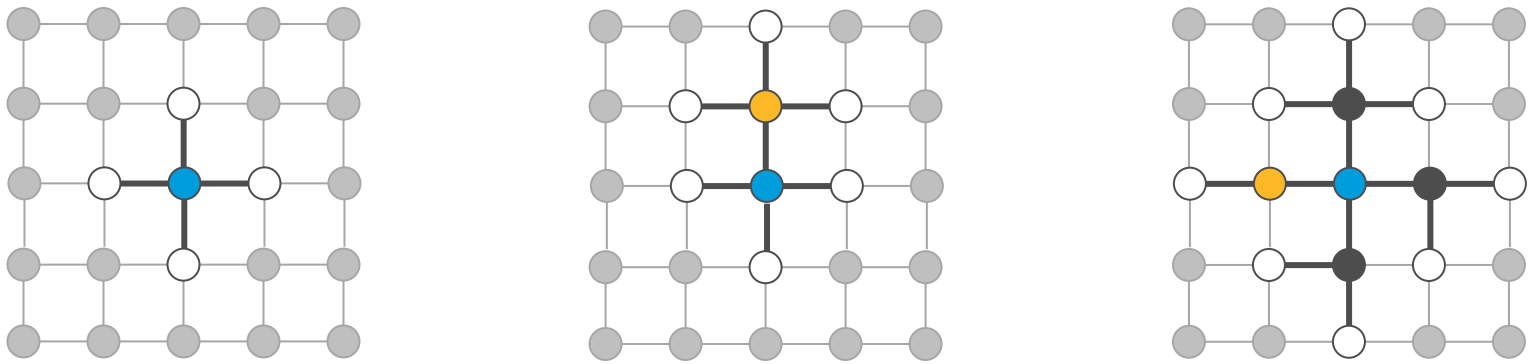
function GRAPH-SEARCH(*problem*, *strategy*)

set of frontier nodes contains the start state of *problem*

loop

- **if** there are no frontier nodes **then return** failure
- choose a frontier node for expansion using *strategy*, and add it to the explored set
- **if** the node contains a goal **then return** the corresponding solution
- **else** expand the node and add the resulting nodes to the set of frontier nodes, only if not in the explored set

GRAPH SEARCH ILLUSTRATED

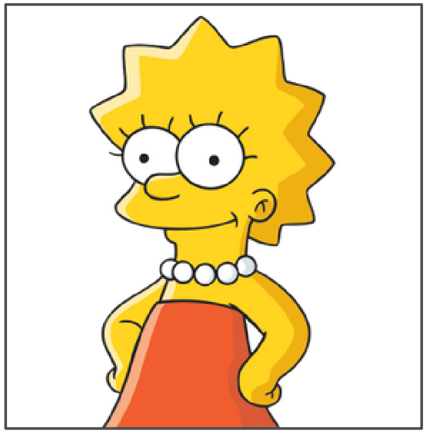


Separation property: Every path from initial state to an unexplored state has to pass through the frontier

UNINFORMED VS. INFORMED

Uninformed

Can only generate successors and distinguish goals from non-goals



Informed

Strategies that know whether one non-goal is more promising than another

MEASURING PERFORMANCE

Completeness

Guaranteed to find a solution when there is one?

Optimality

Finds the cheapest solution?

Time

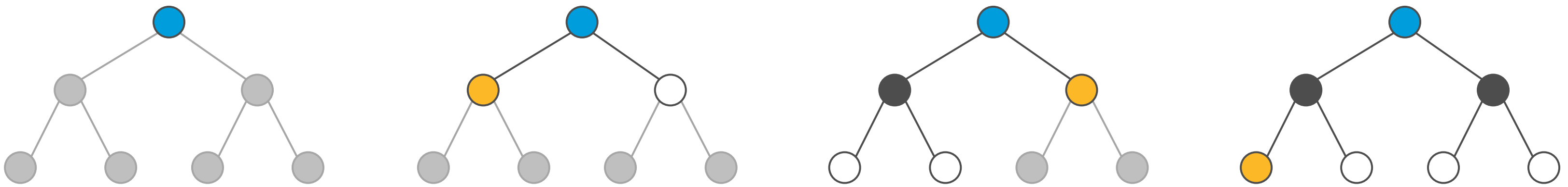
How long does it take to find a solution?

Space

How much memory is needed to perform the search?

BREADTH-FIRST SEARCH

- Strategy: Expand shallowest frontier node
- Can be implemented by using a FIFO queue for the frontier



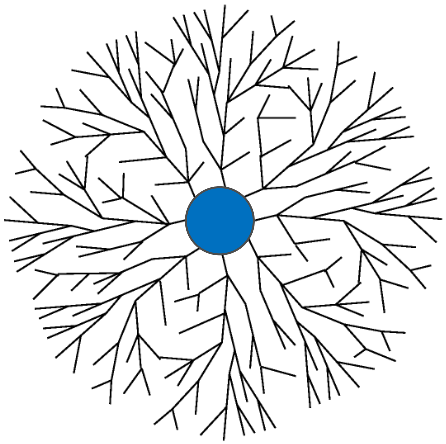
BREADTH-FIRST SEARCH

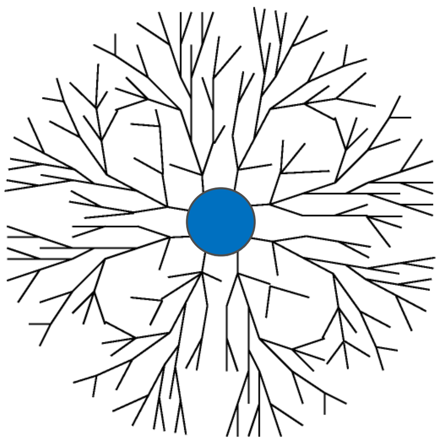
Algorithm	Complete?	Optimal?	Time	Space
BFS	Yes	Not really	$\Theta(b^d)$	$\Theta(b^d)$

- Optimality: If the path cost is a nondecreasing function of the depth (e.g., all actions have the same cost)
- Time complexity: Imagine each node has $b \geq 2$ successors, and solution is at depth d, then generate $\sum_{i=1}^{d} b^i = \Theta(b^d)$ nodes
- Space complexity: The frontier is almost as large as the entire search tree

BIDIRECTIONAL SEARCH

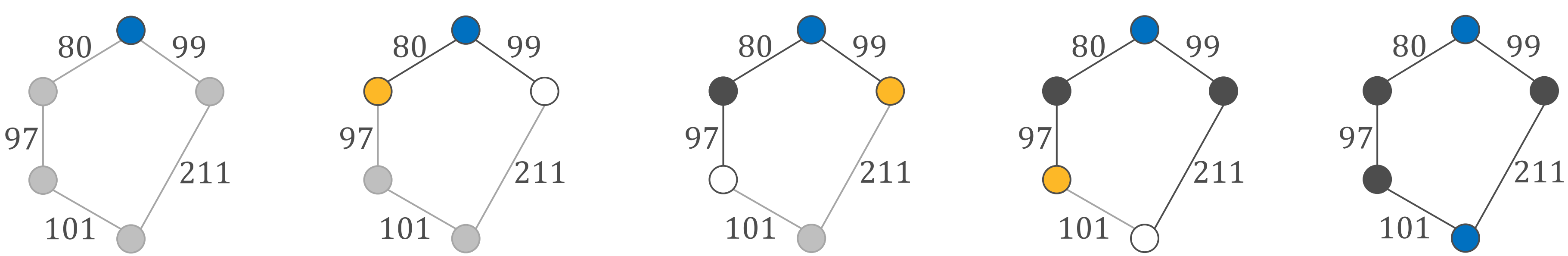
- Idea: Possibly improve the running time of BFS by running two simultaneous searches, forward from the initial state and backward from the goal
- Poll 1: What is the worst-case running time of BIDRECTIONAL SEARCH?
  1. $\Theta(b \cdot d)$
  2. $\Theta((b/2)^d)$
  3. $\Theta(b^{d/2})$ ✓
  4. $\Theta(b^d)$





UNIFORM-COST SEARCH

- Strategy: Expand frontier node n with lowest path cost $g(n)$
- Can be implemented by using a priority queue ordered by $g(n)$ for the frontier
- Other changes from BFS:
  - Goal test applied when node is selected for expansion
  - Need to update cost of nodes on frontier



UNIFORM-COST SEARCH

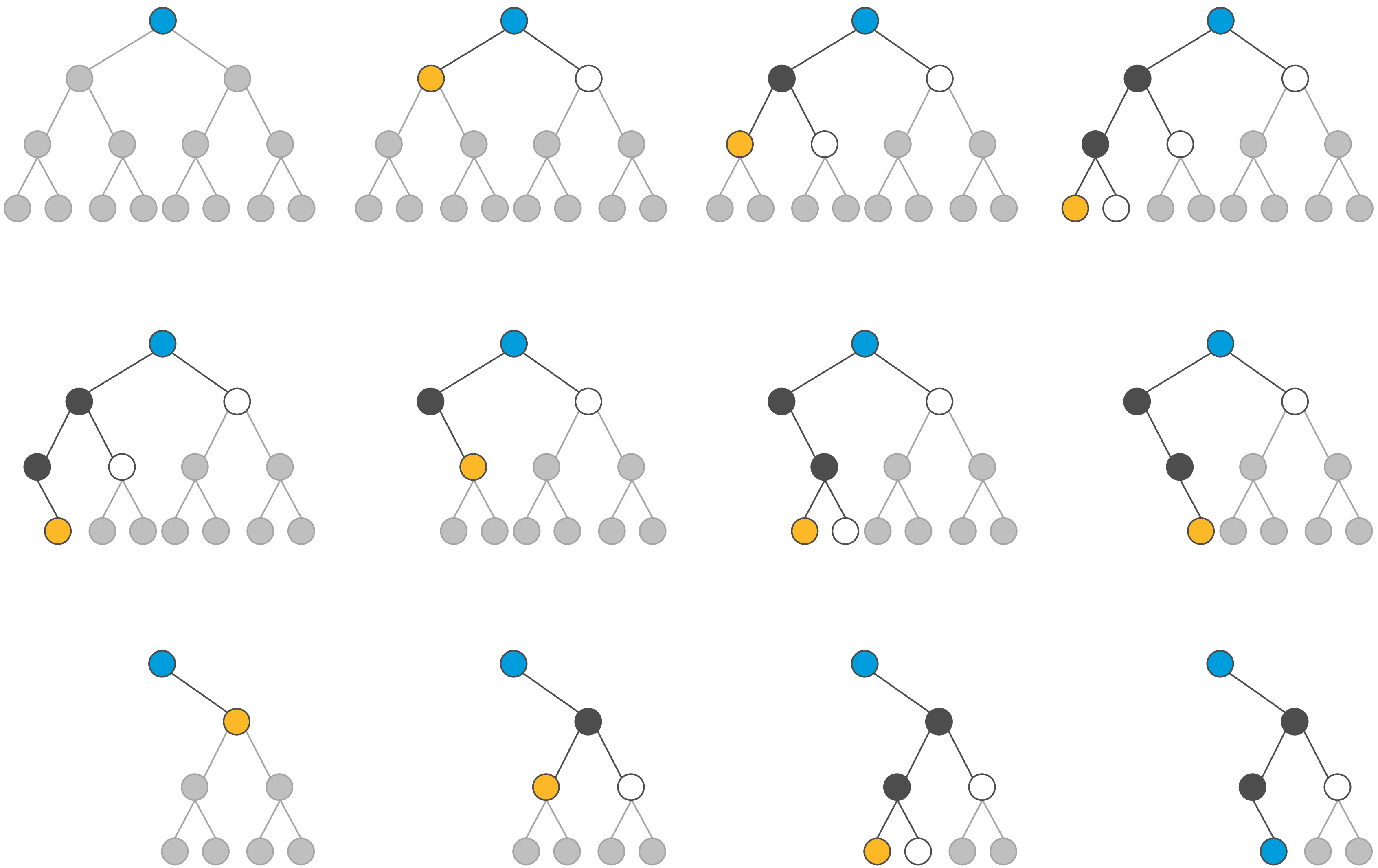
Algorithm	Complete?	Optimal?	Time	Space
UCS	Sorta	Yes	$\Theta\left(b^{C^*/\epsilon}\right)$	$\Theta\left(b^{C^*/\epsilon}\right)$

- Optimality: Yes, but requires proof
- Completeness: If the cost of every action exceeds some $\epsilon > 0$
- Time and space complexity: If C^* is the cost of the optimal solution and ϵ is a lower bound on the action cost, the depth of the search tree is roughly C^*/ϵ in the worst case

DEPTH-FIRST SEARCH

- Strategy: Expand deepest unexpanded node
- Can be implemented by using a stack for the frontier
- Recursive implementation is also common

DEPTH-FIRST SEARCH



DEPTH-FIRST SEARCH

Algorithm	Complete?	Optimal?	Time	Space
DFS	No	No	$\Theta(b^m)$	$\Theta(b \cdot m)$

- Completeness: Clearly not in general
- Poll 2: In a finite state space, which version of DFS is complete?
  1. TREE SEARCH
  2. GRAPH SEARCH ✓
  3. Both
  4. Neither

DEPTH-FIRST SEARCH

Algorithm	Complete?	Optimal?	Time	Space
DFS	No	No	$\Theta(b^m)$	$\Theta(b \cdot m)$

- Time complexity: $\Theta(b^m)$, where m is the maximum depth of the search tree
- Space complexity: DFS tree search needs to store only a single path from the root to a leaf, along with frontier sibling nodes for each node on the path
- Consequently, depth-first tree search is the workhorse of many areas of AI (including CSPs and SAT solving)

ITERATIVE DEEPENING SEARCH

Algorithm	Complete?	Optimal?	Time	Space
IDS	Yes	No	$\Theta(b^d)$	$\Theta(b \cdot d)$

- Run DFS with depth limit $\ell = 1,2, \ldots$
- Combines the best properties of BFS and DFS
- Completeness: Yes, for the same reason BFS is complete
- Time complexity: Seems wasteful but most of the nodes are at the bottom level; total

$$d \cdot b + (d-1)b^2 + \cdots + 1 \cdot b^d = \Theta(b^d)$$

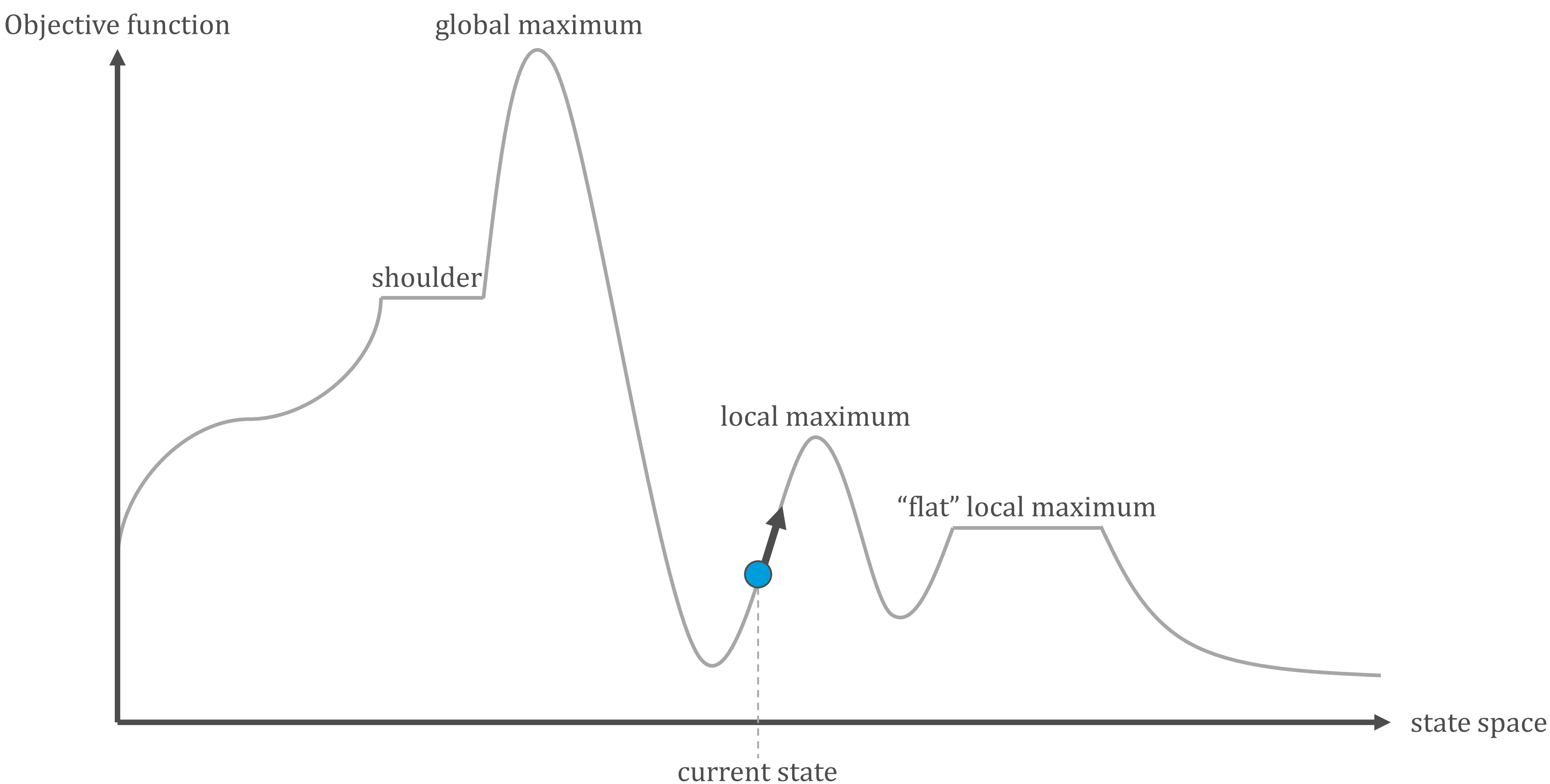
SUMMARY OF ALGORITHMS

Algorithm	Complete?	Optimal?	Time	Space
BFS	Yes	Not really	$\Theta(b^d)$	$\Theta(b^d)$
UCS	Sorta	Yes	$\Theta(b^{C^*/\epsilon})$	$\Theta(b^{C^*/\epsilon})$
DFS	No	No	$\Theta(b^m)$	$\Theta(b \cdot m)$
IDS	Yes	No	$\Theta(b^d)$	$\Theta(b \cdot d)$

OPTIMIZATION AND LOCAL SEARCH

- The algorithms we discussed so far are designed to find a path to the solution
- If the path doesn't matter, can use local search algorithms that consider a single current node, and move to one of its neighbors in the next step
- Local search algorithms are useful for optimization problems, where the goal is to find the best state according to an objective function

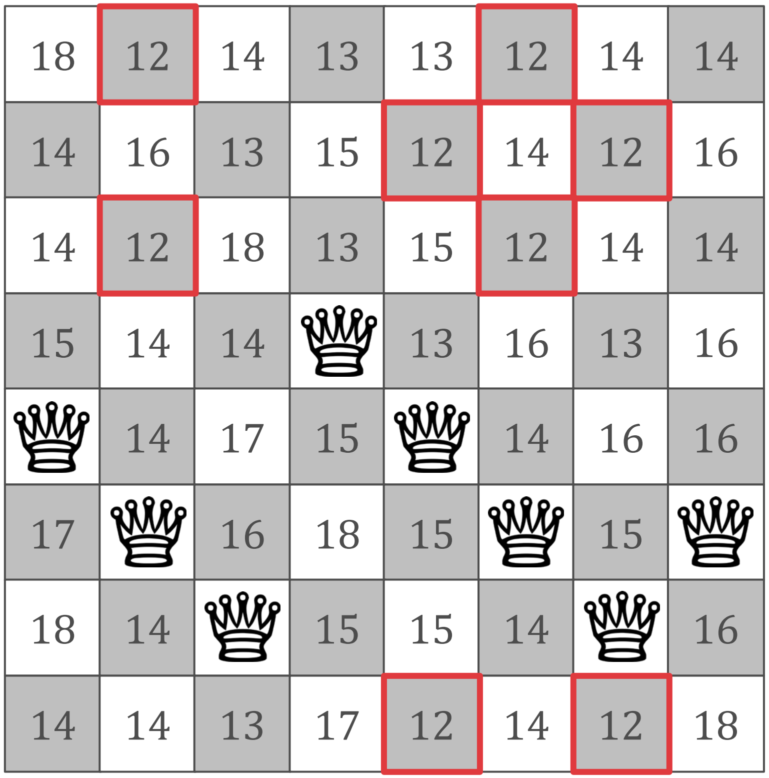
STATE SPACE LANDSCAPE



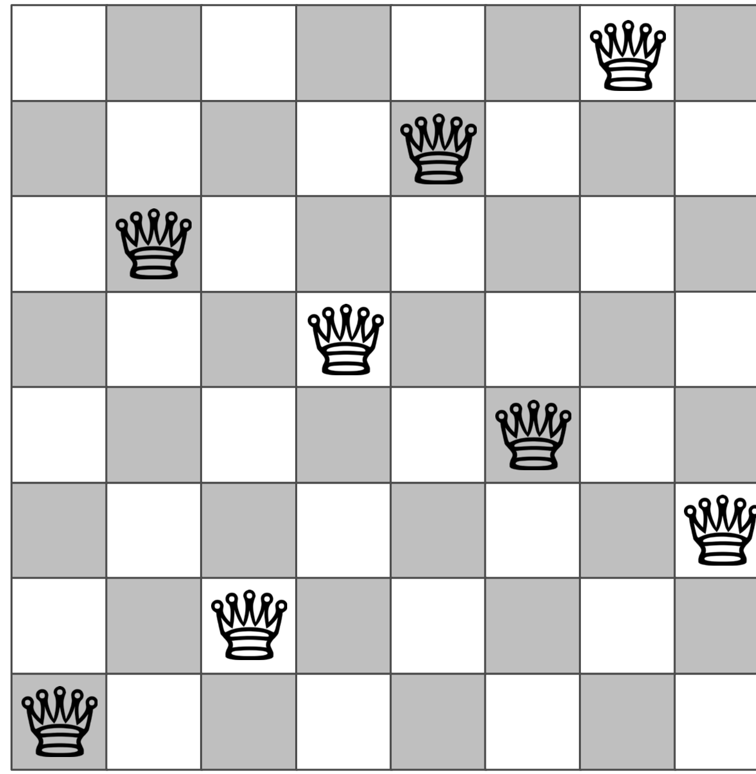
HILL-CLIMBING SEARCH

- Move in the direction of increasing value (up the hill)
- Terminate when no neighbor has higher value

HILL-CLIMBING SEARCH



State with 17 conflicts, showing the #conflicts by moving a queen within its column, with best moves in red



Local optimum: state that has only one conflict, but every move leads to larger #conflicts

HILL-CLIMBING SEARCH

- 8 queens statistics:
  - State space of size ≈17 million
  - Starting from random state, steepest-ascent hill climbing solves 14% of problem instances
  - It takes 4 steps on average when it succeeds and 3 when it gets stuck
  - When "sideways" moves are allowed, solves 94% of instances, but with 21 steps for success and 64 for failure
- Variants:
  - Stochastic hill climbing: Chooses at random among uphill moves, with the probability depending on the improvement
  - Random-restart hill climbing: Conducts a series of hill-climbing searches from random states; obviously complete, and expected number of iterations is roughly 7, with roughly 22 steps overall