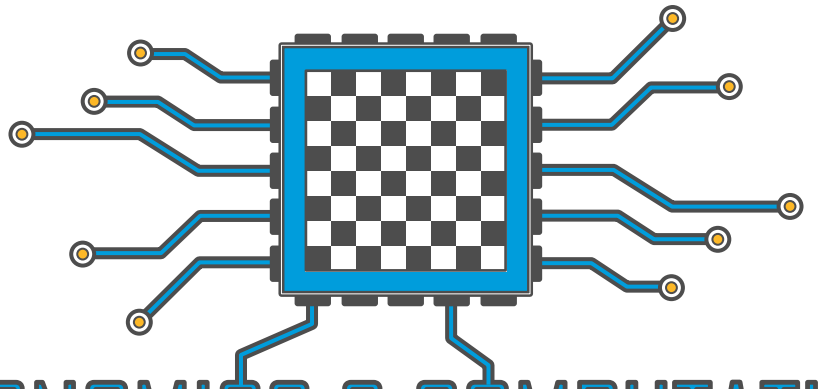


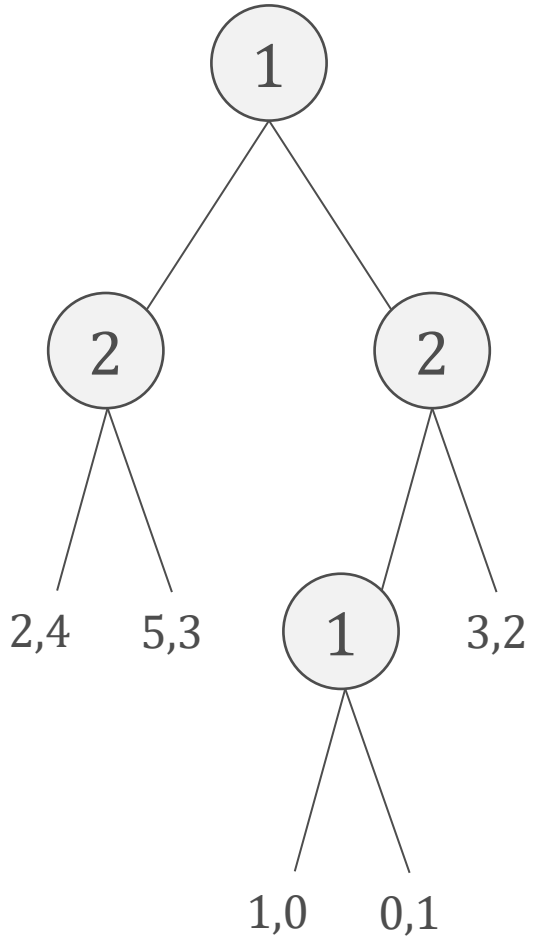

Spring 2026 | Lecture 3

# Extensive-Form Games

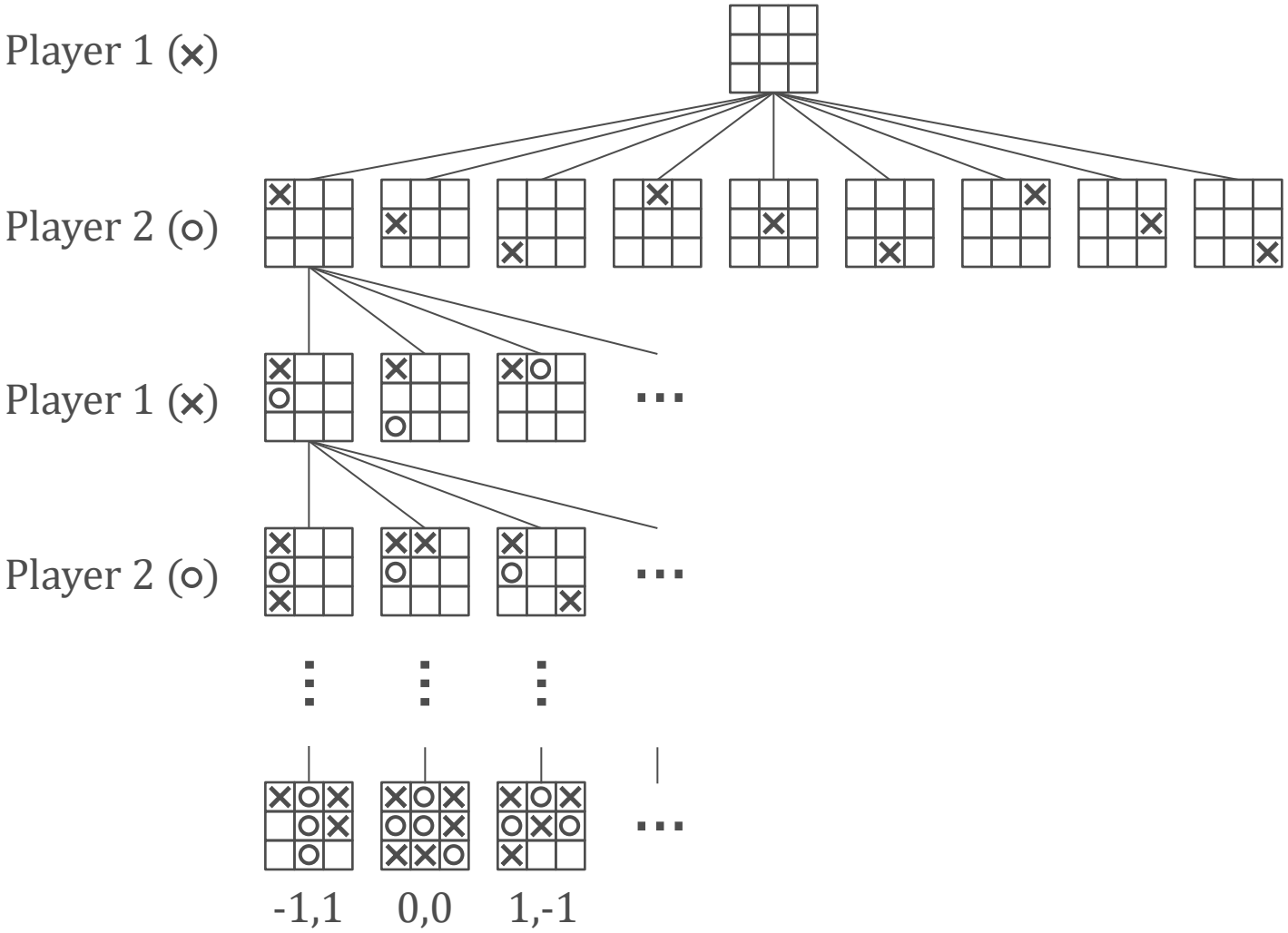
Ariel Procaccia | Harvard University

# EXTENSIVE-FORM GAMES

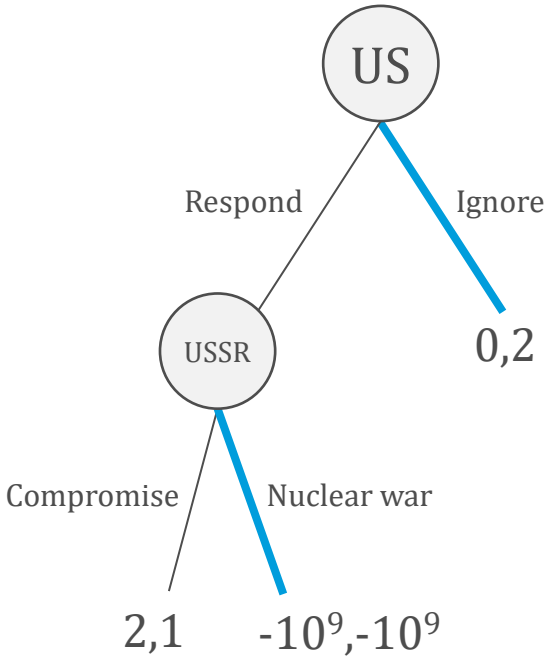
- Moves are done sequentially, not simultaneously
- Game forms a tree
- Nodes are labeled by players
- Leaves show payoffs



# EXAMPLE: TIC-TAC-TOE



# EXTENSIVE VS. NORMAL FORM

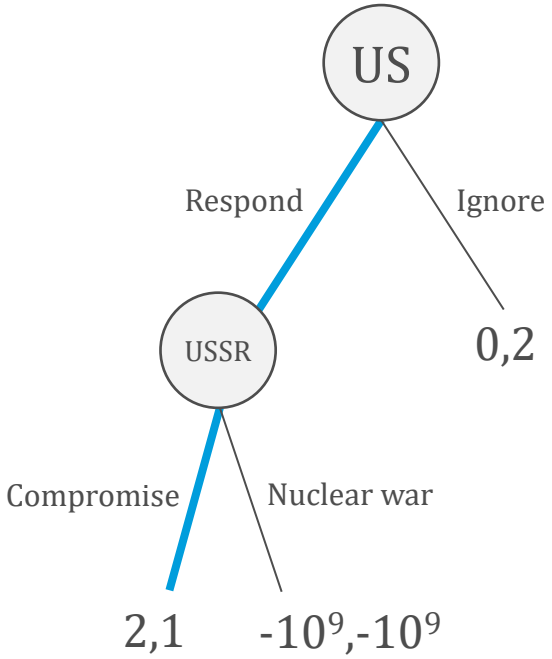



| | Compromise | Nuclear war |
|---|---|---|
| Respond | 2,1 | $-10^9,-10^9$ |
| Ignore | 0,2 | 0,2 |

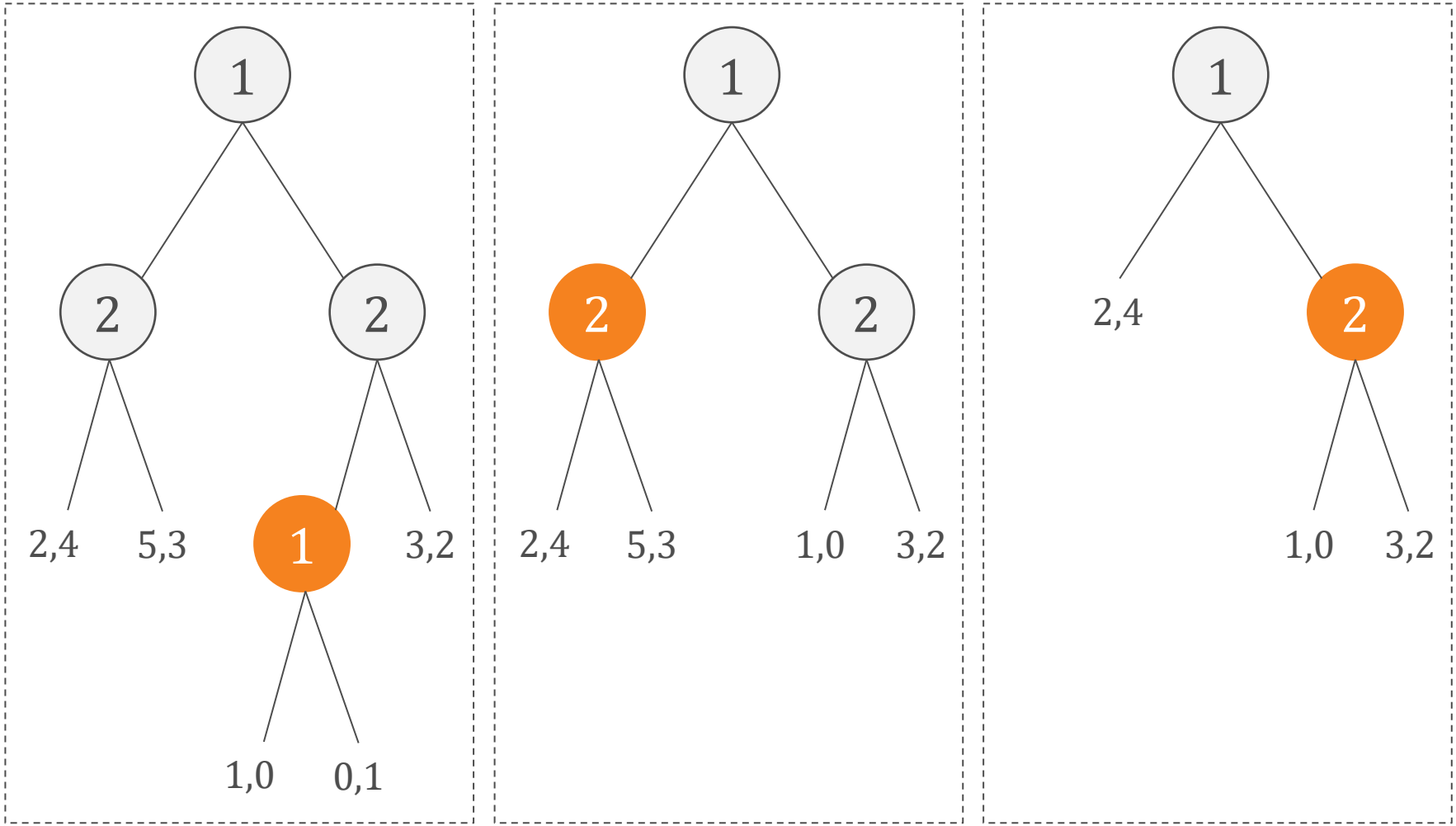
Problem: (ignore, nuclear war) is a Nash equilibrium, but threat isn't credible!

# SUBGAME-PERFECT EQUILIBRIUM

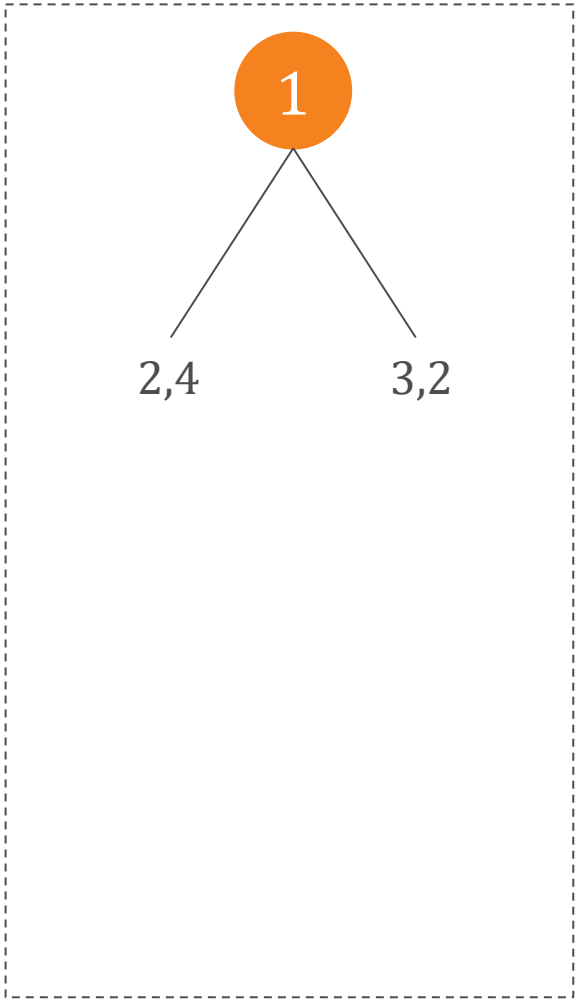
- Each subtree forms a subgame
- A set of strategies is a subgame-perfect equilibrium if it is a Nash equilibrium in each subgame
- Players may be able to improve their equilibrium payoff by eliminating strategies!



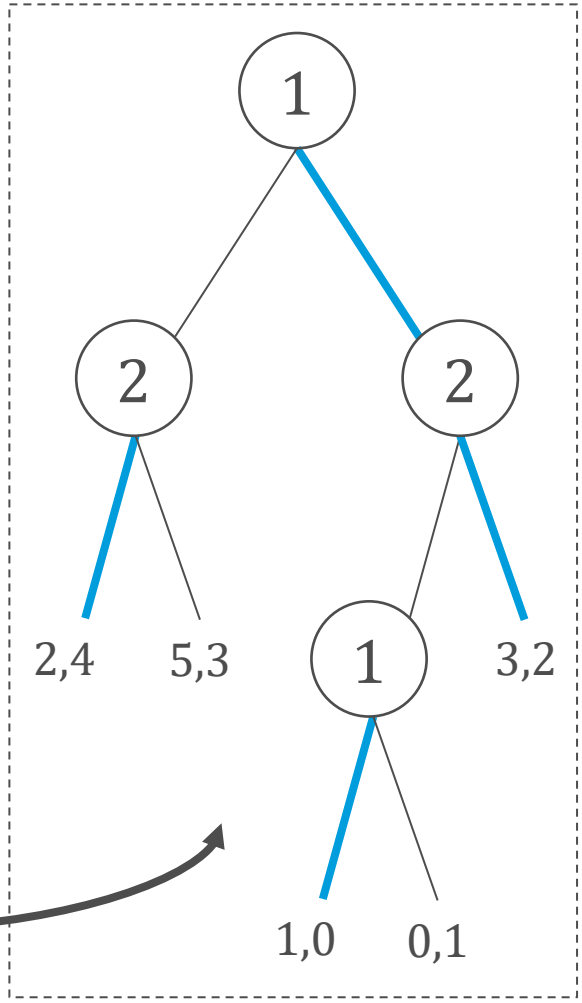
# BACKWARD INDUCTION



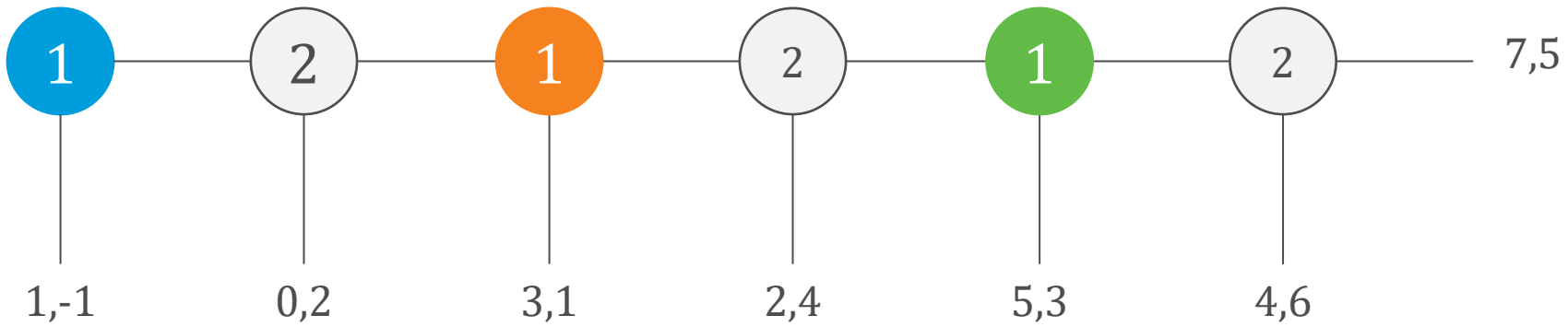
# BACKWARD INDUCTION







# EXAMPLE: CENTIPEDE GAME




## Poll 1

Suppose you are player 1 and you're playing with a random classmate. At which point do you choose down?

○ Blue ○ Orange ○ Green ○ None

# STACKELBERG GAMES

| | |
|---|---|
| 1,1 | 3,0 |
| 0,0 | 2,1 |

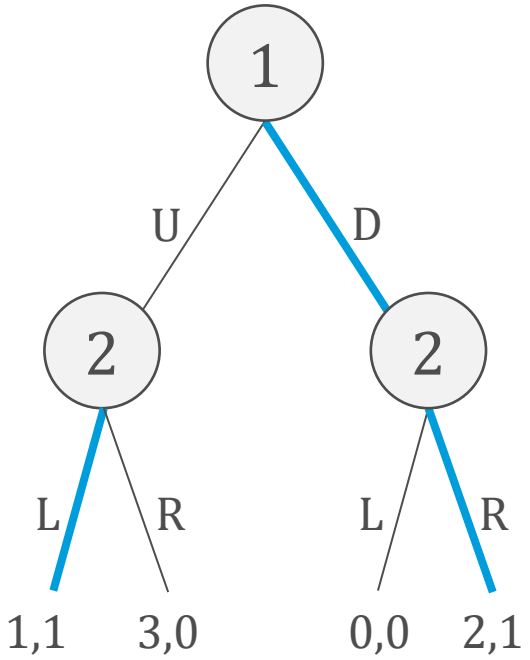
- Playing up is a dominant strategy for row player
- So column player would play left
- Therefore, (1,1) is the only Nash equilibrium outcome

# STACKELBERG GAMES

| | |
|---|---|
| 1,1 | 3,0 |
| 0,0 | 2,1 |

- A Stackelberg game is played as follows:
  - Row player (the leader) commits to playing a row
  - Column player (the follower) observes the commitment and chooses column
- The leader can commit to playing down!

# STACKELBERG GAMES




If the leader announces their commitment, the Stackelberg game can be rewritten as an extensive-form game (of perfect information)

# STACKELBERG GAMES

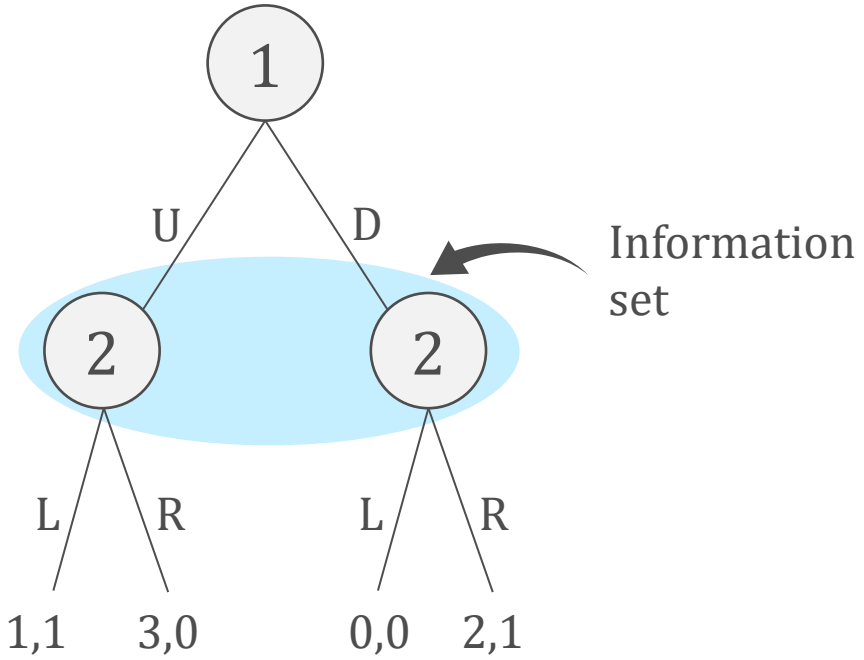
| | |
|---|---|
| 1,1 | 3,0 |
| 0,0 | 2,1 |

## Poll 2

What reward can the leader get by committing to a mixed strategy? (Assume the follower breaks ties in favor of the leader)

○ 1 ○ 1.5 ○ 2 ○ 2.5

# STACKELBERG GAMES




Randomness helps the leader due to imperfect information

# STACKELBERG EQUILIBRIUM

- For a mixed strategy $x_1$ of the leader, define the best response set of the follower as

$$B_2(x_1) = \text{argmax}_{s_2 \in S}\, u_2(x_1, s_2)$$

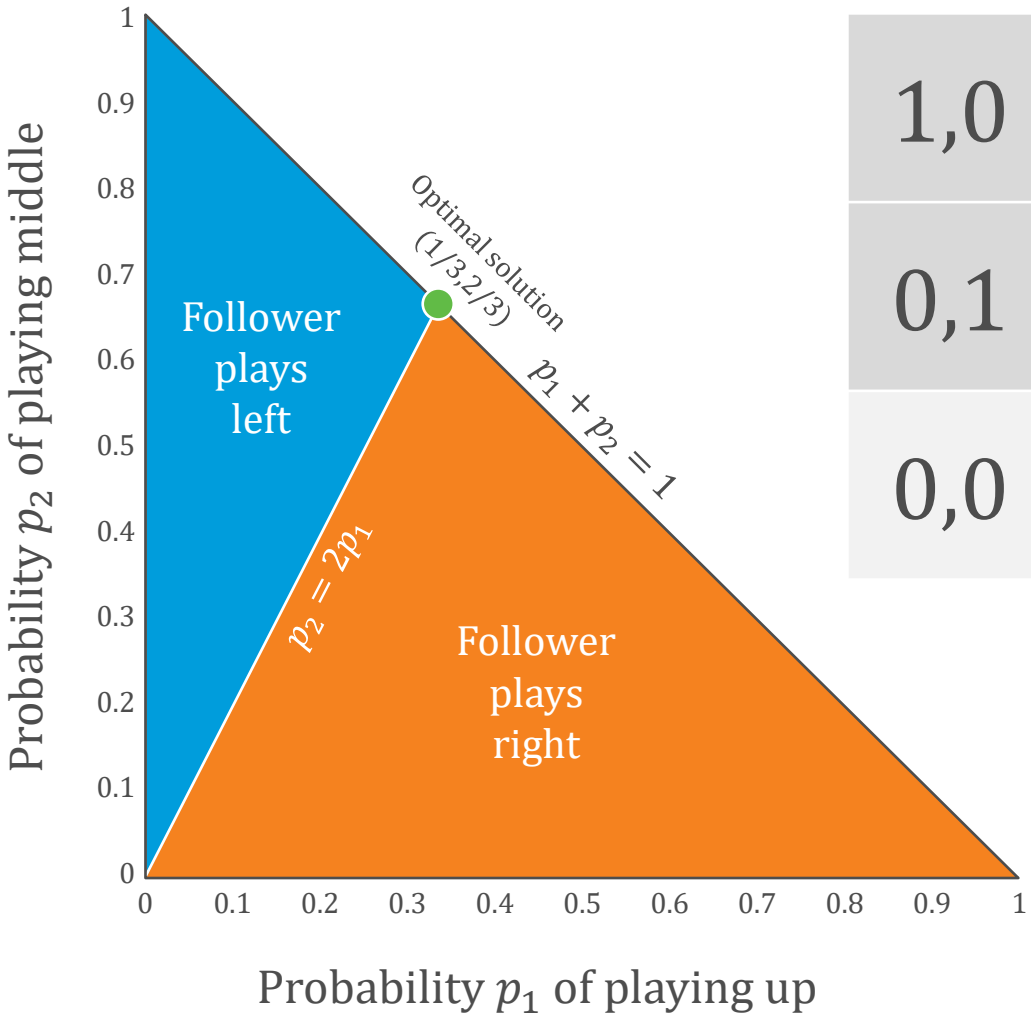
- In a strong Stackelberg equilibrium (SSE), the leader plays a mixed strategy in

$$\text{argmax}_{x_1 \in \Delta(S)} \max_{s_2 \in B_2(x_1)} u_1(x_1, s_2),$$

where $\Delta(S)$ is the set of mixed strategies

- We'll next see that an SSE can be computed via linear programming

# COMPUTING SSE: EXAMPLE

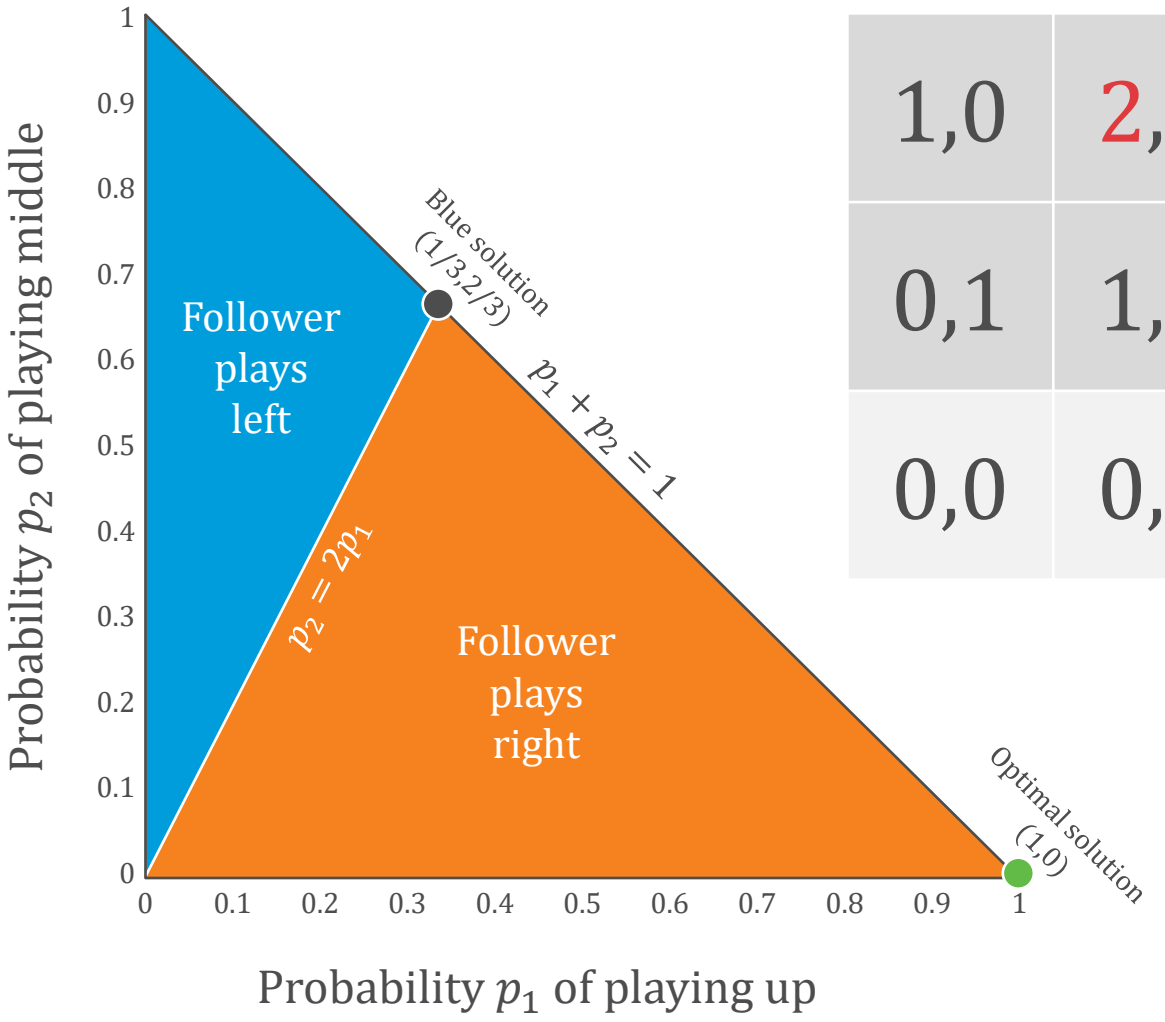



| | |
|---|---|
| 1,0 | 0,2 |
| 0,1 | 1,0 |
| 0,0 | 0,0 |

$$\max p_1$$
$$\text{s.t.} \quad p_2 \geq 2p_1$$
$$p_1 + p_2 \leq 1$$
$$p_1, p_2 \geq 0$$

$$\max p_2$$
$$\text{s.t.} \quad p_2 \leq 2p_1$$
$$p_1 + p_2 \leq 1$$
$$p_1, p_2 \geq 0$$

# COMPUTING SSE: EXAMPLE




| | |
|---|---|
| 1,0 | 2,2 |
| 0,1 | 1,0 |
| 0,0 | 0,0 |

$$\max p_1$$
$$\text{s.t.} \quad p_2 \geq 2p_1$$
$$p_1 + p_2 \leq 1$$
$$p_1, p_2 \geq 0$$

$$\max 2p_1 + p_2$$
$$\text{s.t.} \quad p_2 \leq 2p_1$$
$$p_1 + p_2 \leq 1$$
$$p_1, p_2 \geq 0$$

# COMPUTING SSE: ALGORITHM

- The leader's mixed strategy is defined by variables $x(s_1)$, which give the probability of playing each strategy $s_1 \in S$
- For each follower strategy $s_2^\star$, we compute a strategy $x$ for the leader such that
  - Playing $s_2^\star$ is a best response for the follower
  - Under this constraint, $x$ is optimal

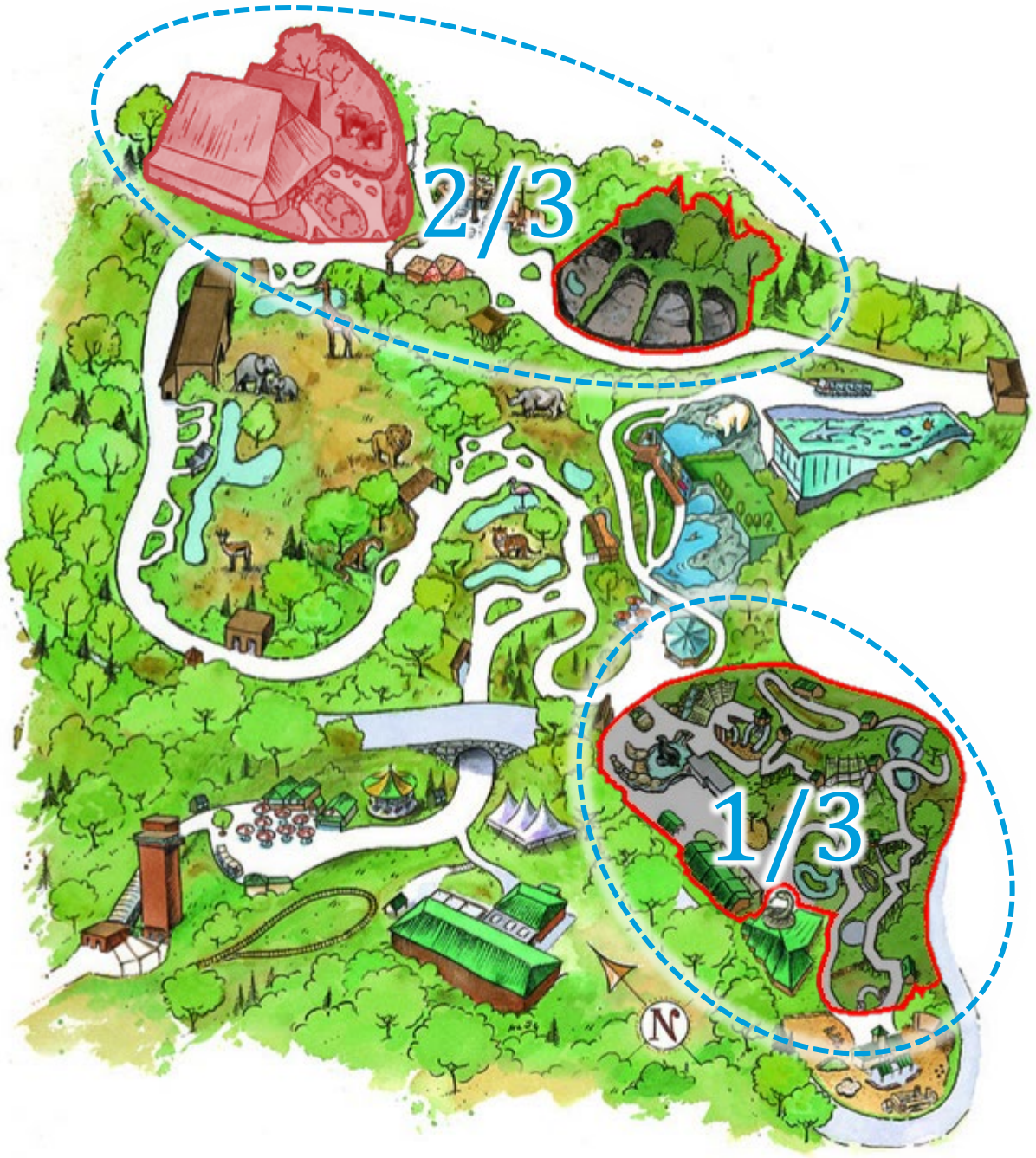
$$\max \sum_{s_1 \in S} x(s_1) u_1(s_1, s_2^\star)$$

$$\text{s.t} \quad \forall s_2 \in S, \sum_{s_1 \in S} x(s_1) u_2(s_1, s_2^\star) \geq \sum_{s_1 \in S} x(s_1) u_2(s_1, s_2)$$

$$\sum_{s_1 \in S} x(s_1) = 1 \qquad \forall s_1 \in S, x(s_1) \geq 0$$

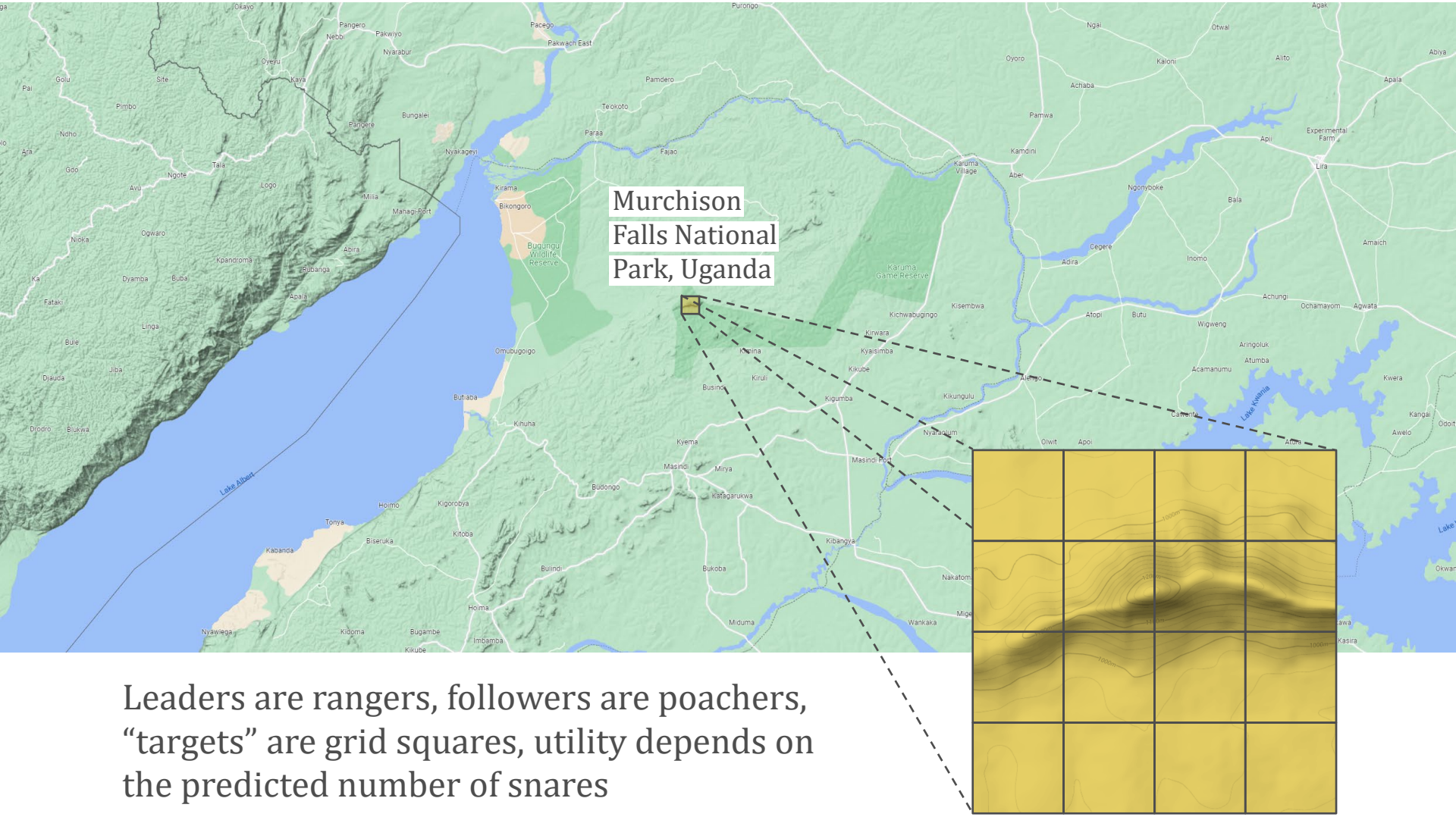
- Take the $x$ resulting from the "best" $s_2^\star$


0
1
N


2/3
1/3
N

# SECURITY GAMES

# WILDLIFE PROTECTION

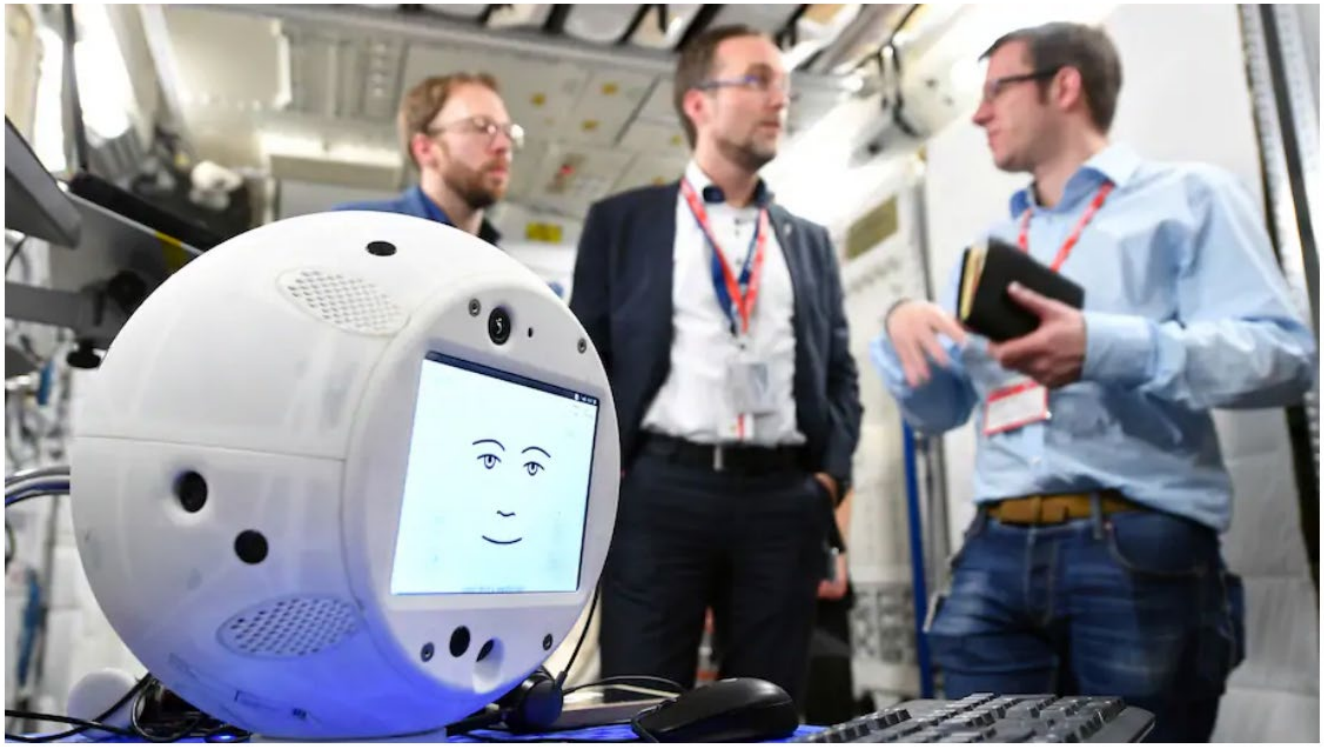



Leaders are rangers, followers are poachers, “targets” are grid squares, utility depends on the predicted number of snares

The Washington Post
Democracy Dies in Darkness

Opinions

# It's time for AI to outgrow gaming



The Cimon (Crew Interactive MObile companioN) robot is shown during a communications test at the ESA European Astronaut Center in Cologne-Porz, Germany, on Jan. 30, 2018. (T. Bourry/AP)

Opinion by **Ariel Procaccia**

March 20, 2020 at 8:52 a.m. EDT