ECONOMICS & COMPUTATION

Spring 2026 | Lecture 1

Nash Equilibrium

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NORMAL-FORM GAME

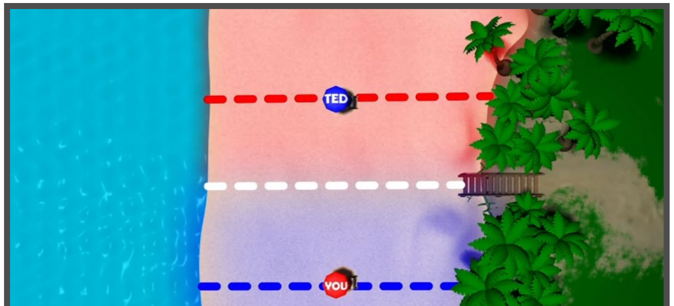
- A game in normal form consists of:
  - Set of players $N = \{1, \dots, n\}$
  - Strategy set S
  - For each $i \in N$, utility function $u_i: S^n \to \mathbb{R}$, which gives the utility of player i, $u_i(s_1, \dots, s_n)$, when each $j \in N$ plays the strategy $s_j \in S$
- Next example created by taking screenshots of `http://youtu.be/jILgxeNBK_8`

THE ICE CREAM WARS

Selling ice cream at the beach.


One day your cousin Ted shows up.

His ice cream is identical!

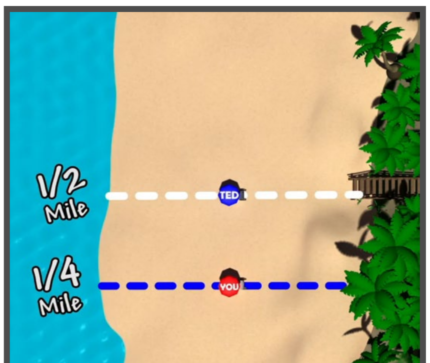


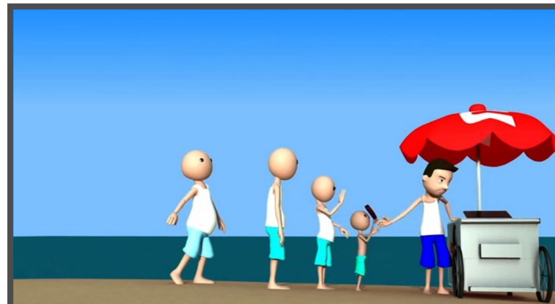
You split the beach in half; you set up at 1/4.


50% of the customers buy from you.


50% buy from Teddy.

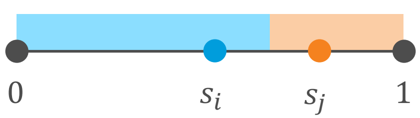

One day Teddy sets up at the 1/2 point!



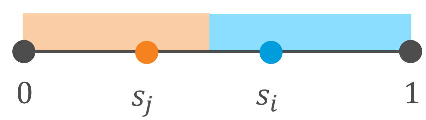

Now you serve only 37.5%!

THE ICE CREAM WARS

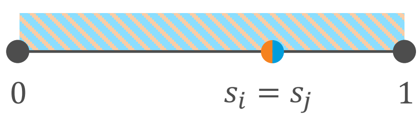
$N = \{1,2\}, S = [0,1],$ and $u_i(s_i, s_j)$ is defined as follows:



$\frac{s_i+s_j}{2}$ if $s_i < s_j$



$1 - \frac{s_i+s_j}{2}$ if $s_i > s_j$



0.5 if $s_i = s_j$

THE PRISONER'S DILEMMA

Reshef Meir • 1st
Associate Prof. at Technion | Game Theory | Social Choice
1w •

Release the prisoners from the Prisoners' Dilemma!

Many of you have heard about the Prisoners' Dilemma, perhaps the most famous example of game theory. So famous, that no one including me can properly remember the contrived story, or explain in a simple way how it relates to the payoff matrix why it even matters. No more.

There is exactly one correct way to present the Dilemma to people, be it colleagues, students, or your Mom:
- You get to choose between getting $100, and some other person that you don't know and will never meet getting $300.
- The other person gets the same choice.

This version is due to Prof. Aumann. That's it. No prisoners. No complex reasoning.
No over-thinking what each player should do.
The dominant strategy is immediate. So is the social optimum.
Generalization to n>2 (Tragedy of the Commons) is obvious.

THE SUCKER'S DILEMMA

	Cooperate	Defect
Cooperate	300,300	0,400
Defect	400,0	100,100

What would you do?

UNDERSTANDING THE DILEMMA

- Defection is a dominant strategy
- But the players can do much better by cooperating
- Related to the tragedy of the commons

THE TRAGEDY OF THE COMMONS

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Opinion
Ariel Procaccia

Tech Giants, Gorging on AI Professors Is Bad for You

If industry keeps hiring the cutting-edge scholars, who will train the next generation of innovators in artificial intelligence?

Eat too much and there won't be grass for anyone. *Photographer: William West/AFP/Getty Images*

By Ariel Procaccia
January 7, 2019 at 6:00 AM EST

THE PRISONER'S DILEMMA ON TV

http://youtu.be/S0qjK3TWZE8

THE PROFESSOR'S DILEMMA

		Class: Listen	Class: Sleep
Professor	Make effort	$10^6,10^6$	-10,0
	Slack off	0,-10	0,0

Dominant strategies?

John Forbes Nash

1928–2015

Mathematician and Nobel laureate in economics. Also remembered as the protagonist in “A Beautiful Mind.”

NASH EQUILIBRIUM

- In a Nash equilibrium, no player wants to unilaterally deviate
- Each player's strategy is a best response to strategies of others
- Formally, a Nash equilibrium is a vector of strategies $\boldsymbol{s} = (s_1 \dots, s_n) \in S^n$ such that for all $i \in N, s_i' \in S$,

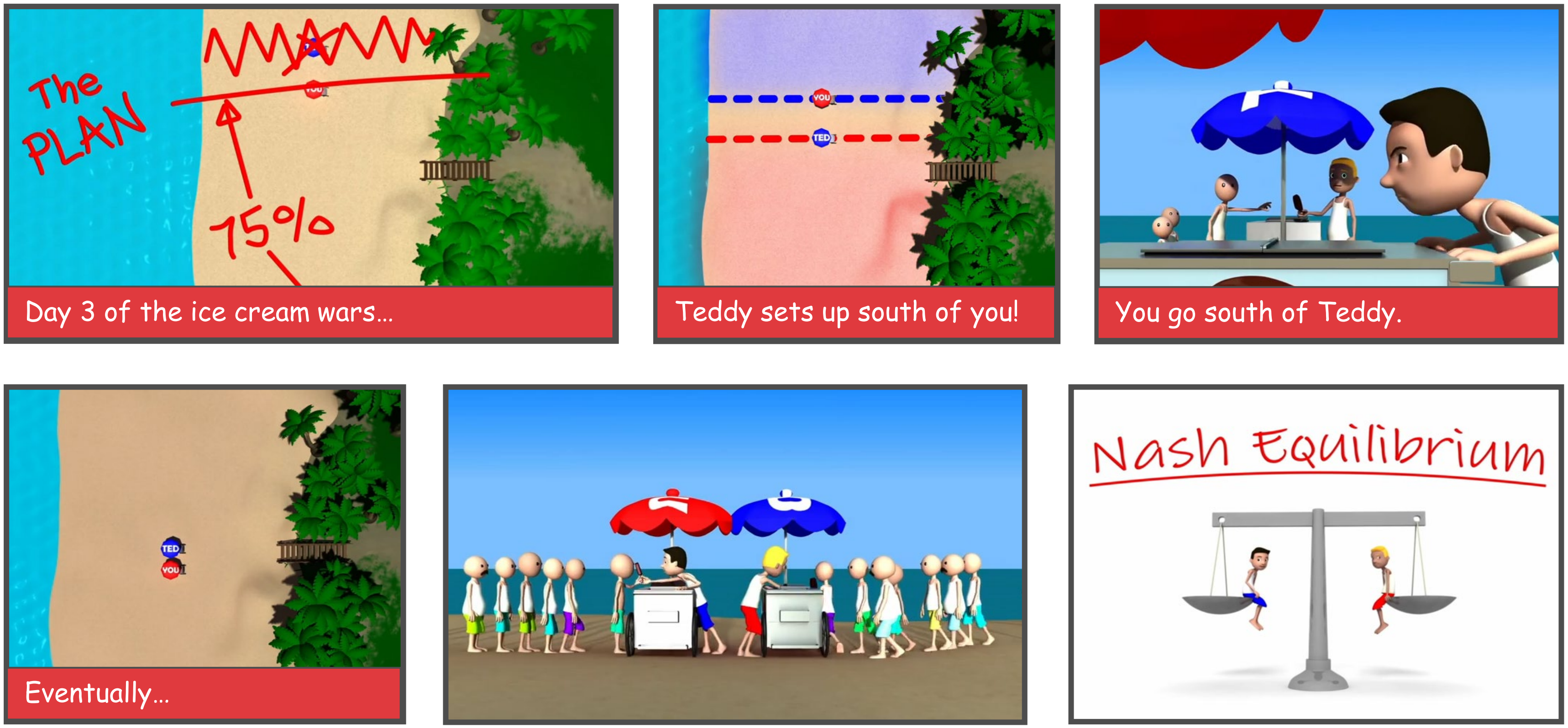
$$u_i(\boldsymbol{s}) \geq u_i(s_i', \boldsymbol{s}_{-i})$$

THE PROFESSOR'S DILEMMA

Professor \ Class	Listen	Sleep
Make effort	$10^6, 10^6$	-10,0
Slack off	0,-10	0,0

Nash equilibria?

END OF THE ICE CREAM WARS



ROCK-PAPER-SCISSORS

			
	0,0	-1,1	1,-1
	1,-1	0,0	-1,1
	-1,1	1,-1	0,0

Nash equilibria?

MIXED STRATEGIES

- A mixed strategy is a probability distribution over (pure) strategies
- The mixed strategy of player $i \in N$ is x_i, where

$$x_i(s_i) \;= \Pr[i \text{ plays } s_i]$$

- The utility of player $i \in N$ is

$$u_i(x_1, \dots, x_n) = \sum_{(s_1,\dots,s_n)\in S^n} u_i(s_1, \dots, s_n) \cdot \prod_{j=1}^{n} x_j(s_j)$$

EXERCISE: MIXED NE

- **Exercise:** player 1 plays $\left(\frac{1}{2}, \frac{1}{2}, 0\right)$, player 2 plays $\left(0, \frac{1}{2}, \frac{1}{2}\right)$. What is u_1?
- **Exercise:** Both players play $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$. What is u_1?

			
	0,0	-1,1	1,-1
	1,-1	0,0	-1,1
	-1,1	1,-1	0,0

EXERCISE: MIXED NE

Poll 1

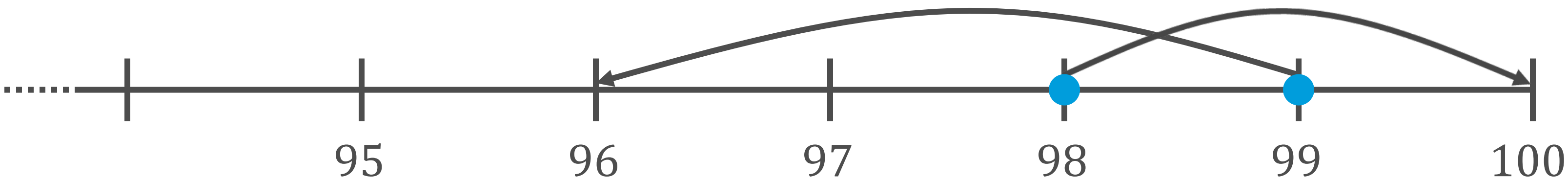
Which is a NE?

- $\left(\left(\frac{1}{2},\frac{1}{2},0\right),\left(\frac{1}{2},\frac{1}{2},0\right)\right)$
- $\left(\left(\frac{1}{2},\frac{1}{2},0\right),\left(\frac{1}{2},0,\frac{1}{2}\right)\right)$
- $\left(\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right),\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)\right)$
- $\left(\left(\frac{1}{3},\frac{2}{3},0\right),\left(\frac{2}{3},0,\frac{1}{3}\right)\right)$

			
	0,0	-1,1	1,-1
	1,-1	0,0	-1,1
	-1,1	1,-1	0,0

Theorem [Nash, 1950]: In any (finite) game there exists at least one (possibly mixed) Nash equilibrium

CAVEAT: NE PREDICTS OUTCOMES?



Two players, strategies are $\{2, \dots, 100\}$. If both choose the same number, that is what they get. If one chooses s, the other t, and $s < t$, the former player gets $s + 2$, and the latter gets $s - 2$.

Poll 2

Suppose you are paired with another random student, and you must play this game with them (for real money) without communicating. What would you choose?