Economics and Computation (Spring 2026)
Assignment #5

Due: 4/28/2026 11:59pm ET

Problem 1: Random assignment

[**20 points**] A random assignment P is *envy free* if for all $i, j \in N$ and $x \in G$,

$$\sum_{y \succeq_{\sigma_i} x} p_{iy} \geq \sum_{y \succeq_{\sigma_i} x} p_{jy}.$$

Prove that the Probabilistic Serial Mechanism produces an envy-free random assignment.

Problem 2: Cascade models

[**15 points**] In Lecture 17 we discussed the coordination game. Consider a similar game, called the *local public goods game*, which is defined using the notation used in Slide 3. The possible actions are again $a_i \in \{0, 1\}$, but here 1 corresponds to investing in a good that is useful to the neighbors of i, and 0 corresponds to not investing. The utility of i is

$$u_i(\mathbf{a}) = \begin{cases} 1 - c & a_i = 1 \\ 1 & a_i = 0 \text{ and } n_{i,1}(\mathbf{a}_{-i}) \geq 1 \\ 0 & \text{otherwise} \end{cases}$$

That is, i gets a payoff of 1 if at least one player in their neighborhood (including themselves) invests, but investment has a cost of $c \in (0, 1)$.

Design a polynomial-time algorithm that computes a pure Nash equilibrium in a given local public goods game.

Problem 3: Influence maximization

[**20 pts**] Given an undirected graph $G = (V, E)$, define the following set function over subsets $S \subseteq V$:

$$f(S) = |\{(u, v) \in E : \ u \in S, v \notin S\}|.$$

Is f monotone? Is it submodular? Prove or disprove each property.

Problem 4: No-regret learning

[20 pts] Consider the analysis of (deterministic) weighted majority in Lecture 19, Slides 11–14. Assume that there is a perfect expert that never makes mistakes. Show that there is a value of ϵ such that the modified weighted majority algorithm (Slide 14) makes at most $\log_2 n$ mistakes, where n is the number of experts.

Problem 5: Feature attribution

[25 points] The Shapley value is hard to compute, but it is easy to estimate accurately using a Monte Carlo algorithm. Specifically, given a player i whose Shapley value we wish to estimate, consider the following algorithm: For $t = 1, \ldots, m$, sample a random permutation π_t and compute $v(S^i_{\pi_t} \cup \{i\}) - v(S^i_{\pi_t})$; then return the marginal contribution of i averaged across the m samples.

Assume that $v(S) \in [0, 1]$ for all $S \subseteq N$ and v is monotonic. Show that, given $\epsilon, \delta > 0$ and $m = O(\ln(1/\delta)/\epsilon^2)$, the above algorithm outputs an estimate $\hat{\sigma}_i$ of the Shapley value of i such that $|\sigma_i - \hat{\sigma}_i| < \epsilon$ with probability at least $1 - \delta$.

Guidance: Each sample is a random variable; what can you say about their expectations? Plug these random variables into Hoeffding's Inequality: Let $X_1, \ldots, X_m$ be i.i.d. random variables bounded in $[0, 1]$ with $\mathbb{E}[X_j] = \mu$ for $j = 1, \ldots, m$, then

$$\Pr\left[\left|\frac{1}{m}\sum_{t=1}^{m} X_t - \mu\right| \geq \epsilon\right] \leq 2 \cdot e^{-2m\epsilon^2}.$$