Economics and Computation (Spring 2026)
Assignment #1

Due: 2/12/2026 11:59pm ET

Problem 1: Nash equilibrium

Recall the ice cream wars setting from Lecture 1. Suppose that there are n ice cream sellers on a beach which can be modeled as the interval $[0,1]$ and that each ice cream seller chooses a real number within that interval where they will set up their cart to sell ice cream. The ice cream being sold from all carts is identical, that is, customers are indifferent about which cart's ice cream they buy and will always buy from the seller closest to them. Customers are uniformly distributed along the beach (assume distance only matters to customers in the horizontal direction along the beach's length, the width of the beach is negligible). We saw that when $n = 2$, there is a Nash equilibrium in pure strategies where both sellers are located at $1/2$.

1. [**15 points**] Prove that when $n = 3$, a Nash equilibrium in pure strategies does not exist.
2. [**15 points**] Prove that when n is even, a Nash equilibrium in pure strategies exists.

Problem 2: Equilibrium computation

In a normal-form game, we say that strategy s_i is *strictly dominated* for player i if there exists a mixed strategy x_i of player i such that $u_i(x_i, \mathbf{s}_{-i}) > u_i(\mathbf{s})$ for all pure strategy profiles $\mathbf{s}_{-i}$ of the other players.

For example, in the following game, M is dominated by $(0.5, 0, 0.5)$:

U	2,0	-1,0
M	0,0	0,0
D	-1,0	2,0

A process of *iterated elimination of strictly dominated strategies* starts by identifying a strictly dominated strategy, if one exists. That strategy is then removed from the strategy set of the relevant player, and the process is repeated for the smaller game. The process terminates when strictly dominated strategies no longer exist.

1. [**25 points**] Prove that if $\mathbf{x}$ is a mixed strategy Nash equilibrium, and $x_i(s_i) > 0$, then s_i cannot be eliminated in a process of iterated elimination of strictly dominated strategies.

   Guidance: Use induction on the order of elimination. If s_i was eliminated, it is strictly dominated by a mixed strategy x_i'; in x_i, transfer the probability weight of s_i to x_i', thus obtaining a strictly better mixed strategy that violates the assumption that $\mathbf{x}$ is a Nash equilibrium.

2. **[15 points]** For the case of two players, design an algorithm that performs iterated elimination of strictly dominated strategies in time that is polynomial in the size of the strategy set.

   Hint: Use linear programming. You may assume that LPs can be solved in polynomial time and, although this is a bit messy, you may include strict inequalities in your LP.

Taken together, these two facts suggest that iterated elimination of dominated strategies can be a useful *preprocessing* step when attempting to compute a mixed Nash equilibrium.

Problem 3: Extensive-form games

1. **[15 points]** Consider a 2-player game in normal form. Let (x_1, x_2) be a (possibly mixed) Nash equilibrium. Show that if $(x_1^\star, s_2)$ is a strong Stackelberg equilibrium (where player 1 is the leader) then $u_1(x_1^\star, s_2) \geq u_1(x_1, x_2)$.

2. **[15 points]** Prove that in the game of chess, precisely one of the following holds: white has a strategy that guarantees a win no matter how black plays, black has a strategy that guarantees a win no matter how white plays, or each player has a strategy that guarantees a tie no matter how the other plays.

   Hint: Backward induction.