CS 1360 Spring 2025
Final Exam

Name: ______________________________

Harvard ID: __________________________

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Problem 1: Social Choice

1. **[5 pts]** Describe in words the concepts *Condorcet winner* and *Condorcet consistency*.

2. **[15 pts]** Define the *score* of an alternative $x \in A$ in a preference profile $\boldsymbol{\sigma}$ to be $\min_{y \in A \setminus \{x\}} |\{i \in N : x \succ_{\sigma_i} y\}|$. In words, the score of an alternative is the result of its worst head-to-head comparison with another alternative. The *maximin rule* chooses an alternative with maximum score (breaking ties if needed). Prove that the maximin rule is Condorcet consistent.

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Problem 2: The VCG Mechanism

1. **[5 pts]** Define (using mathematical notation) the *VCG Mechanism*.

2. **[15 pts]** Prove that under the VCG Mechanism, when $A = A^{-i}$ for all players $i \in N$, the payment of each player is always non-negative.

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Problem 3: Rent Division

1. **[5 pts]** In the context of rent division with quasi-linear utilities, define (using mathematical notation) the concept of *envy-free solution* $(\pi, \mathbf{p})$.

2. **[15 pts]** Prove that if $(\pi, \mathbf{p})$ is an envy-free solution, then π is a welfare-maximizing assignment.

   Note: This was done in class.

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Problem 4: Stable Matching

1. **[5 pts]** Define (using mathematical notation) the concept of *student-optimal* stable matching in bipartite matching with two-sided preferences. (There is no need to define stable matching.)

2. **[15 pts]** Prove that a given instance of bipartite matching with two-sided preferences has a unique stable matching if and only if the student-proposing deferred acceptance algorithm and the course-proposing deferred acceptance algorithm output the same matching.

   Note: Prove both directions. You may rely on any result stated in class.

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Problem 5: Submodular Functions

1. **[5 pts]** Define (using mathematical notation) the concept of a *submodular* set function.

2. **[15 pts]** For $B \geq 0$, $w_1, \ldots, w_n \geq 0$, and $S \subseteq \{1, \ldots, n\}$, define the following two functions:

   - $f_1(S) = \min\{B, \sum_{i \in S} w_i\}$.
   - $f_2(S) = \max\{B, \sum_{i \in S} w_i\}$.

   Which of these two functions is submodular? For each, prove submodularity or give a counterexample.

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