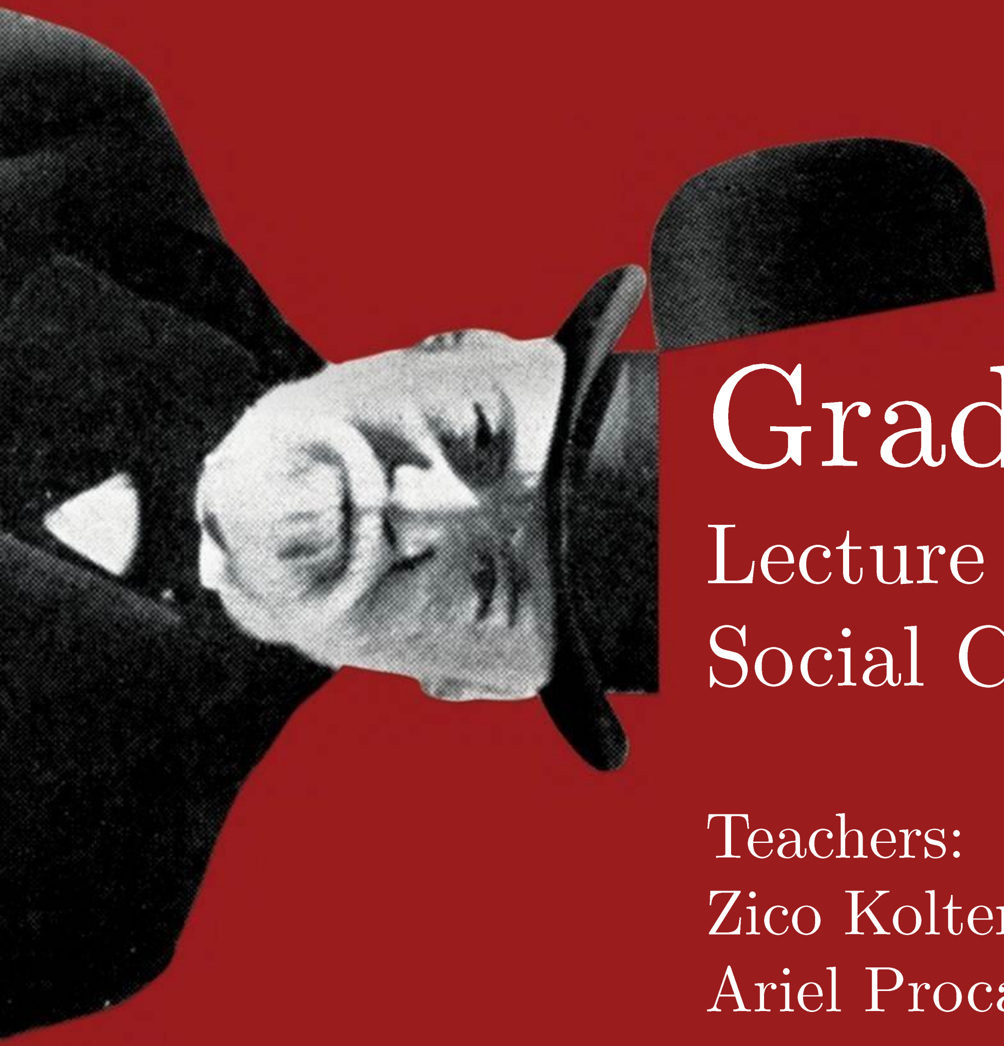


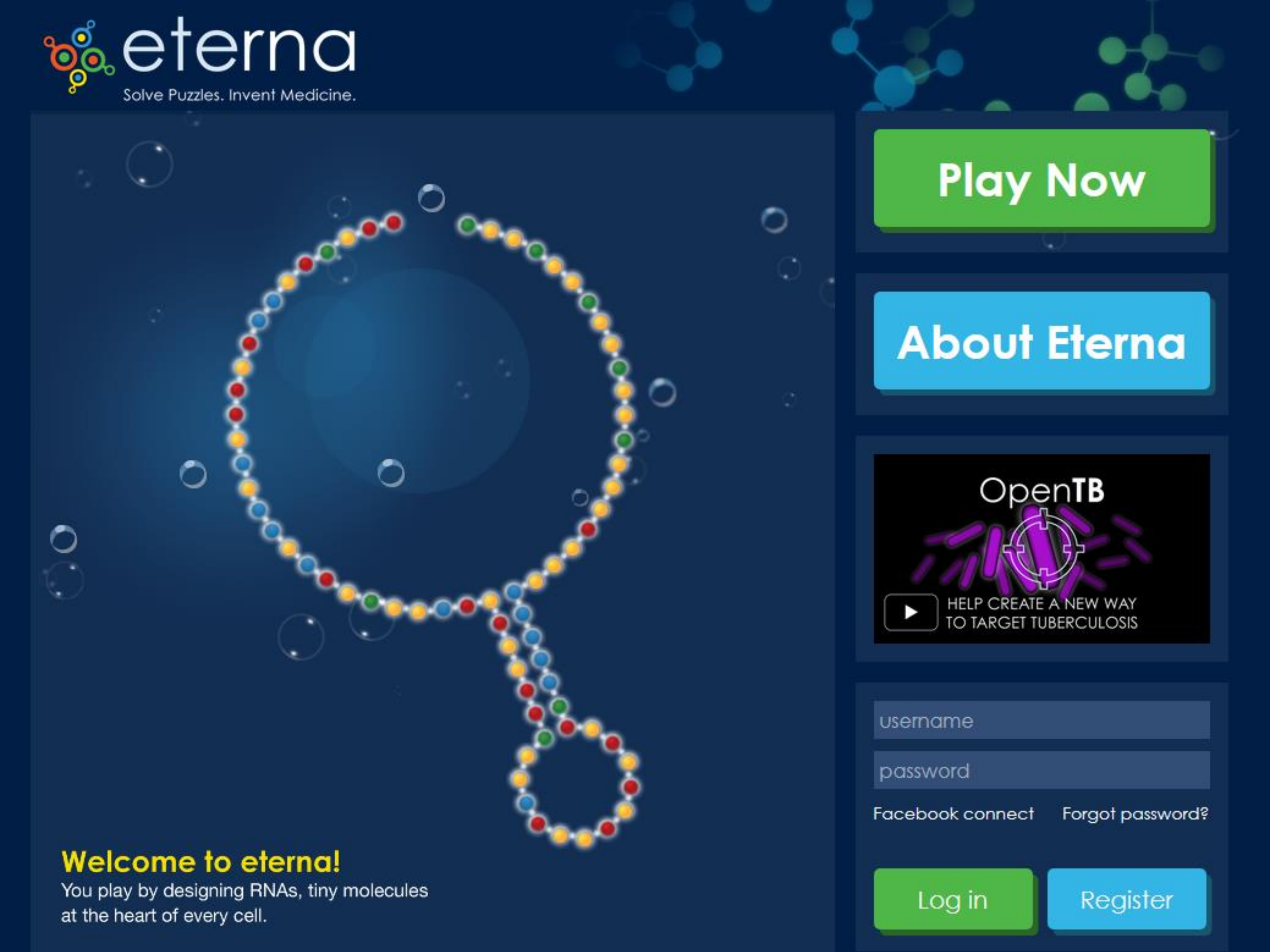
Graduate AI

Lecture 25: Social Choice III

Teachers:
Zico Kolter
Ariel Procaccia (this time)

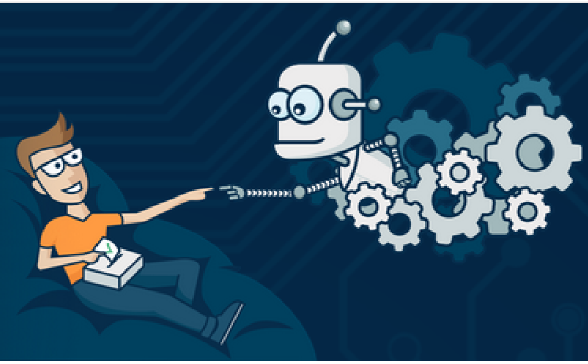
Condorcet strikes again

- For Condorcet [1785], the purpose of voting is not merely to balance subjective opinions; it is a collective quest for the truth
- Enlightened voters try to judge which alternative best serves society
- For $m = 2$ the majority opinion will very likely be correct
- Realistic in trials by jury, but also in the pooling of expert opinions, or in human computation!


eterna
Solve Puzzles. Invent Medicine.
Play Now
About Eterna
OpenTB
HELP CREATE A NEW WAY
TO TARGET TUBERCULOSIS
username
password
Facebook connect
Forgot password?
Log in
Register
Welcome to eterna!
You play by designing RNAs, tiny molecules
at the heart of every cell.

AI-Driven Decisions

RoboVote is a free service that helps users combine their preferences or opinions into optimal decisions. To do so, RoboVote employs state-of-the-art voting methods developed in artificial intelligence research. Learn More



Poll Types

RoboVote offers two types of polls, which are tailored to different scenarios; it is up to users to indicate to RoboVote which scenario best fits the problem at hand.

Objective Opinions

In this scenario, some alternatives are objectively better than others, and the opinion of a participant reflects an attempt to estimate the correct order. RoboVote's proposed outcome is guaranteed to be as close as possible — based on the available information — to the best outcome. Examples include deciding which product prototype to develop, or which company to invest in, based on a metric such as projected revenue or market share. Try the demo.

Subjective Preferences

In this scenario participants' preferences reflect their subjective taste; RoboVote proposes an outcome that mathematically makes participants as happy as possible overall. Common examples include deciding which restaurant or movie to go to as a group, which destination to choose for a family vacation, or whom to elect as class president. Try the demo.

Ready to get started?

CREATE A POLL

Condorcet's noise model

- True ranking of the alternatives
- Voting pairwise on alternatives, each comparison is correct with prob. $p > 1/2$
- Results are tallied in a voting matrix

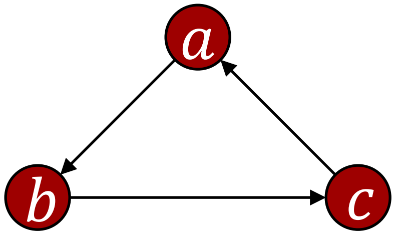
	a	b	c
a	-	8	6
b	5	-	11
c	7	2	-

- Poll 1: What is the Borda score of alternative b?
  1. 5
  2. 8
  3. 10
  4. 16

CONDORCET'S 'SOLUTION'

- Condorcet's goal: find "the most probable" ranking
- Condorcet suggested: take the majority opinion for each comparison; if a cycle forms, "successively delete the comparisons that have the least plurality"
- In example, we delete $c \succ a$ to get $a \succ b \succ c$

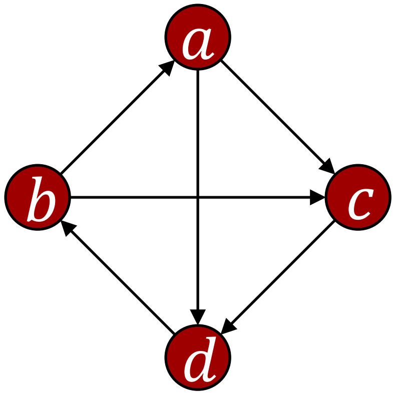
	a	b	c
a	-	8	6
b	5	-	11
c	7	2	-



Condorcet's 'solution'

- With four alternatives we get ambiguities
- In example, order of strength is $c \succ d$, $a \succ d$, $b \succ c$, $a \succ c$, $d \succ b$, $b \succ a$
- Delete $b \succ a \Rightarrow$ still cycle
- Delete $d \succ b \Rightarrow$ either a or b could be top-ranked

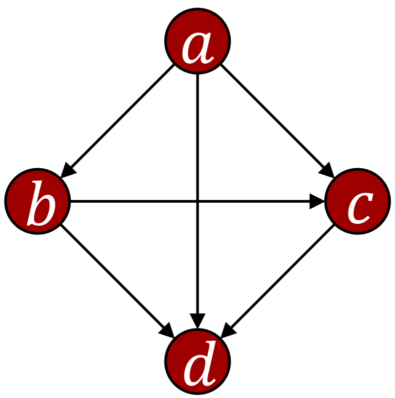
	a	b	c	d
a	-	12	15	17
b	13	-	16	11
c	10	9	-	18
d	8	14	7	-



Condorcet's 'solution'

- Did Condorcet mean we should reverse the weakest comparisons?
- Reverse $b \succ a$ and $d \succ b \Rightarrow$ we get $a \succ b \succ c \succ d$, with 89 votes
- $b \succ a \succ c \succ d$ has 90 votes (only reverse $d \succ b$)

	a	b	c	d
a	-	12	15	17
b	13	-	16	11
c	10	9	-	18
d	8	14	7	-



Exasperation?

- "The general rules for the case of any number of candidates as given by Condorcet are stated so briefly as to be hardly intelligible . . . and as no examples are given it is quite hopeless to find out what Condorcet meant" [Black 1958]
- "The obscurity and self-contradiction are without any parallel, so far as our experience of mathematical works extends …

Young's solution

- M = matrix of votes
- Suppose true ranking is $a \succ b \succ c$; prob of observations $\Pr[M \mid \succ]$:
  $\binom{13}{8} p^8(1-p)^5 \cdot \binom{13}{6} p^6(1-p)^7 \cdot \binom{13}{11} p^{11}(1-p)^2$
- For $a \succ c \succ b$, $\Pr[M \mid \succ]$ is
  $\binom{13}{8} p^8(1-p)^5 \cdot \binom{13}{6} p^6(1-p)^7 \cdot \binom{13}{2} p^2(1-p)^{11}$
- Coefficients are identical, so
  $\Pr[M \mid \succ] \propto p^{\#agree}(1-p)^{\#disagree}$

	a	b	c
a	-	8	6
b	5	-	11
c	7	2	-

Young's solution

- $\Pr[\succ \mid M] = \frac{\Pr[M|\succ] \cdot \Pr[\succ]}{\Pr[M]}$
- Assume uniform prior over $\succ$, $\Pr[\succ] = \frac{1}{m!}$
- Maximum a posteriori probability (MAP) estimate maximizes $\Pr[M \mid \succ]$
- The optimal rule maximizes #agreements with voters on pairs of alternatives
- This rule is called the Kemeny rule

The Kemeny rule

- The Kendall tau distance between $\succ$ and $\succ'$ is
  $d_{KT}(\succ, \succ') = |\{(a,b) \in A^2 \,|(a \succ b) \wedge (b \succ' a)|$
- The Kemeny rule chooses the ranking that minimizes the sum of Kendall tau distances to the preference profile
- Theorem [Bartholdi, Tovey, Trick 1989]: Computing the Kemeny ranking is NP-hard

The Kemeny rule

- Typically formulated as an IP: for every $a, b \in A$, $x_{(a,b)} = 1$ iff a is ranked above b, and

$$w_{(a,b)} = |\{i \in N \mid a \succ_i b\}|$$

Minimize $\sum_{(a,b)} x_{(a,b)} w_{(b,a)}$

Subject to

For all distinct $a, b \in A, x_{(a,b)} + x_{(b,a)} = 1$

For all distinct $a, b, c \in A, x_{(a,b)} + x_{(b,c)} + x_{(c,a)} \leq 2$

For all distinct $a, b \in A, x_{(a,b)} \in \{0,1\}$

The Mallows model

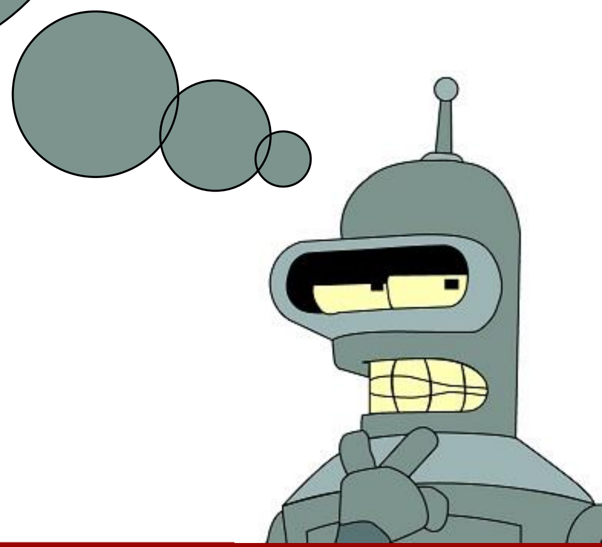
- Same as Condorcet's model, but votes are rankings
- Defined by parameter $\phi \in (0,1]$
- Probability of a voter casting the vote $\succ'$ given true ranking $\succ$ is

$$\Pr[\succ' \mid \succ] = \frac{\phi^{d_{KT}(\succ',\succ)}}{\sum_{\succ''} \phi^{d_{KT}(\succ'',\succ)}}$$

- Kemeny still gives the MAP ranking

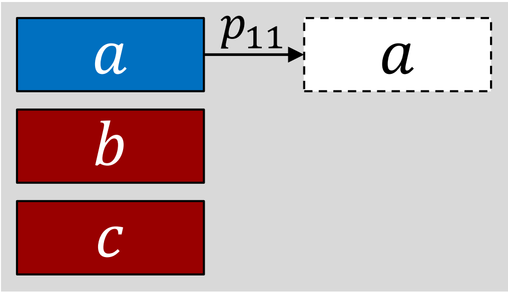
$$\Pr[\succ' \mid \succ] = \frac{\phi^{d_{KT}(\succ',\succ)}}{\sum_{\succ''} \phi^{d_{KT}(\succ'',\succ)}}$$

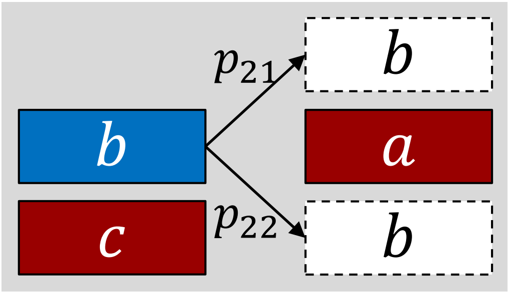
What is the relation between ϕ in the Mallows model, and p in the Condorcet model?

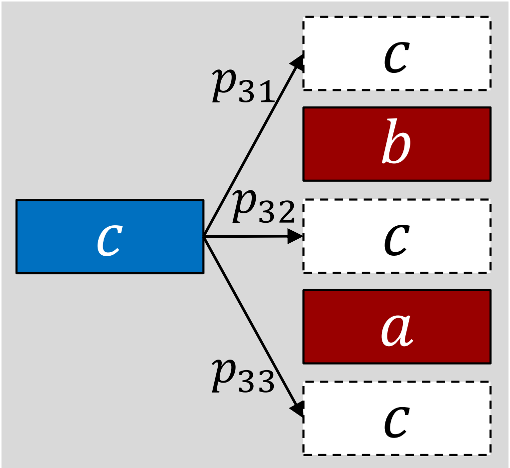


The Mallows model

- How can we sample a vote?
- Suppose the true ranking is $a \succ b \succ c$
- Repeated insertion model:



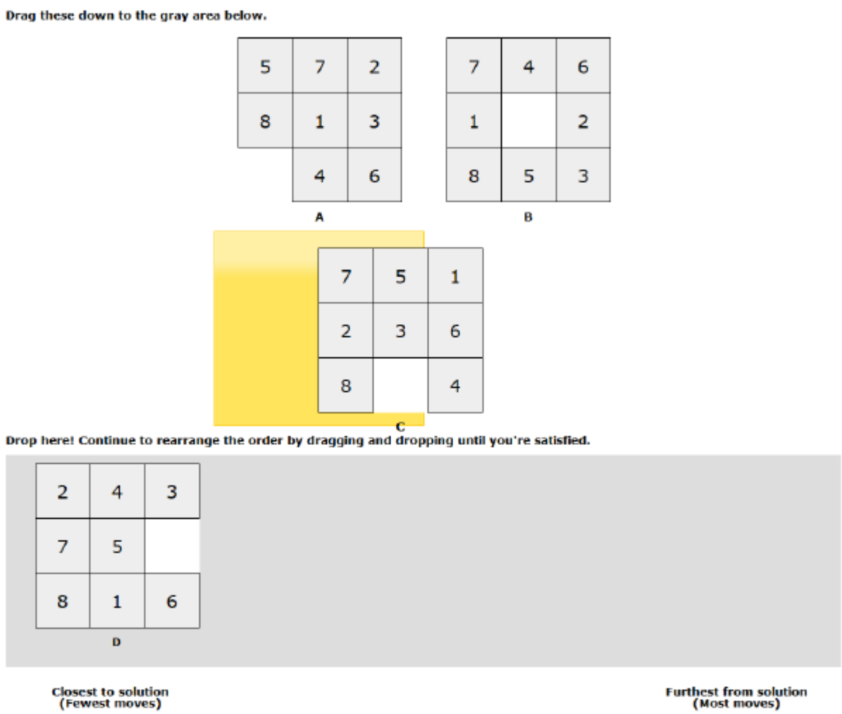


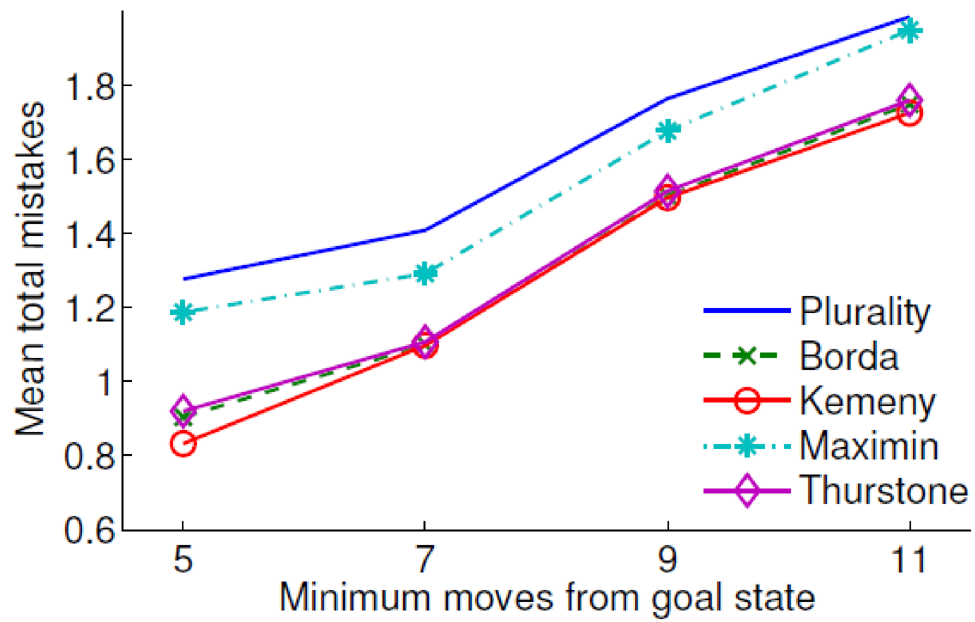


- Theorem [Doignon et al. 2004]:

  By setting $p_{ij} = \phi^{i-j} \cdot \frac{1-\phi}{1-\phi^i}$ for $j \le i \le m$, RIM induces the same distribution over rankings as Mallows

Is Mallows realistic?





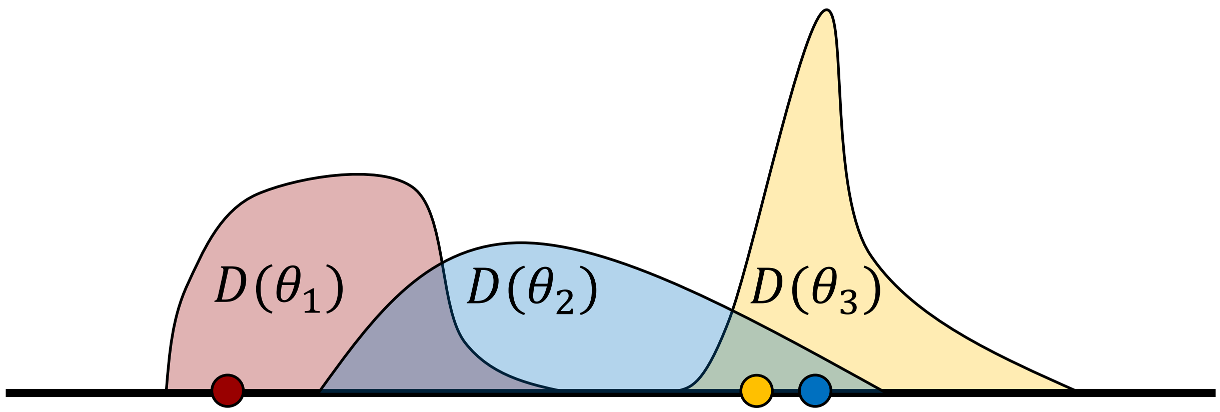
[Mao et al. 2013]

Random utility models

- Parameters $\boldsymbol{\theta} = (\theta_1, \ldots, \theta_m)$
  - m = number of alternatives
  - Each alternative x_j modeled by utility distribution $D(\theta_j)$
- A voter's utility U_j for alternative x_j is drawn independently from $D(\theta_j)$
- Voters rank alternatives by $U_1, \ldots, U_m$:

$$\Pr[x_2 \succ x_1 \succ x_3 \mid \theta_1, \theta_2, \theta_3] = \Pr_{U_j \sim D(\theta_j)}[U_2 > U_1 > U_3]$$

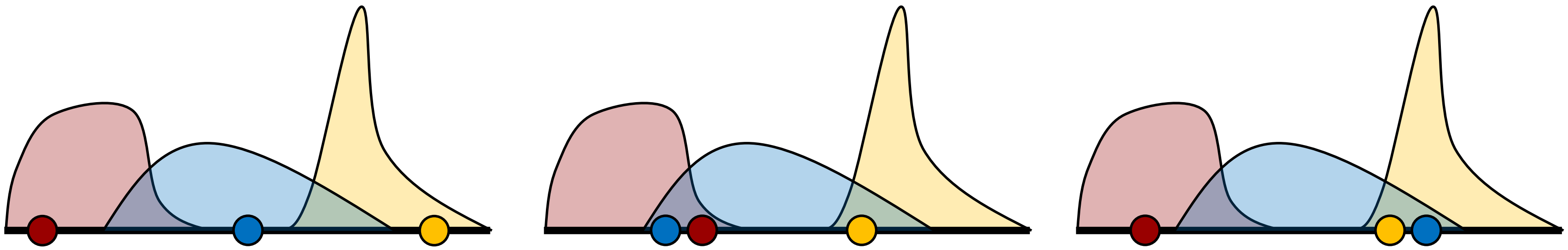
Random utility models



Generating a single vote

$$x_2 \succ x_3 \succ x_1$$

Random utility models



Voter 1
$x_3 \succ x_2 \succ x_1$

Voter 2
$x_3 \succ x_1 \succ x_2$

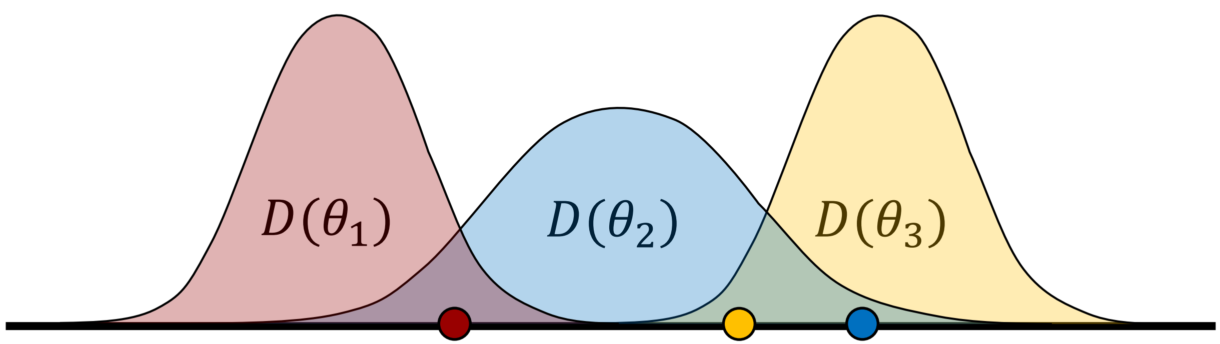
Voter 3
$x_2 \succ x_3 \succ x_1$

Generating a preference profile

$$\Pr[\succ_1, \dots, \succ_n \mid \boldsymbol{\theta}] = \prod_{i \in N} \Pr[\succ_i \mid \boldsymbol{\theta}]$$

The Thurstone Model

- Defined by a normal distribution
  - For each x_j, $\theta_j = (\mu_j, \sigma_j)$
  - $D(\theta_j) = \mathcal{N}(\mu_j, \sigma_j^2)$
- Computing $\Pr[\succ \mid \theta]$ believed to be hard

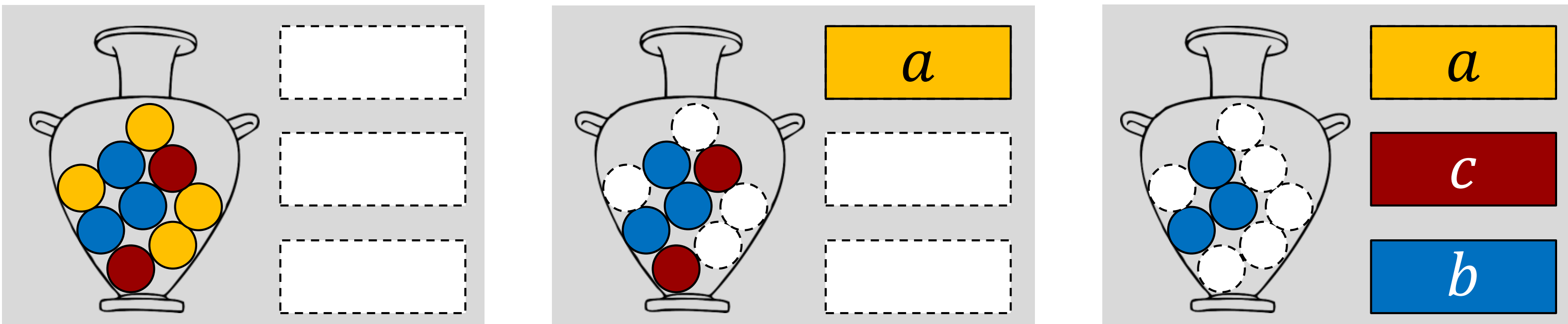


The Plackett-Luce model

- Defined by a Gumbel distribution
  - For each x_j, $\theta_j = (\mu_j, \beta_j)$
  - $D(\theta_j) = \mathcal{G}(\mu_j, \beta_j)$
- Equivalently, there exist weights $w_1, \ldots, w_m$ such that $\Pr[x_{j_1} \succ x_{j_2} \cdots \succ x_{j_m} \mid \boldsymbol{w}]$ is given by

$$\frac{w_{j_1}}{w_{j_1} + \cdots w_{j_m}} \cdot \frac{w_{j_2}}{w_{j_2} + \cdots w_{j_m}} \cdots \frac{w_{j_{m-1}}}{w_{j_{m-1}} + w_{j_m}}$$

The Plackett-Luce model



Urn interpretation

$$\Pr[a \succ c \succ b \mid (4,3,2)] = \frac{4}{9} \cdot \frac{2}{5}$$

Beyond social choice

- We previously interpreted pairwise comparisons as voters comparing alternatives
- But these comparisons can be the results of competitions between players
- In these situations, we typically want to update our estimates of player ratings online
- The famous Elo system originally used the Thurstone model

TrueSkill^{TM} system used to rank Halo players
Also based on the Thurstone model
[Herbrich et al. 2006]

Summary

- Terminology:
  - Models: Condorcet, Mallows, random insertion, Thurstone, Plackett-Luce
  - Kendall tau distance
  - The Kemeny rule
- Algorithms:
  - IP for Kemeny
- Big ideas:
  - Voting as search for truth