Graduate AI

Lecture 24: Social Choice II

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Reminder: Voting

- Set of voters $N = \{1, \ldots, n\}$
- Set of alternatives A, $|A| = m$
- Each voter has a ranking over the alternatives
- $x \succ_i y$ means that voter i prefers x to y
- Preference profile $\vec{\succ}$ = collection of all voters' rankings
- Voting rule f = function from preference profiles to alternatives
- Important: so far voters were honest!

Manipulation

- Using Borda count
- Top profile: b wins
- Bottom profile: a wins
- By changing his vote, voter 3 achieves a better outcome!

1	2	3
b	b	a
a	a	b
c	c	c
d	d	d

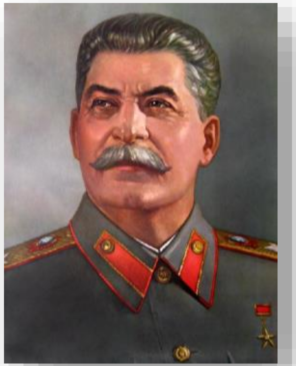
1	2	3
b	b	a
a	a	c
c	c	d
d	d	b

Strategyproofness

- A voting rule is strategyproof (SP) if a voter can never benefit from lying about his preferences:
  $\forall \vec{\prec}, \forall i \in N, \forall \prec_i', f(\vec{\prec}) \succcurlyeq_i f(\prec_1, \dots, \prec_i', \dots, \prec_n)$
- Poll 1: Maximum value of m for which plurality is SP?
  1. 2
  2. 3
  3. 4
  4. ∞

STRATEGYPROOFNESS

- A voting rule is dictatorial if there is a voter who always gets his most preferred alternative
- A voting rule is constant if the same alternative is always chosen
- Constant functions and dictatorships are SP



Dictatorship

Constant function

Gibbard-Satterthwaite

- A voting rule is onto if any alternative can win
- Theorem (Gibbard-Satterthwaite): If $m \geq 3$ then any voting rule that is SP and onto is dictatorial
- In other words, any voting rule that is onto and nondictatorial is manipulable

Gibbard

Satterthwaite

Circumventing G-S

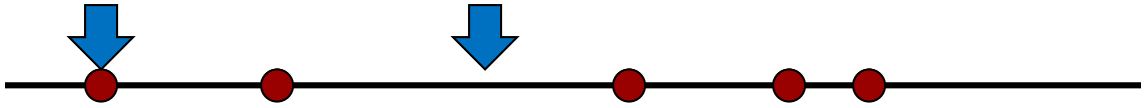
- Restricted preferences (this lecture)
- Money ⇒ mechanism design (not covered)
- Computational complexity (this lecture)

Single peaked preferences

- We want to choose a location for a public good (e.g., library) on a street
- Alternatives = possible locations
- Each voter has an ideal location (peak)
- The closer the library is to a voter's peak, the happier he is

Single peaked preferences

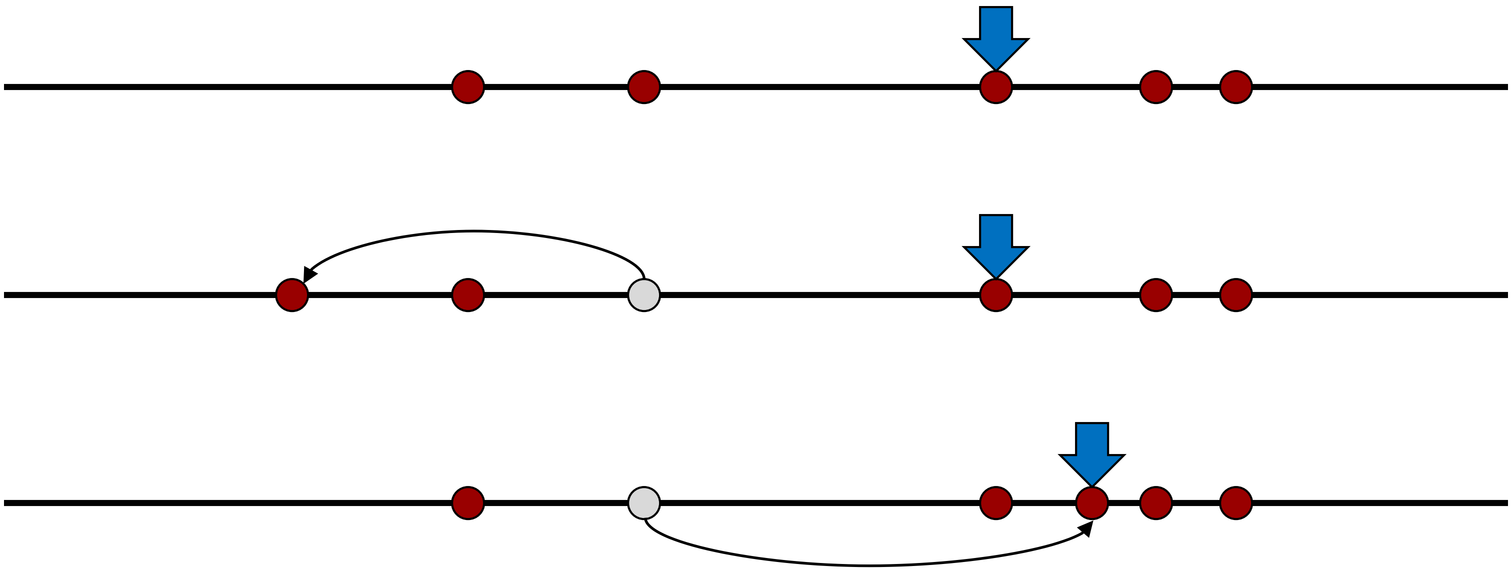
- Leftmost point mechanism: return the leftmost point
- Midpoint mechanism: return the average of leftmost and rightmost points
- Poll 2: Which mechanism is SP?
  1. Only leftmost point
  2. Only midpoint
  3. Both
  4. Neither



The median

- Select the median peak
- The median is a Condorcet winner!
- The median is onto
- The median is nondictatorial

The median is SP



Complexity of manipulation

- Manipulation is always possible in theory
- But can we design voting rules where it is difficult in practice?
- Are there "reasonable" voting rules where manipulation is a hard computational problem? [Bartholdi et al. 1989]

The computational problem

- f-Manipulation problem:
  - Given votes of nonmanipulators and a preferred alternative p
  - Can manipulator cast vote that makes p uniquely win under f?
- Example: Borda, $p = a$

1	2	3
b	b	
a	a	
c	c	
d	d	

1	2	3
b	b	a
a	a	c
c	c	d
d	d	b

A greedy algorithm

- Rank p in first place
- While there are unranked alternatives:
  - If there is an alternative that can be placed in next spot without preventing p from winning, place this alternative
  - Otherwise return false

Example: Borda

1	2	3
b	b	a
a	a	
c	c	
d	d	

1	2	3
b	b	a
a	a	b
c	c	
d	d	

1	2	3
b	b	a
a	a	c
c	c	
d	d	

1	2	3
b	b	a
a	a	c
c	c	b
d	d	

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b	b	a
a	a	c
c	c	d
d	d	

1	2	3
b	b	a
a	a	c
c	c	d
d	d	b

Example: Copeland

1	2	3	4	5
a	b	e	e	a
b	a	c	c	
c	d	b	b	
d	e	a	a	
e	c	d	d	

Preference profile

	a	b	c	d	e
a	-	2	3	5	3
b	3	-	2	4	2
c	2	2	-	3	1
d	0	0	1	-	2
e	2	2	3	2	-

Pairwise elections

Example: Copeland

1	2	3	4	5
a	b	e	e	a
b	a	c	c	c
c	d	b	b	
d	e	a	a	
e	c	d	d	

Preference profile

	a	b	c	d	e
a	-	2	3	5	3
b	3	-	2	4	2
c	2	3	-	4	2
d	0	0	1	-	2
e	2	2	3	2	-

Pairwise elections

Example: Copeland

1	2	3	4	5
a	b	e	e	a
b	a	c	c	c
c	d	b	b	d
d	e	a	a	
e	c	d	d	

Preference profile

	a	b	c	d	e
a	-	2	3	5	3
b	3	-	2	4	2
c	2	3	-	4	2
d	0	1	1	-	3
e	2	2	3	2	-

Pairwise elections

Example: Copeland

1	2	3	4	5
a	b	e	e	a
b	a	c	c	c
c	d	b	b	d
d	e	a	a	e
e	c	d	d	

Preference profile

	a	b	c	d	e
a	-	2	3	5	3
b	3	-	2	4	2
c	2	3	-	4	2
d	0	1	1	-	3
e	2	3	3	2	-

Pairwise elections

Example: Copeland

1	2	3	4	5
a	*b*	*e*	*e*	*a*
b	*a*	*c*	*c*	*c*
c	*d*	*b*	*b*	*d*
d	*e*	*a*	*a*	*e*
e	*c*	*d*	*d*	*b*

Preference profile

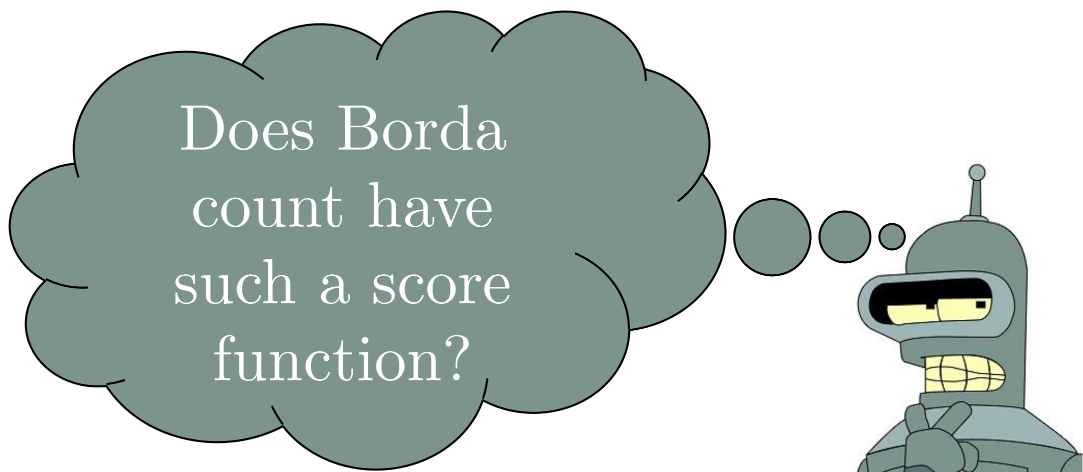
	a	*b*	*c*	*d*	*e*
a	-	2	3	5	3
b	3	-	2	4	2
c	2	3	-	4	2
d	0	1	1	-	3
e	2	3	3	2	-

Pairwise elections

When does the alg work?

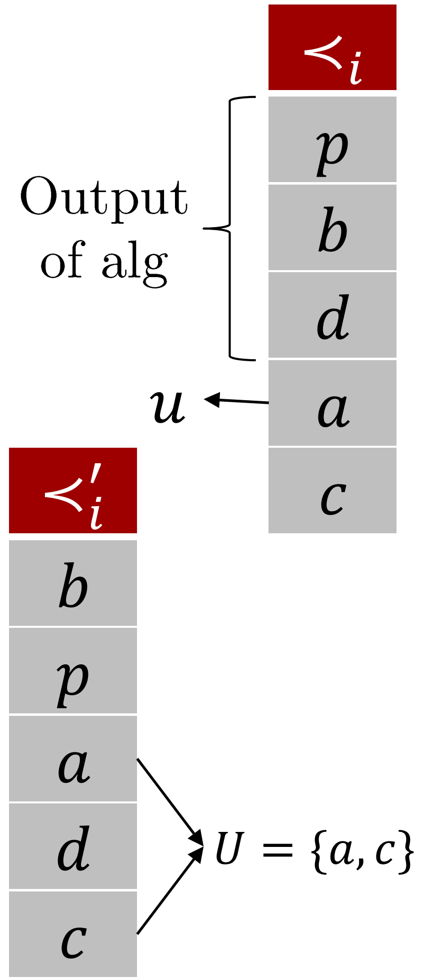
- Theorem [Bartholdi et al., SCW 89]:
Fix $i \in N$ and the votes of other voters. Let f be a rule s.t. $\exists$function $s(\prec_i, x)$ such that:
    1. For every $\prec_i$, f chooses a alternative that uniquely maximizes $s(\prec_i, x)$
    2. $\{y: y \prec_i x\} \subseteq \{y: y \prec_i' x\} \Rightarrow s(\prec_i, x) \leq s(\prec_i', x)$

Then the algorithm always decides f-Manipulation correctly



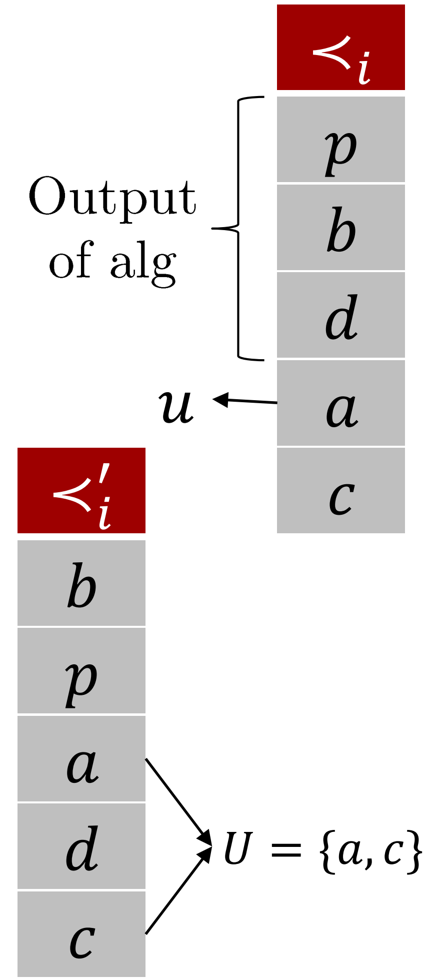
Proof of theorem

- Suppose the algorithm failed, producing a partial ranking $\prec_i$
- Assume for contradiction $\prec_i'$ makes p win
- $U \leftarrow$ alternatives not ranked in $\prec_i$
- $u \leftarrow$ highest ranked alternative in U according to $\prec_i'$
- Complete $\prec_\mathrm{i}$ by adding u first, then others arbitrarily



Proof of theorem

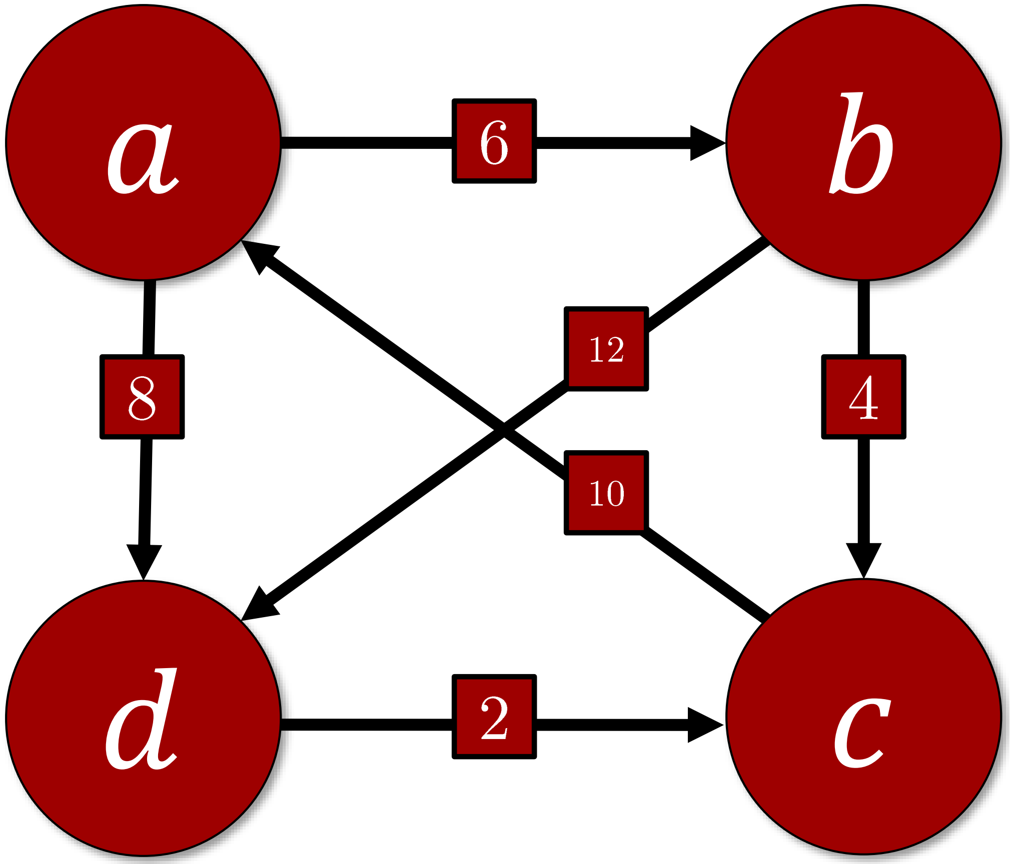
- Property 2 $\Rightarrow s(\prec_i, p) \geq s(\prec_i', p)$
- Property 1 and $\prec'$ makes p the winner $\Rightarrow s(\prec_i', p) > s(\prec_i', u)$
- Property 2 $\Rightarrow s(\prec_i', u) \geq s(\prec_i, u)$
- Conclusion: $s(\prec_i, p) > s(\prec_i, u)$, so the alg could have inserted u next ■



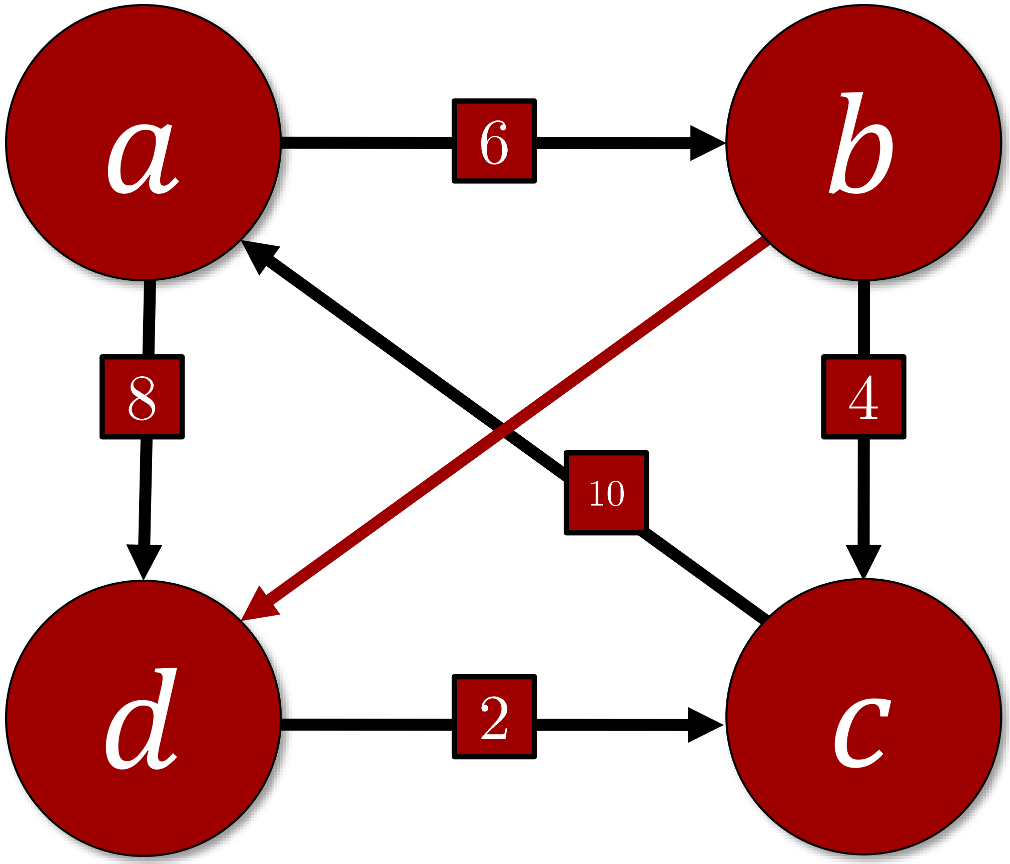
Hard-to-manipulate rules

- Copeland with second order tie breaking [Bartholdi et al. 1989]
- STV [Bartholdi and Orlin 1991]
- Ranked Pairs [Xia et al. 2009]
  - Sort pairwise comparisons by strength
  - Lock in pairwise comparisons in that order, unless a cycle is created, in which case the opposite edge is locked in
  - Return the alternative at the top of the induced order

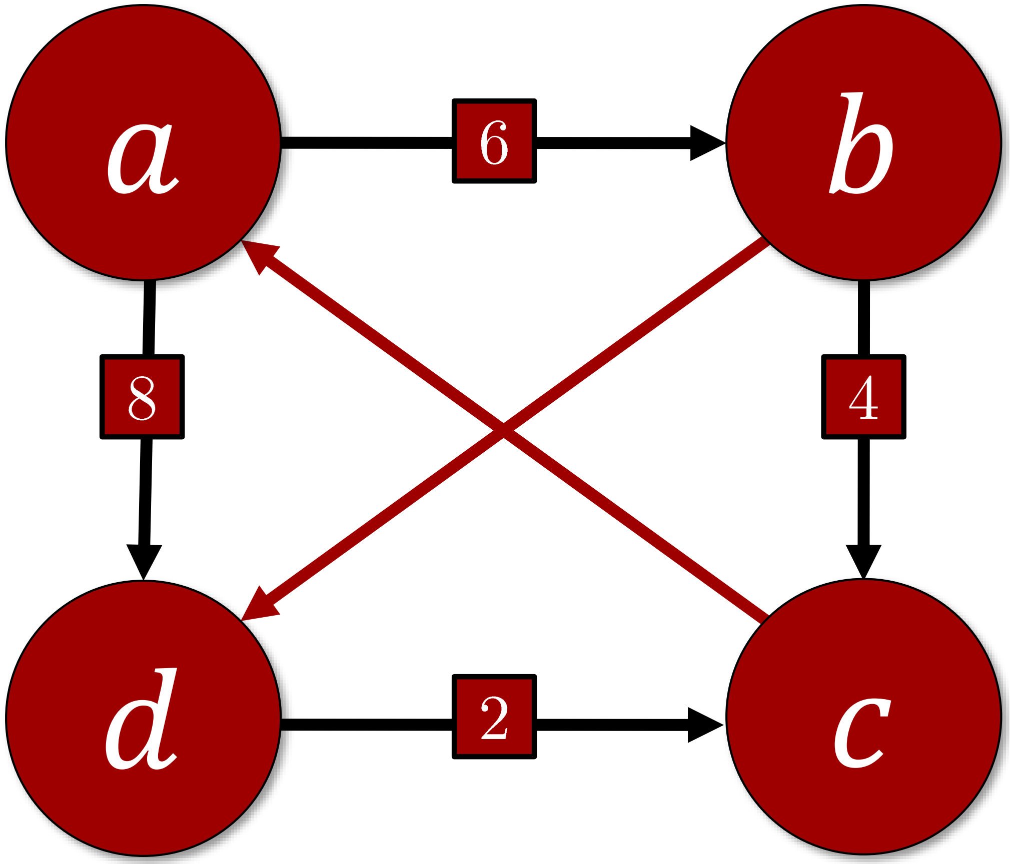
Example: ranked pairs



Example: ranked pairs



Example: ranked pairs



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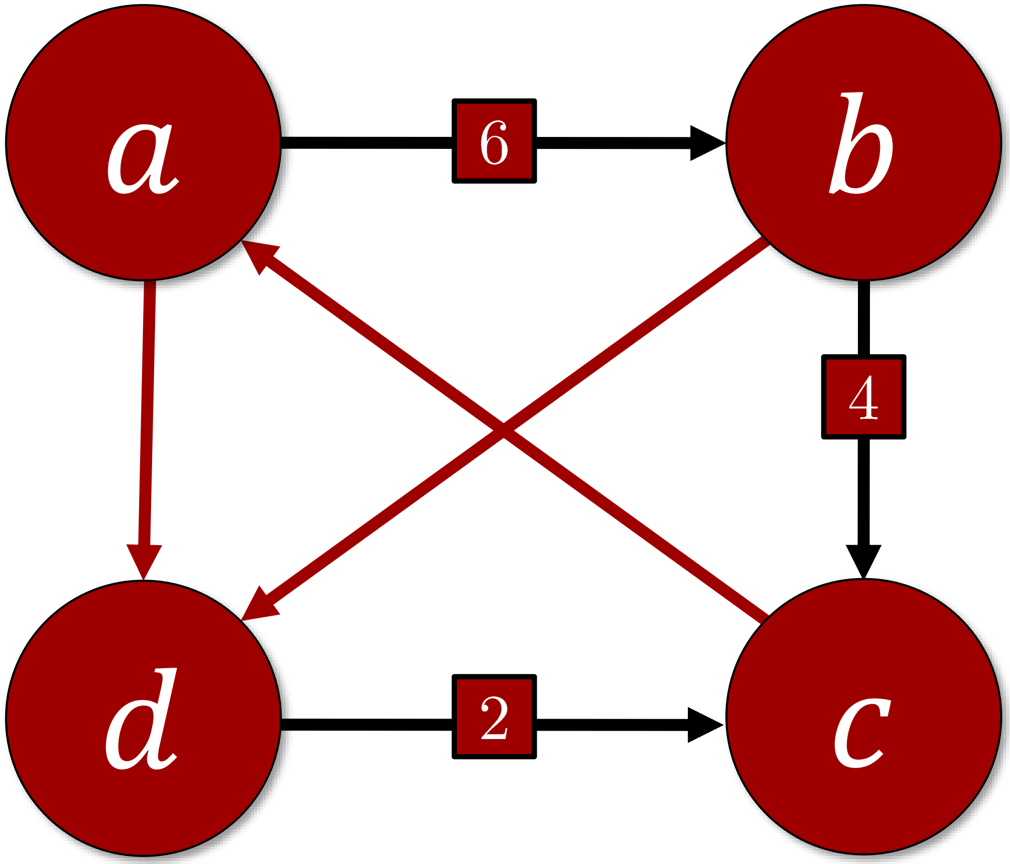
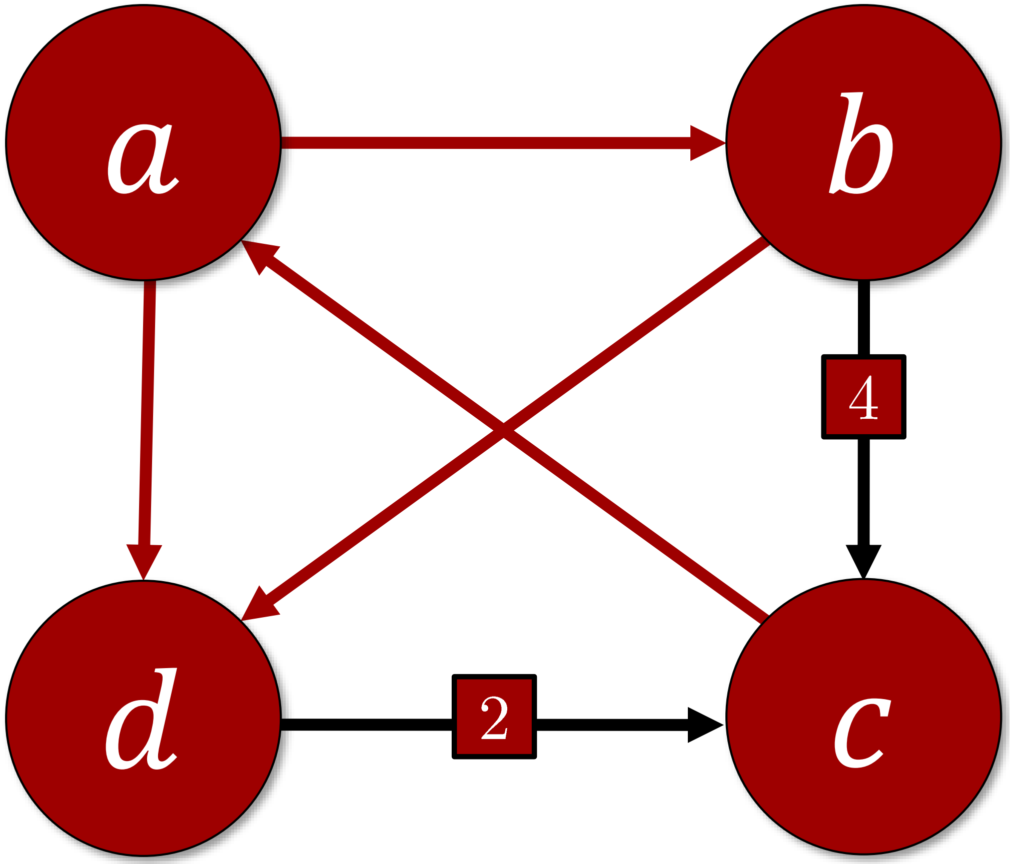


Example: ranked pairs



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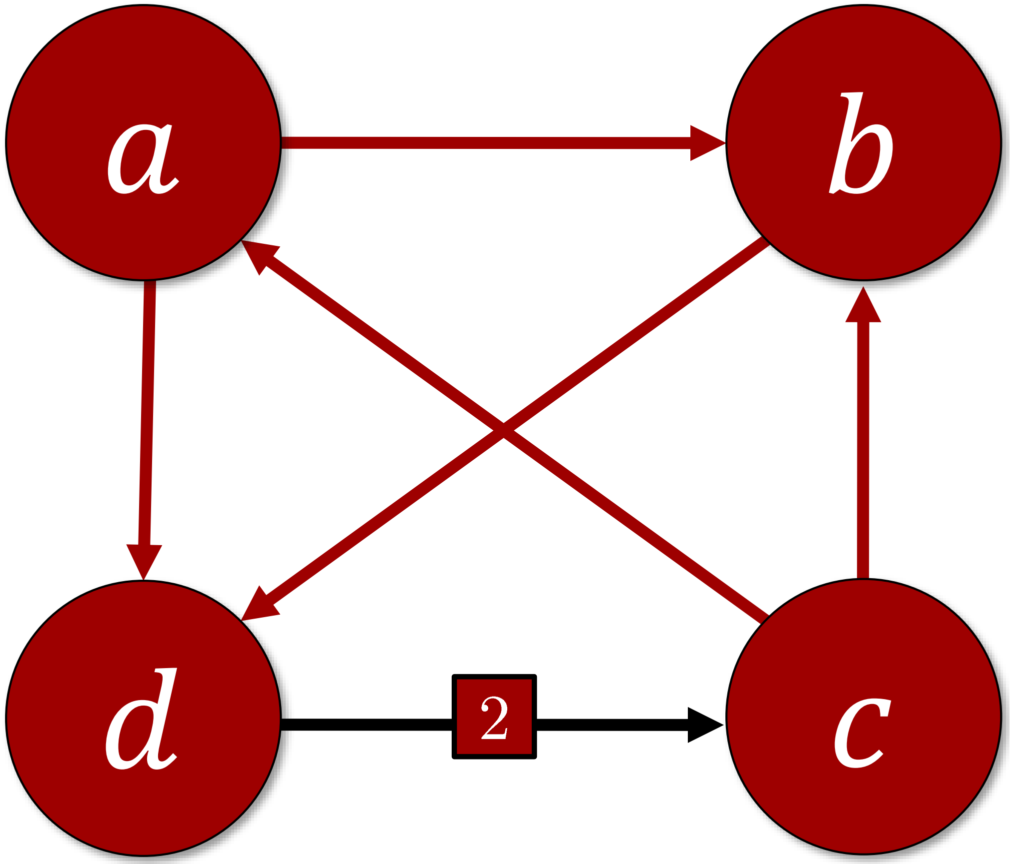

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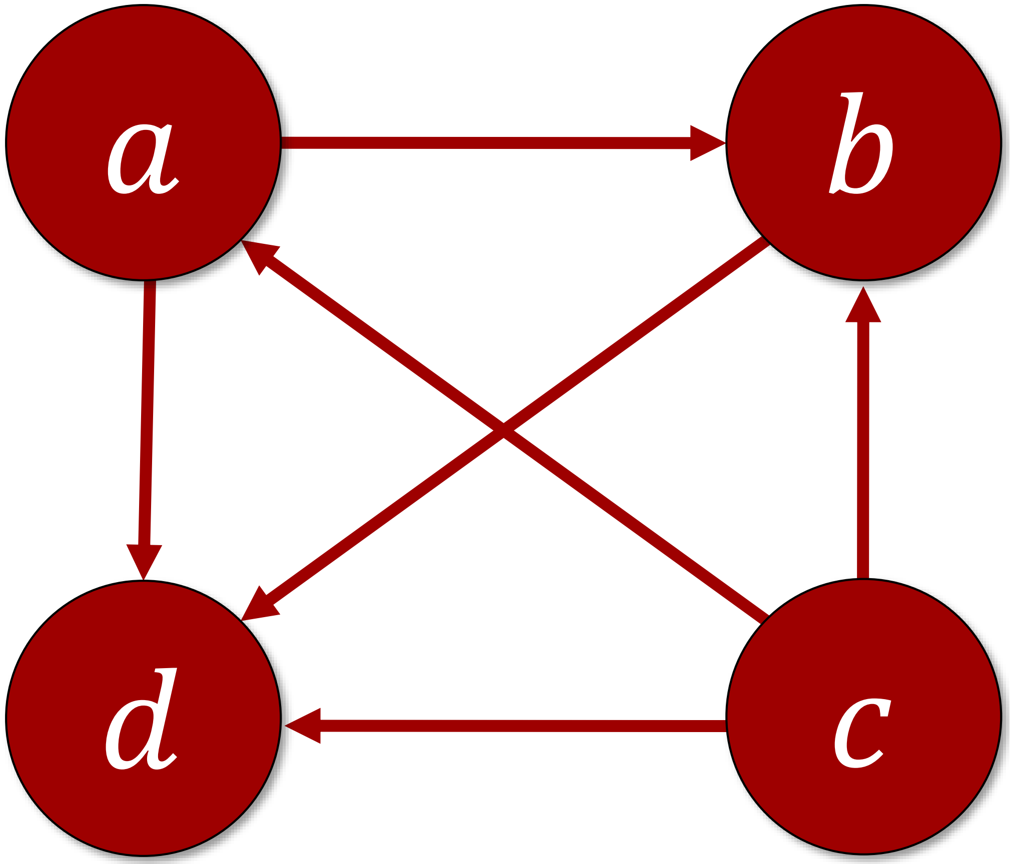

Summary

- Definitions, theorems, algorithms:
  - Strategyproof voting rules
  - The Gibbard-Satterthwaite Theorem
  - Greedy manipulation algorithm
- Big ideas:
  - Voting rules are provably manipulable
  - Circumvent via restricted preferences
  - Circumvent via computational complexity