

# Automated Design of Scoring Rules by Learning from Examples

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## ABSTRACT

Scoring rules are a broad and concisely-representable class of voting rules which includes, for example, Plurality and Borda. Our main result asserts that the class of scoring rules, as functions from preferences into candidates, is efficiently learnable in the PAC model. We discuss the applications of this result to automated design of scoring rules. We also investigate possible extensions of our approach, and (along the way) we establish a lemma of independent interest regarding the number of distinct scoring rules.

## Categories and Subject Descriptors

F.2 [Theory of Computation]: Analysis of Algorithms and Problem Complexity;

I.2.11 [Artificial Intelligence]: Distributed Artificial Intelligence—*Multiagent Systems*;

J.4 [Computer Applications]: Social and Behavioral Sciences—*Economics*

## General Terms

Algorithms, Theory, Economics

## Keywords

Voting, PAC learning

## 1. INTRODUCTION

Voting is a well-studied method of preference aggregation, in terms of its theoretical properties, as well as its computational aspects [5, 13]; various practical, implemented applications that use voting exist [8, 7]. In an election,  $n$  voters express their preferences over a set of  $m$  candidates or alternatives. To be precise, each voter is assumed to reveal linear preferences—a ranking of the candidates. The outcome of the election is determined according to a *voting rule*.

### 1.1 Scoring Rules

The predominant—ubiquitous, even—voting rule in real-life elections is the *Plurality* rule. Under Plurality, each

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voter awards one point to the candidate it ranks first, i.e., its most preferred alternative. The candidate that accumulated the most points, summed over all voters, wins the election. Another example of a voting rule is the *Veto* rule: each voter “vetoes” a single candidate; the candidate that was vetoed by the fewest voters wins the election. Yet a third example is the *Borda* rule: every voter awards  $m - 1$  points to its top-ranked candidate,  $m - 2$  points to its second choice, and so forth—the least preferred candidate is not awarded any points. Once again, the candidate with the most points is elected.

The abovementioned three voting rules all belong to an important family of voting rules known as *scoring rules*. A scoring rule can be expressed by a vector of parameters  $\vec{\alpha} = \langle \alpha_1, \alpha_2, \dots, \alpha_m \rangle$ , where each  $\alpha_i$  is a real number and  $\alpha_1 \geq \alpha_2 \geq \dots \geq \alpha_m$ . Each voter awards  $\alpha_1$  points to its most-preferred alternative,  $\alpha_2$  to its second-most-preferred alternative, etc. Predictably, the candidate with the most points wins. Under this unified framework, we can express our three rules as:

- *Plurality*:  $\vec{\alpha} = \langle 1, 0, \dots, 0 \rangle$ .
- *Borda*:  $\vec{\alpha} = \langle m - 1, m - 2, \dots, 0 \rangle$ .
- *Veto*:  $\vec{\alpha} = \langle 1, \dots, 1, 0 \rangle$ .

Voting rules are often compared on the basis of various criteria that define potentially desirable properties. We outline below several important criteria, some economic, and some computational.

1. *Anonymity*: The voting rule is symmetric with regard to the voters.
2. *Neutrality*: The voting rule is symmetric with regard to the candidates.
3. *Consistency*: If two disjoint sets of voters elect the same candidate, this candidate is also elected by the union of the two sets.
4. *Majority*: A candidate that is most preferred by a majority of voters wins the election.
5. *Robustness* [14]: The worst-case probability of the outcome of the election *not* changing as a result of a random mistake/fault in the preferences of the voters.













