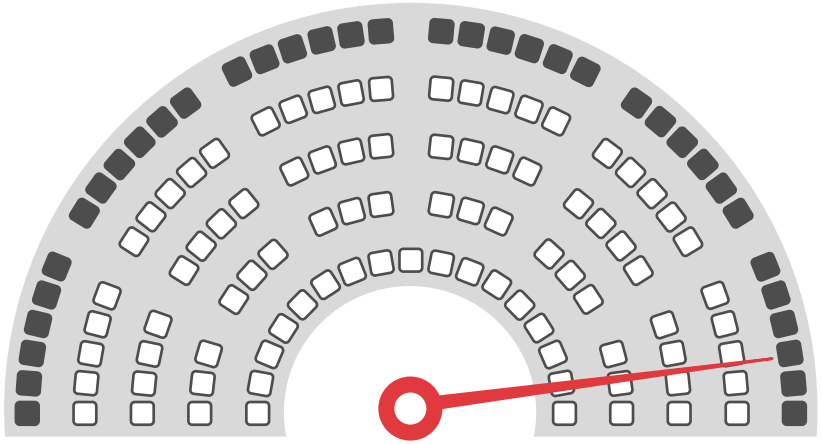


Optimized Democracy

Fall 2026 | Lecture 13

Participatory Budgeting

Ariel Procaccia | Harvard University

PARTICIPATORY BUDGETING



Allocation of a city's budget based on the votes of residents

PARTICIPATORY BUDGETING



Porto Alegre
Brazil
Since 1989



Paris
France
€81M (2025)



Madrid
Spain
€50M (2025)



Boston
USA
$2.2M (2026)

EXAMPLE BALLOT: BOSTON

You have selected **2 / 4** projects.

Water Bottle Refill Stations at Parks
Multiple installations of the water fountain and bottle refill designs at different parks in Boston.

Estimated Cost: $260,000

Location: Parks around Boston, West roxbury, Dorchester,South Boston, Roslindale, Downtown, Fenway

✔ Selected | Remove



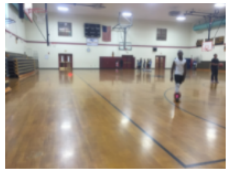
Click image for slide show

BLA Gym Renovations
No more netless hoops and broken bleachers--the BLA gym will be revamped to include newly painted hoops and floors, and repaired bleachers.

Estimated Cost: $475,000

Location: 205 Townsend Street, Roxbury

Select



Click image for slide show

Ringer Park Renovation
Renovate the main path in Ringer Playground so it is no longer cracked and safer to walk on.

Estimated Cost: $280,000

Location: Ringer Park, Brighton

✔ Selected | Remove



Click image for slide show

Bike Lane Installation
After a study, bike lanes will begin to be installed around Charlestown Navy Yard, Bunker Hill housing, and Charlestown High.

Estimated Cost: $200,000

Location: Charlestown

Select



Click image for slide show

Wicked Free Wifi 2.0
Wicked free Wi-Fi 2.0 provides Wi-fi at locations with young people.

Estimated Cost: $119,000

Location: Various High Schools and Community Centers, Dorchester, Roxbury, East Boston, Charlestown

Select



Click image for slide show

Track at Walker Park
Build a track that goes around the field that is behind Walker Playground.

Estimated Cost: $240,000

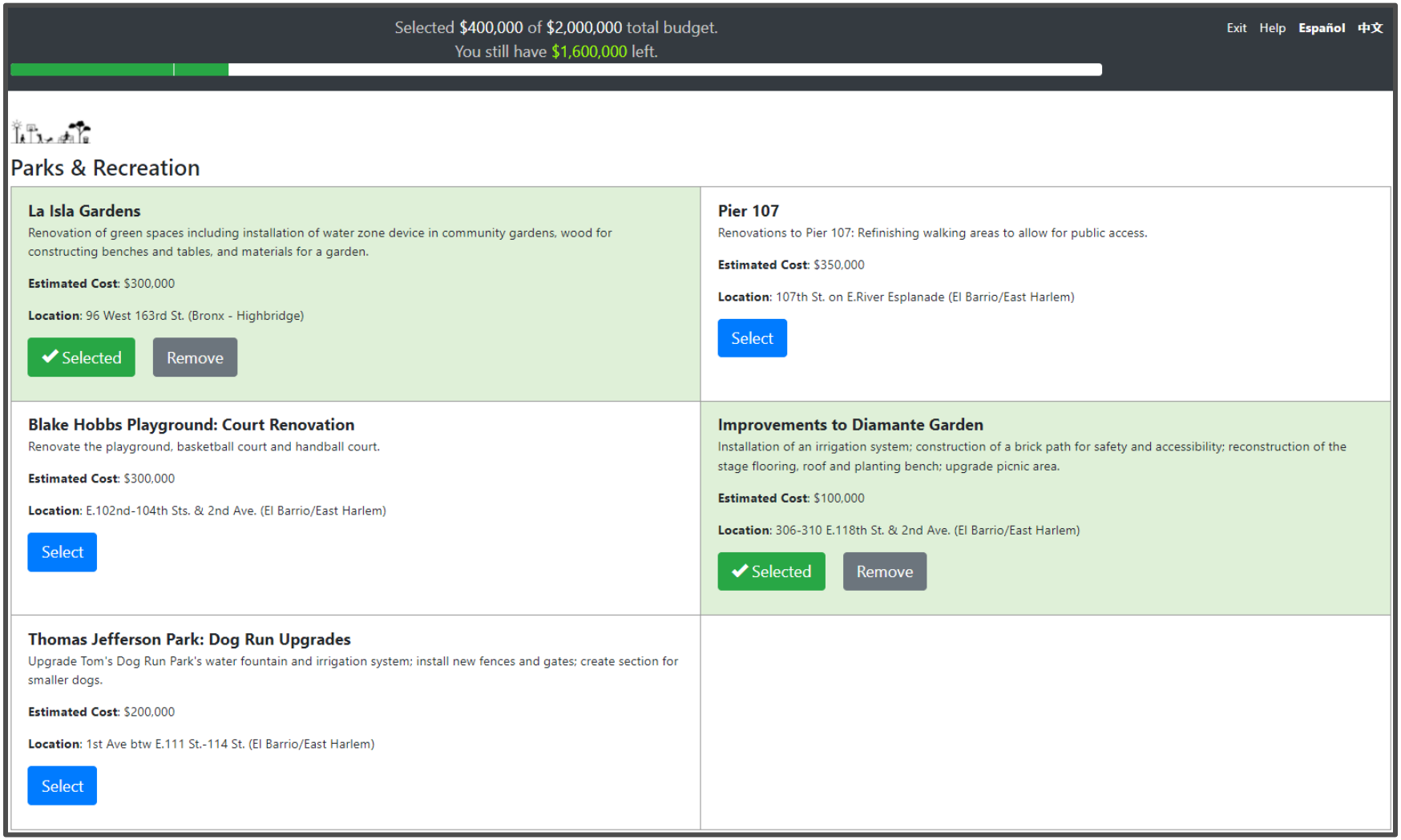
Location: Walker Playground on Norfolk St. and Astoria St., Mattapan

Select



Click image for slide show

EXAMPLE BALLOT: NYC



PARTICIPATORY BUDGETING MODEL

- Each voter $i \in N$ still casts an approval vote $\alpha_i \subseteq A$
- Each $x \in A$ has a cost $c(x)$, and there is a budget B
- The outcome is a budget-feasible subset $W \subseteq A$ such that $c(W) = \sum_{x \in W} c(x) \leq B$
- For now, we still assume that the utility of voter $i \in N$ for $W \subseteq A$ is $u_i(W) = |\alpha_i \cap W|$

APPROVAL VOTING, REVISITED

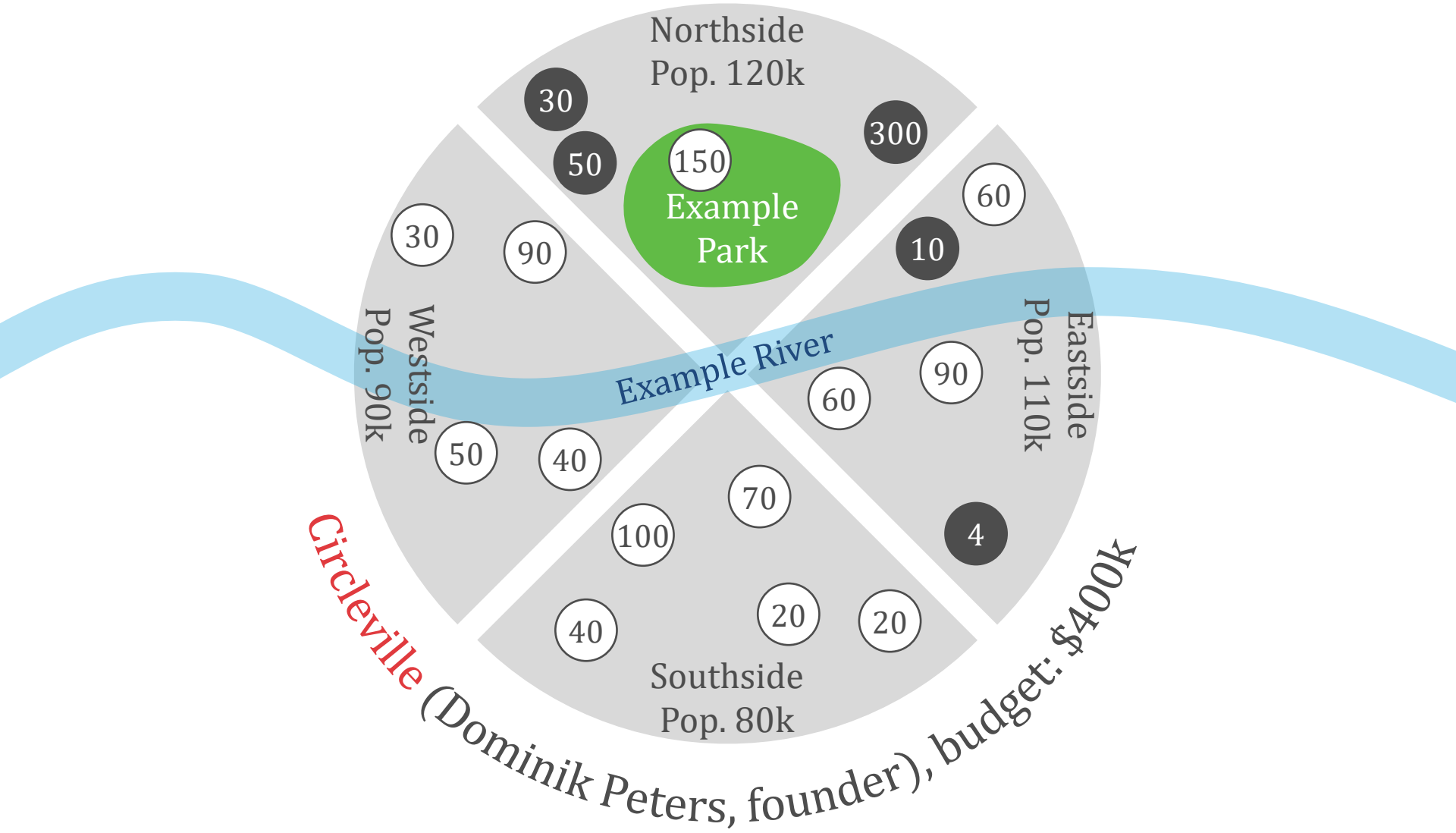
- A natural interpretation of approval voting is to maximize $\sum_{i \in N} u_i(W)$ subject to the budget constraint, which amounts to a knapsack problem
  - The "value" of each $x \in A$ is $|\{i \in N : x \in \alpha_i\}|$
  - The "weight" of each $x \in A$ is $c(x)$
- In practice, a greedy algorithm is often used, which adds alternatives in order of approval score, skipping those that are unaffordable

GREEDY AV: EXAMPLE

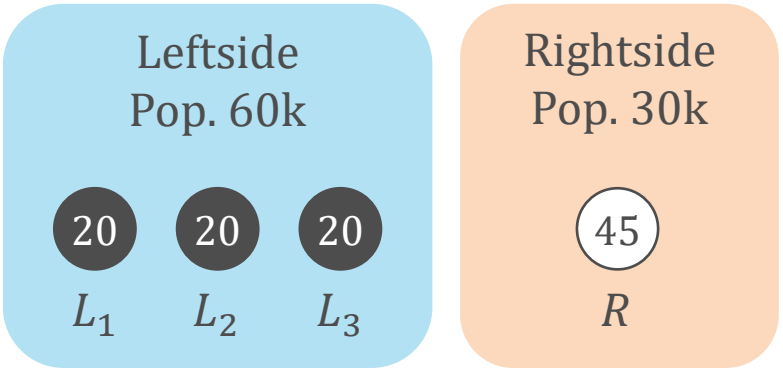
Paris, 4th District, 2019

Votes	Cost (€)	Greedy	Optimal
788	30,000	✓	✓
706	15,000	✓	✓
702	300,000	✓	✓
655	1,000,000	✓	
649	120,000	✓	✓
630	200,000		✓
528	20,000		✓
491	15,000		✓
473	20,000		✓
453	5,000		✓
410	150,000		✓
315	350,000		✓
265	30,000		✓
240	10,000		✓
228	120,000		✓
	Total:	1,465,000	1,385,000

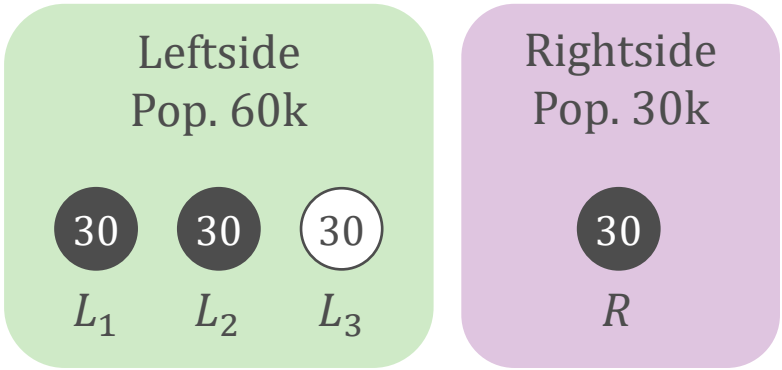
GREEDY AV: EXAMPLE



PAV TO THE RESCUE?



Rectangleville, budget: \$90k



New Rectangleville, budget: \$90k

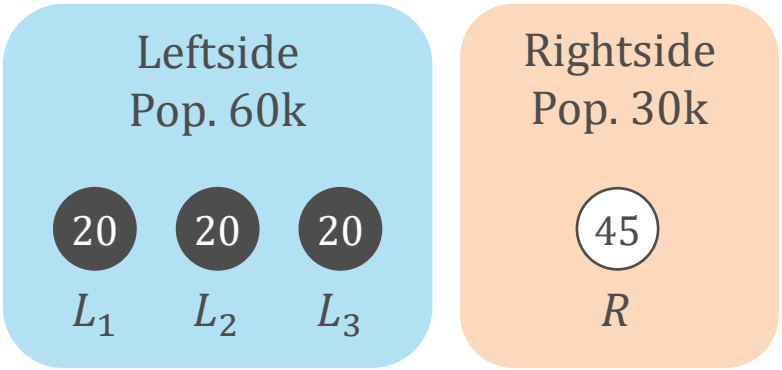
Poll

In which of the two towns (R and NR) does PAV give the proportional outcome shown in black?

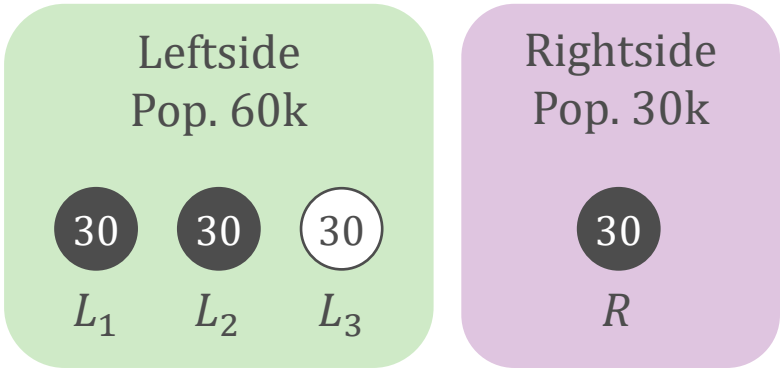
- Only R
- Only NR
- Both
- Neither



PAV TO THE RESCUE?



Rectangleville, budget: \$90k

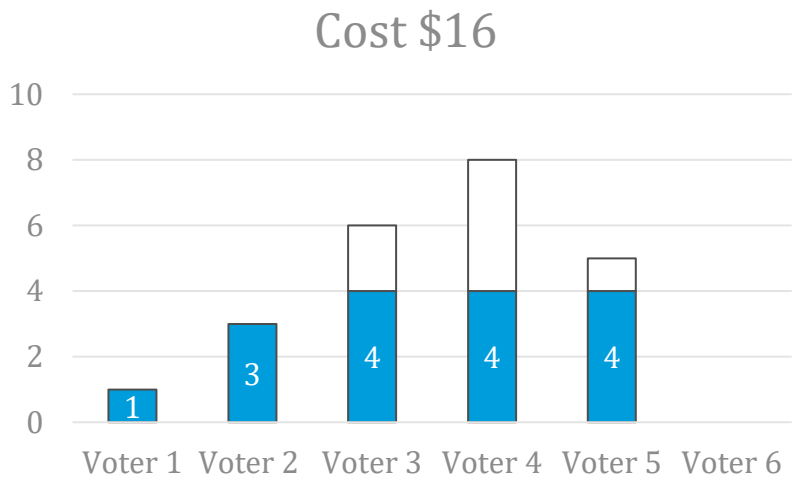


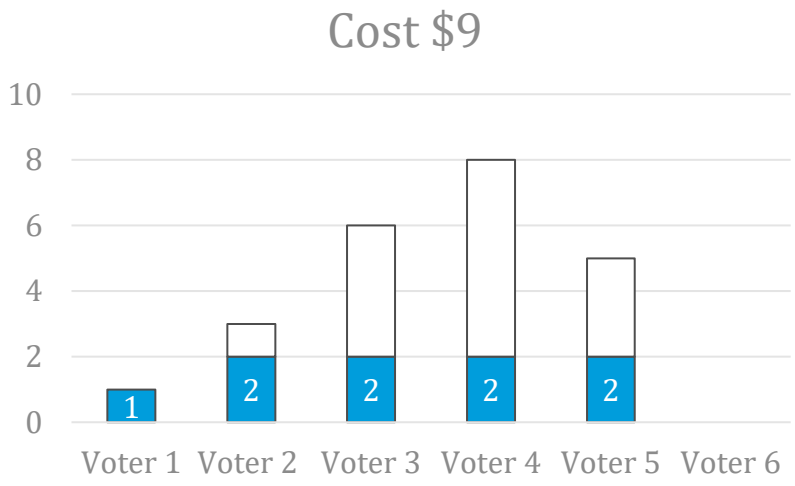
New Rectangleville, budget: \$90k

- PAV can't distinguish between the two examples and therefore cannot identify which outcomes are proportional
- Theorem: Every voting rule that only depends on the collection of budget-feasible subsets must fail proportionality, even on instances with a district structure

METHOD OF EQUAL SHARES

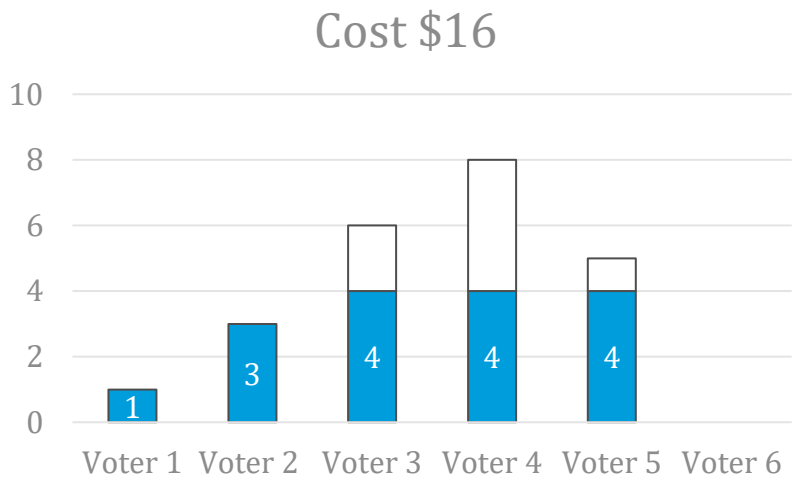
- Give a budget of B/n to each voter
- Do until the budget runs out:
  - For each alternative, divide its cost as evenly as possible among its supporters
  - Fund an affordable alternative with the lowest max payment

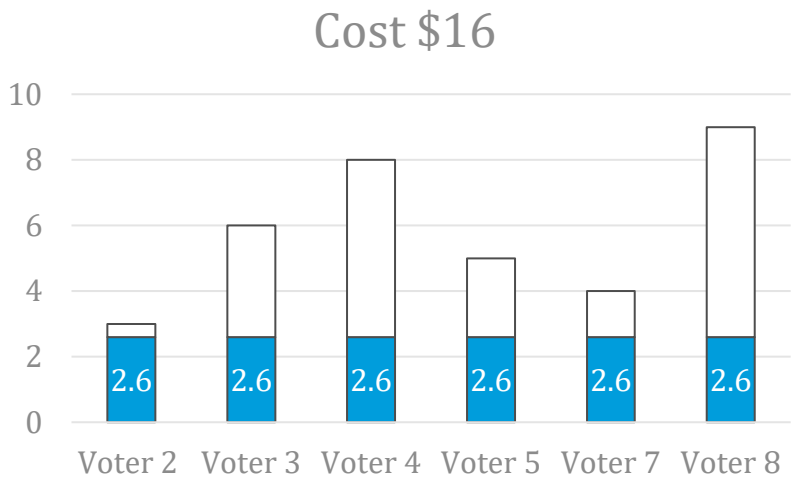




METHOD OF EQUAL SHARES

- Give a budget of B/n to each voter
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METHOD OF EQUAL SHARES

- Give a budget of B/n to each voter
- Do until the budget runs out:
  - For each alternative, divide its cost as evenly as possible among its supporters
  - Fund an affordable alternative with the lowest max payment
- Extended justified representation (for participatory budgeting): For all $S \subseteq N$ such that $|S| \geq \gamma \cdot n$ and $T \subseteq \bigcap_{i \in S} \alpha_i$ such that $c(T) \leq \gamma \cdot B$ there is $i \in S$ such that $u_i(W) \geq u_i(T)$
- Theorem: The Method of Equal Shares satisfies EJR
- Used in Aarau and Winterthur (Switzerland), Wieliczka and Świecie (Poland), and Assen (Netherlands)

A DIFFERENT PERSPECTIVE

- In the 2019 PB election of Paris' 16th District, a refurbishment of a sports facility received 775 votes and cost €560k, and materials for a school project received 670 votes and cost €3k
- Let us consider general additive utilities, i.e., $u_i(W) = \sum_{x \in W} u_i(x)$, where $u_i(x) \in \mathbb{R}^+$
- The goal is to find $W \subseteq A$ that maximizes the social welfare $\text{sw}(W, \boldsymbol{u}) = \sum_{i \in N} u_i(W)$ subject to the budget constraint $c(W) \leq B$
- But we don't necessarily want to ask voters to explicitly report utilities
- Instead, we'll ask voters to cast votes in some input format

INPUT FORMATS

Ranking by value

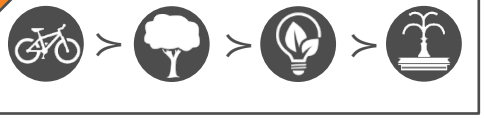


Knapsack voting



Budget 9

Ranking by VFM



Threshold approval



Threshold 5



Utility 6
Cost 6



Utility 3
Cost 2



Utility 2
Cost 1



Utility 8
Cost 9



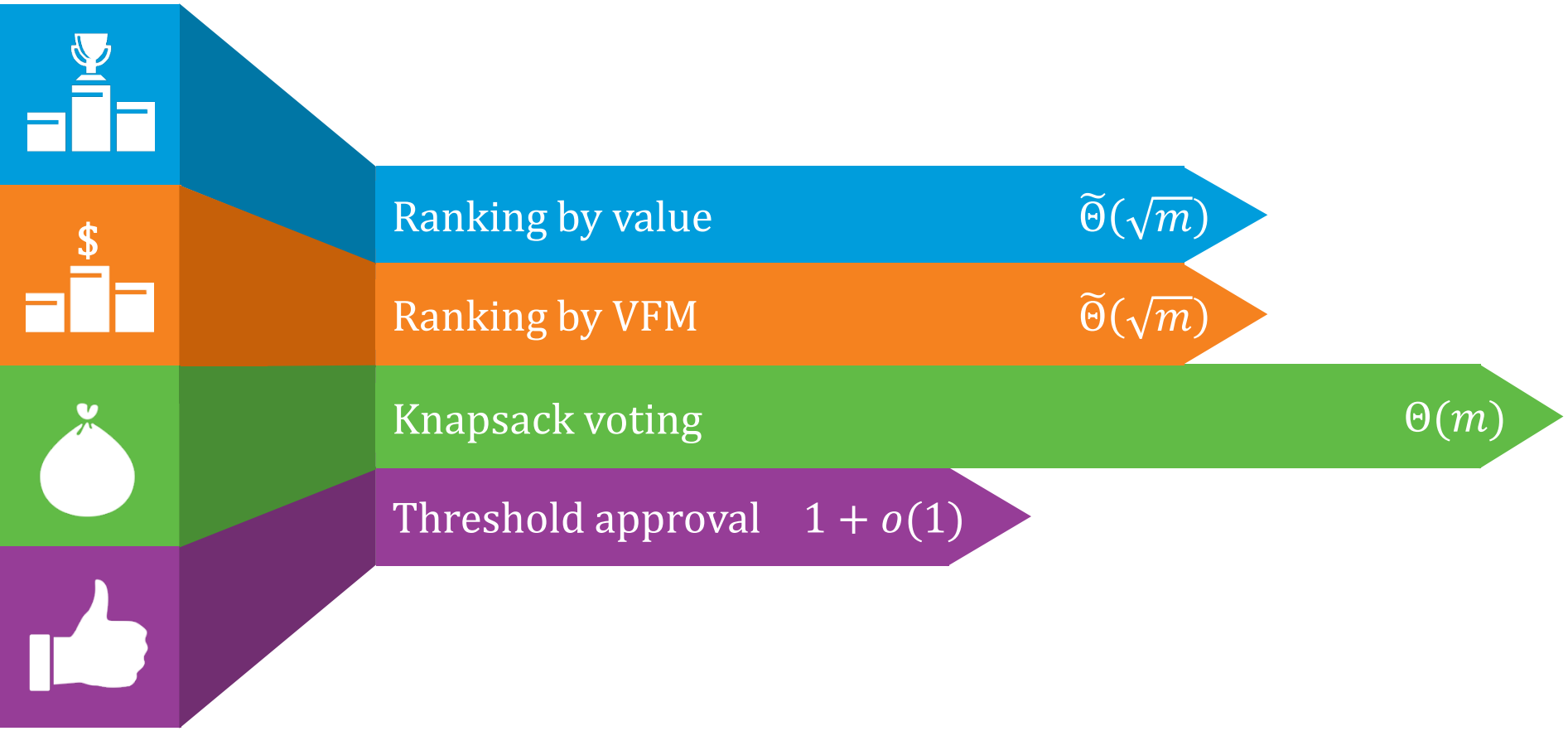
DISTORTION, REVISITED

- Voter i reports a vote σ_i that is consistent with u_i; denote $u_i \rhd \sigma_i$
- A randomized voting rule f maps an input profile $\boldsymbol{\sigma}$ to a distribution over budget-feasible subsets of alternatives
- The distortion of f on $\boldsymbol{\sigma}$ is

$$\max_{\boldsymbol{u} \rhd \boldsymbol{\sigma}} \frac{\max_{W \subseteq A : c(W) \leq B} \mathrm{SW}(W, \boldsymbol{u})}{\mathbb{E}[\mathrm{SW}(f(\boldsymbol{\sigma}), \boldsymbol{u})]}$$

- Associate an input format with the worst-case distortion of the best voting rule

DISTORTION BY INPUT FORMAT



BIBLIOGRAPHY

D. Peters, G. Pierczyński, and P. Skowron. Proportional Participatory Budgeting with Cardinal Utilities. NeurIPS 2021.

Gerdus Benadè, Swaprava Nath, A. D. Procaccia and N. Shah. Preference Elicitation for Participatory Budgeting. Management Science, 2021.

