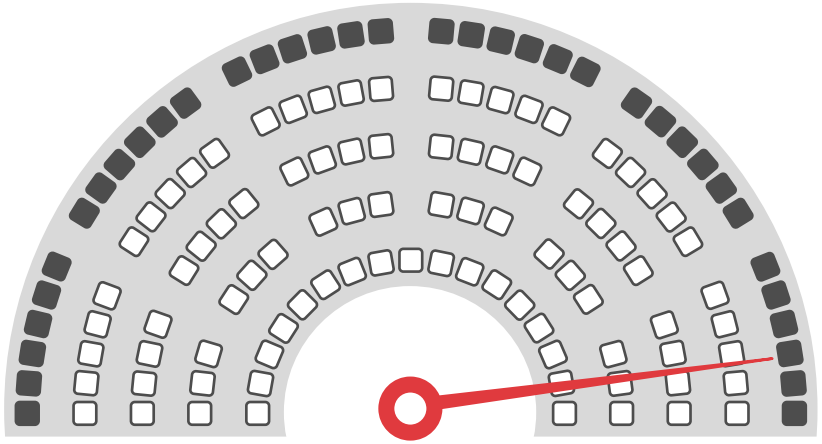


Optimized Democracy

Fall 2026 | Lecture 12

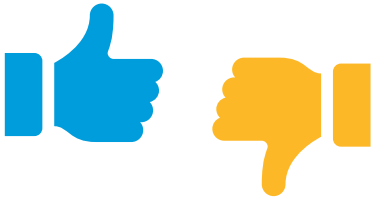
Committee Elections

Ariel Procaccia | Harvard University

SOME BALLOT TYPES, REVISITED



Rankings



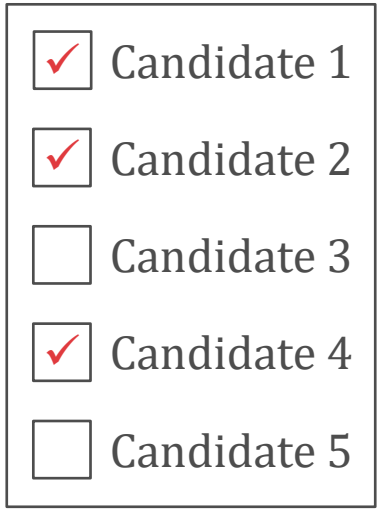
Approvals



Scores/stars

Let’s talk about approvals!

APPROVAL VOTING

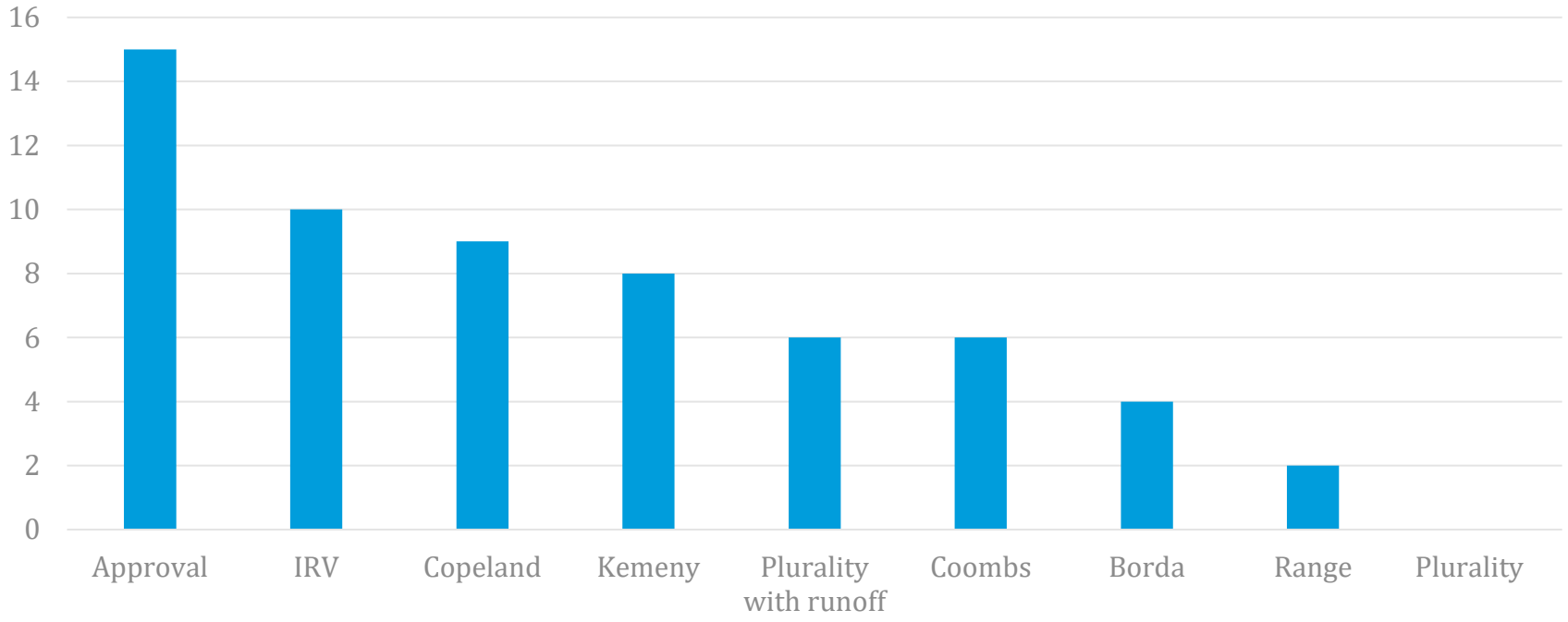


Ballots: Approve as many alternatives as you like

Aggregation: Elect an alternative that is approved by the most voters

APPROVAL OF APPROVAL

What is the best voting rule for electing a mayor?



This election was held using approval voting

APPROVAL VOTING IN THE USA

Vox

BIDEN ADMINISTRATION CORONAVIRUS RECODE THE GOODS MORE

This city just approved a new election system never tried before in America

Fargo just switched to an "approval voting" system, which allows you to mark all the candidates on the ballot that you like.

By Kelsey Piper | Nov 15, 2018, 9:20am EST

SHARE



FiveThirtyEight

Politics Sports Science Podcasts Video

MAR. 1, 2021, AT 6:00 AM

In St. Louis, Voters Will Get To Vote For As Many Candidates As They Want

Tuesday is the biggest test yet for "approval voting."

By Nathaniel Rakich

Filed under Voting Reform



APPROVAL-BASED COMMITTEES

- Denote the approved set of voter $i \in N$ by $\alpha_i \subseteq A$
- The outcome is a committee $W \subseteq A$ such that $|W| = k$
- The utility of voter $i \in N$ for $W \subseteq A$ is $u_i(W) = |\alpha_i \cap W|$

THIELE'S METHODS

- Given a sequence $s_1, s_2, \ldots$ select a committee W that maximizes

$$\sum_{i \in N} \left(s_1 + s_2 + \cdots + s_{u_{i(W)}} \right)$$

- Examples:
  - Approval voting (AV): $1,1,1, \cdots$
  - Chamberlin-Courant (CC): $1,0,0, \cdots$
  - Proportional approval voting (PAV): $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$



Om Flerfoldsvalg.

Af

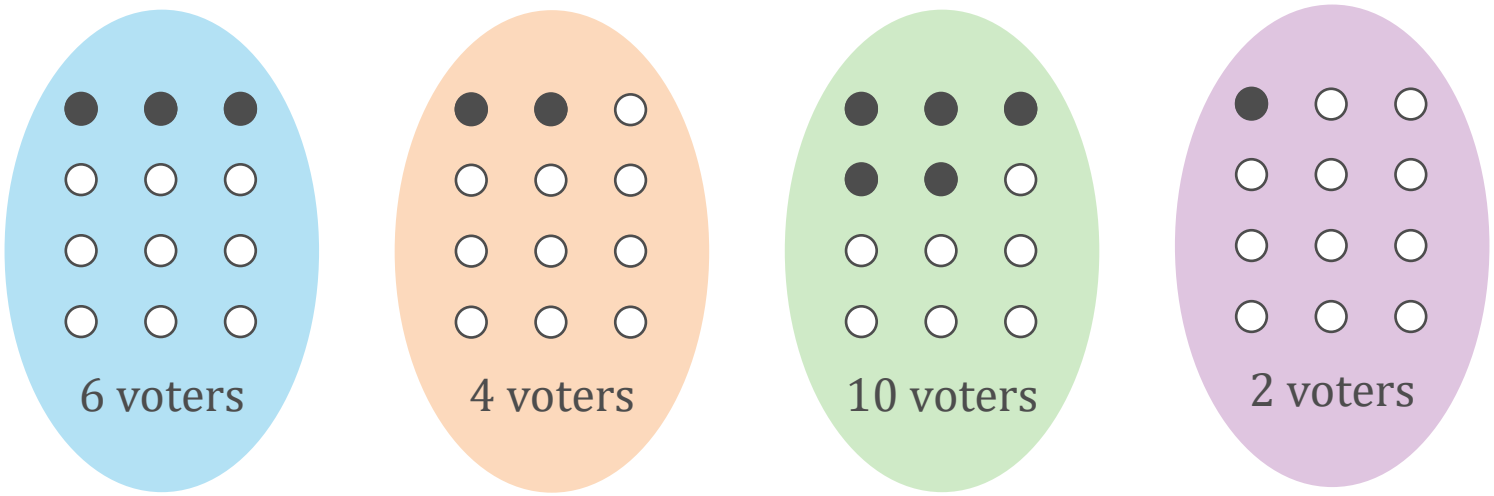
Dr. **T. N. Thiele,**
Prof. astron.

(Meddelt i Mødet den 29. November 1895.)

Den forholdsmæssige Valgmaade,

$$f(n) = 1 + \frac{1}{2} + \ldots\ldots + \frac{1}{n}.$$

WHY HARMONIC NUMBERS?



$$k = 11$$

+6	+4	+10	+2
+3	+2	+5	+1
+2	+1.33	+3.33	+0.66
+1.5	+1	+2.5	+0.5
+1.2	+0.8	+2	+0.4
+1	+0.66	+1.66	+0.33

WHY HARMONIC NUMBERS?

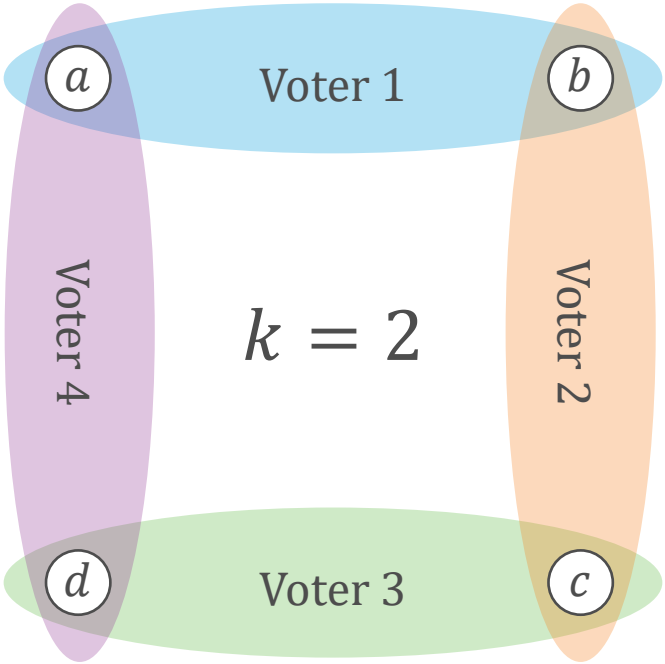
- Proportionality: Suppose a party list has x supporters with $x \geq \ell \cdot \frac{n}{k}$, then it deserves ℓ seats
- It holds that

$$\frac{x}{1} > \frac{x}{2} > \cdots \frac{x}{\ell} \geq \frac{n}{k}$$

- There can't be more than k alternatives with marginal increase at least n/k
- But how do we define proportionality when approval sets intersect?

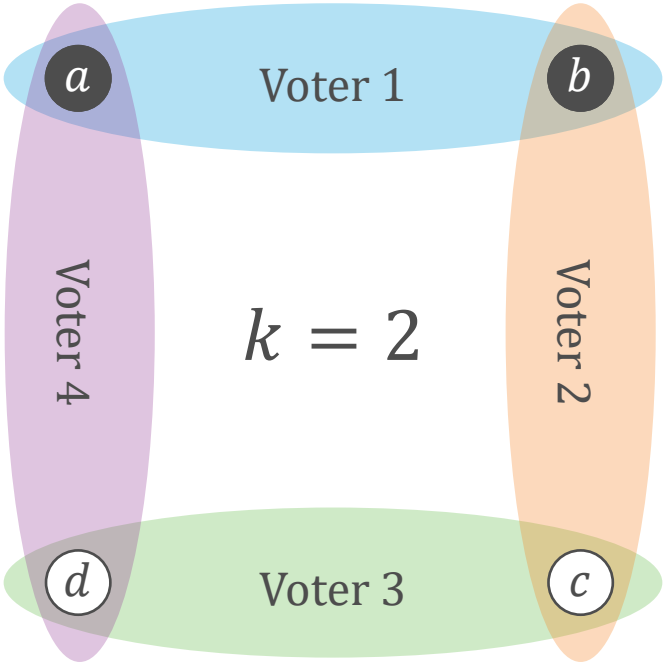
FIRST ATTEMPT

If there is $S \subseteq N$ such that $|S| \geq n/k$ and $|\bigcap_{i \in S} \alpha_i| \geq 1$ then $(\bigcap_{i \in S} \alpha_i) \cap W \neq \emptyset$

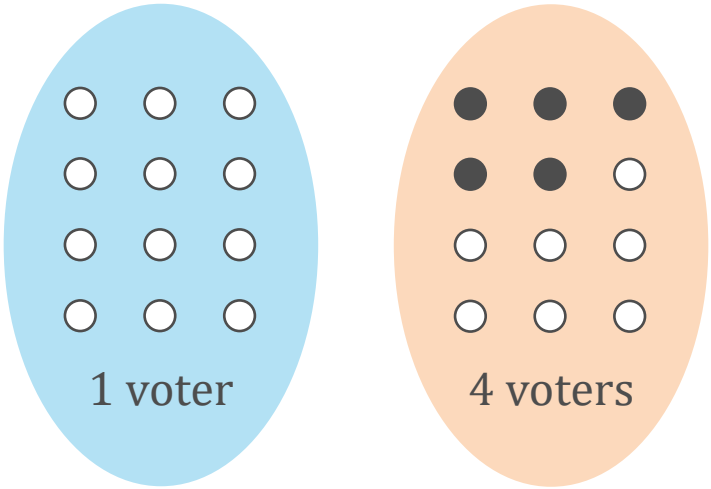


JUSTIFIED REPRESENTATION

Justified representation: If there is $S \subseteq N$ such that $|S| \geq n/k$ and $|\bigcap_{i \in S} \alpha_i| \geq 1$ then $\exists i \in S$ such that $u_i(W) \geq 1$



JUSTIFIED REPRESENTATION



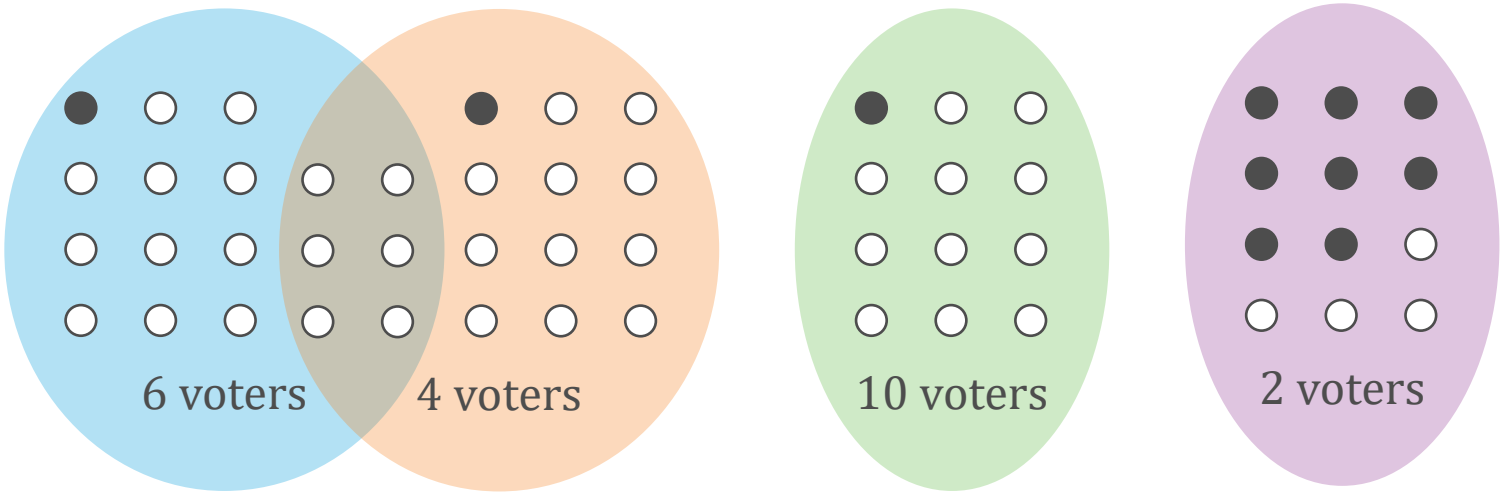
AV fails justified representation

JUSTIFIED REPRESENTATION

- Theorem: Chamberlin-Courant satisfies justified representation
- Proof:
  - Let W be the CC committee violating JR, and let S be the subset witnessing the violation
  - The number of voters covered by W is less than n
  - There must be $x \in W$ whose marginal contribution is less than n/k voters
  - Remove x and add the alternative approved by S — this gives higher CC score ■

EXTENDED JUSTIFIED REPRESENTATION

Extended justified representation: If there is $S \subseteq N$ such that $|S| \geq \ell \frac{n}{k}$ and $|\bigcap_{i \in S} \alpha_i| \geq \ell$ then $\exists i \in S$ such that $u_i(W) \geq \ell$



Chamberlin-Courant fails EJR

EXTENDED JUSTIFIED REPRESENTATION

- EJR is clearly stronger than JR, so AV also fails EJR
- Theorem: PAV satisfies EJR (proof on the next slide)

Poll

What is the relation between JR, EJR, and proportionality in the case of party lists (disjoint approval sets)?

- JR ⇒ Prop.
- EJR ⇒ Prop.
- Both
- Neither



PROOF OF THEOREM

- Let W be the PAV committee, and suppose for contradiction that $S \subseteq N$ is such that $|S| \geq \ell \cdot \frac{n}{k}$ and $|\bigcap_{i \in S} \alpha_i| \geq \ell$ but $u_i(W) < \ell$ for all $i \in S$
- Let $x^\star \in \bigcap_{i \in S} \alpha_i \setminus W$ and $W' = W \cup \{x^\star\}$, then

$$\text{PAV}-\text{score}(W') \geq \text{PAV}-\text{score}(W) + |S|\frac{1}{\ell} \geq \text{PAV}-\text{score}(W) + \frac{n}{k}$$

- We claim that we can remove an alternative from W' and decrease PAV-score by less than n/k
- The average loss of PAV score after removal is

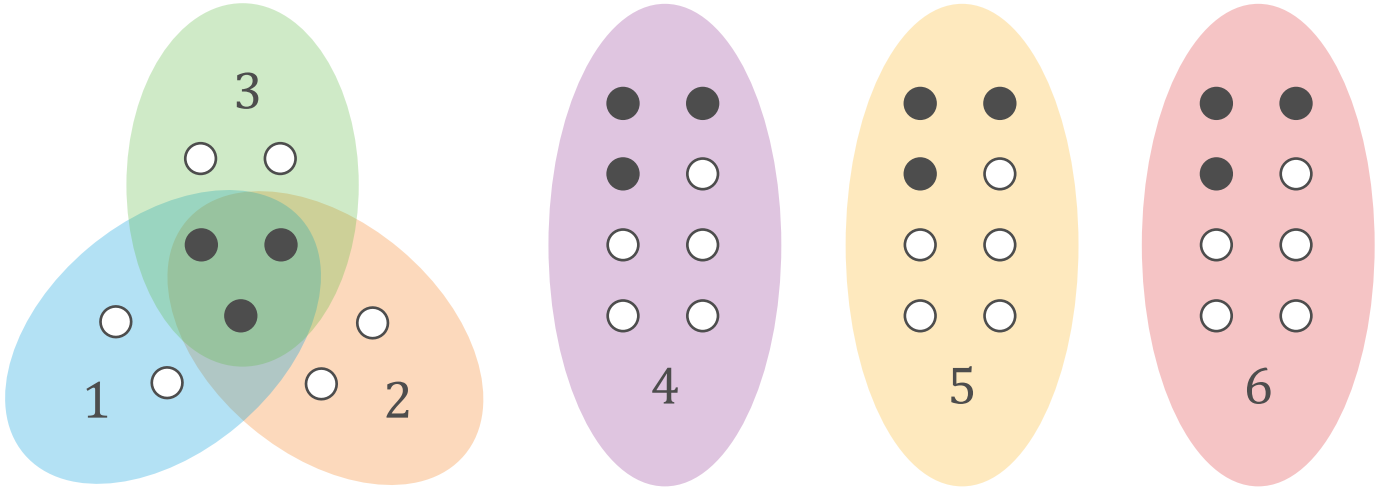
$$\frac{1}{k+1}\sum_{x \in W'}\sum_{i : x \in \alpha_i}\frac{1}{u_i(W')} = \frac{1}{k+1}\sum_{i \in N}\sum_{x \in \alpha_i \cap W'}\frac{1}{u_i(W')} \leq \frac{1}{k+1}\sum_{i \in N} 1 < \frac{n}{k}$$

- Hence there is some $x' \in W$ such that

$$\text{PAV}-\text{score}\big(W' \setminus \{x'\}\big) > \text{PAV}-\text{score}(W),$$

in contradiction to the optimality of W ■

IS EJR ENOUGH?



PAV selects an outcome satisfying EJR that doesn't quite feel proportional

THE CORE

- W is in the core if for all $S \subseteq N$ and $W' \subseteq A$ such that $|W'| \leq k \cdot |S|/n$, there exists $i \in S$ such that $u_i(W) \geq u_i(W')$
- Theorem (Becker et al., September 2026): The core is nonempty!

BIBLIOGRAPHY

S. J. Brams and P. C. Fishburn. Approval Voting (2nd edition). Springer, 2007.

H. Aziz, M. Brill, V, Conitzer, E. Elkind, R. Freeman, and T. Walsh. Justified Representation in Approval-Based Committee Voting. Social Choice and Welfare, 2017.

Becker, Greger, and Peters. Existence of the Core in Approval-Based Committee Elections. arXiv, 2026.