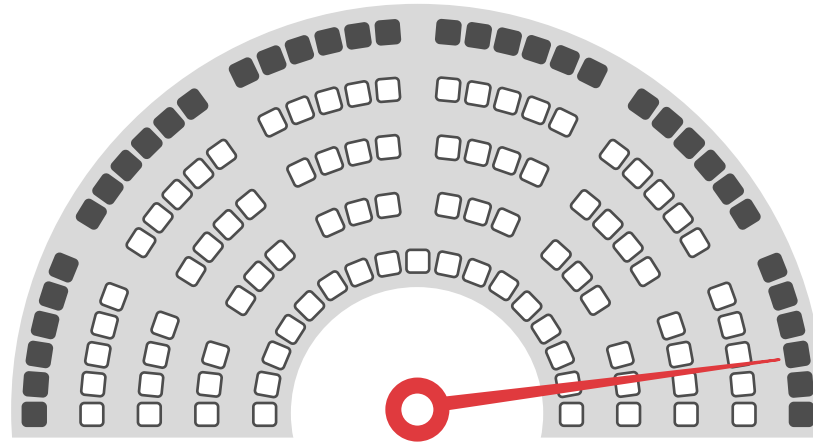




# *Optimized Democracy*

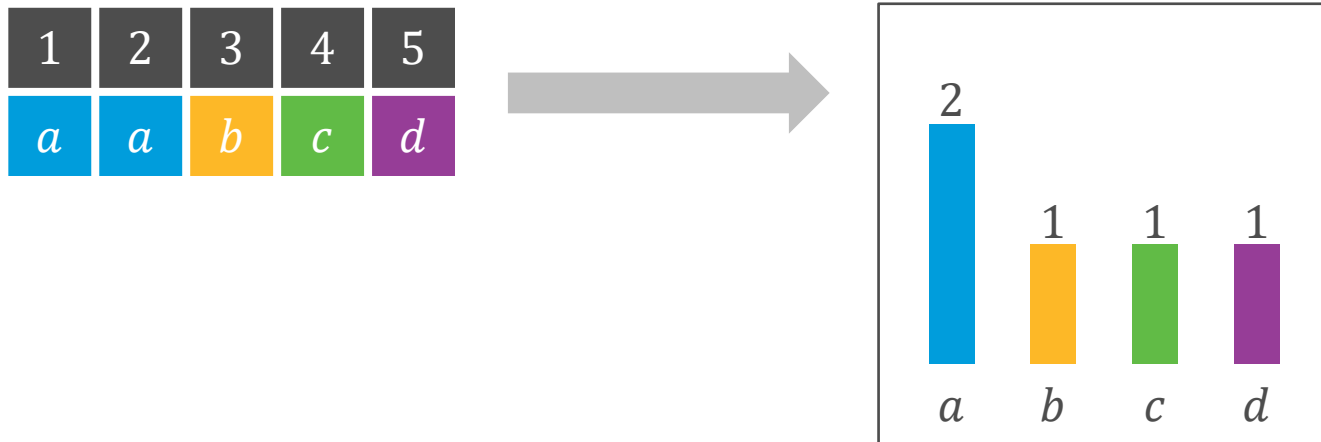
Fall 2026 | Lecture 1

**Voting Rules**

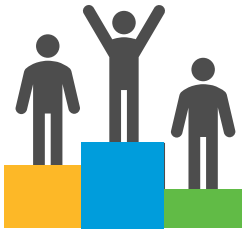
Ariel Procaccia | Harvard University

# PLURALITY

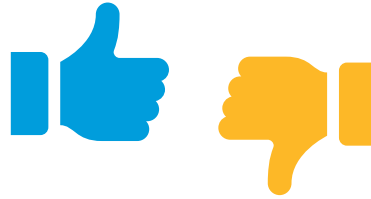
- Each person votes for a single alternative, and the alternative with most points wins
- A highly problematic voting rule!



# SOME BALLOT TYPES



Rankings



Approvals



Scores/stars

For now, we'll focus on rankings!



## Jean-Charles de Borda

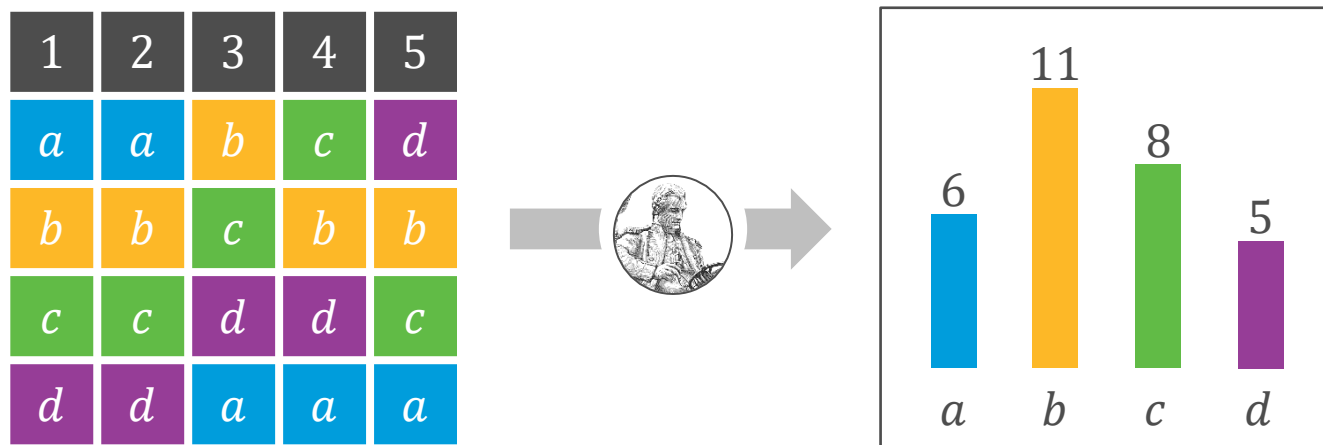
1733–1799

Mathematician, engineer, and naval officer. Also remembered as an instigator of the metric system.



# BORDA COUNT

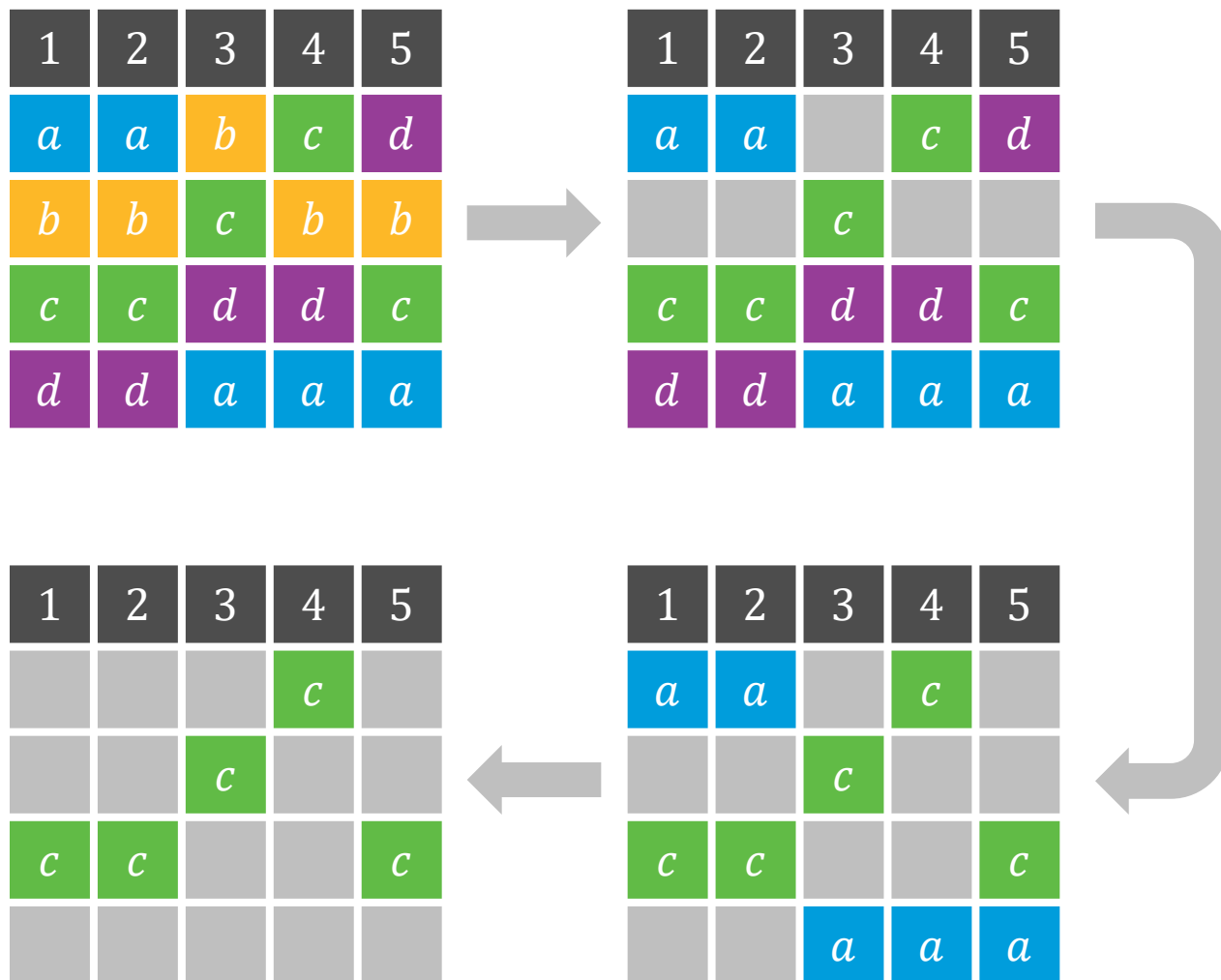
- Each voter awards  $m - k$  points to the alternative placed in the  $k$ 'th position, where  $m$  is the number of alternatives



# INSTANT-RUNOFF VOTING

- Also known as “alternative vote” and (misleadingly) “ranked-choice voting”
- Votes are tabulated in rounds, where in each round the alternative with the lowest plurality score is eliminated; last alternative left standing is the winner

# INSTANT-RUNOFF VOTING



# IRV AROUND THE WORLD



## ■ Ireland

Used for all public elections

## ■ Canada

Used in Ontario for municipal elections

## ■ Australia

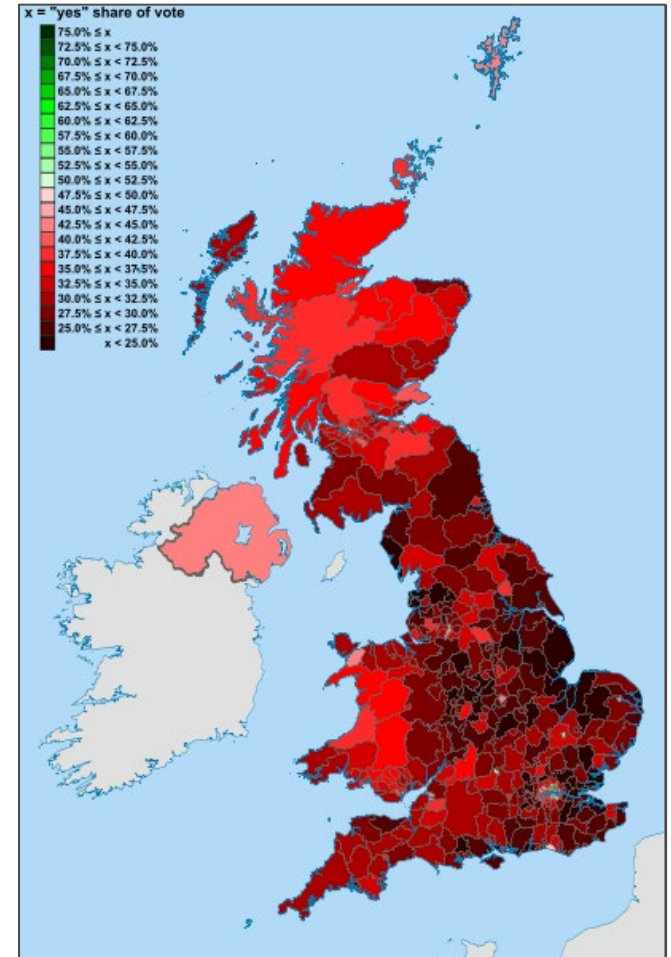
Used for parliamentary elections

## ■ USA

Used in ME and AK, and in cities like NYC and Cambridge

# BARRIERS TO ADOPTION

- UK referendum (2011): Choose between plurality and IRV as a method for electing MPs
- Academics agreed IRV is better
- But IRV was seen as beneficial to a particular politician





## Marquis de Condorcet

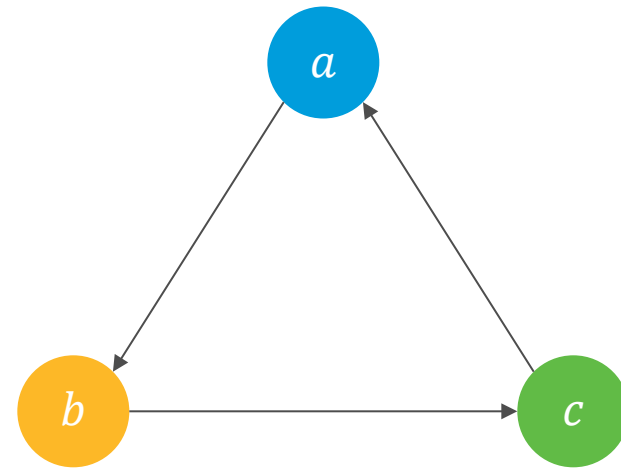
1743–1794

Philosopher, mathematician,  
enlightened nobleman. Also known  
for dying mysteriously in prison.



# THE CONDORCET PARADOX

| 1        | 2        | 3        |
|----------|----------|----------|
| <i>a</i> | <i>c</i> | <i>b</i> |
| <i>b</i> | <i>a</i> | <i>c</i> |
| <i>c</i> | <i>b</i> | <i>a</i> |



The preferences of the majority may be cyclical!

# CONDORCET CONSISTENT RULES

- A **Condorcet winner** is an alternative that defeats every other alternative in a head-to-head comparison
- A rule is **Condorcet consistent** if it always selects a Condorcet winner whenever it is presented with a profile that contains one

## Poll

Which rule is Condorcet consistent?

- Plurality
- Borda Count
- Both rules
- Neither one





## Ramon Llull

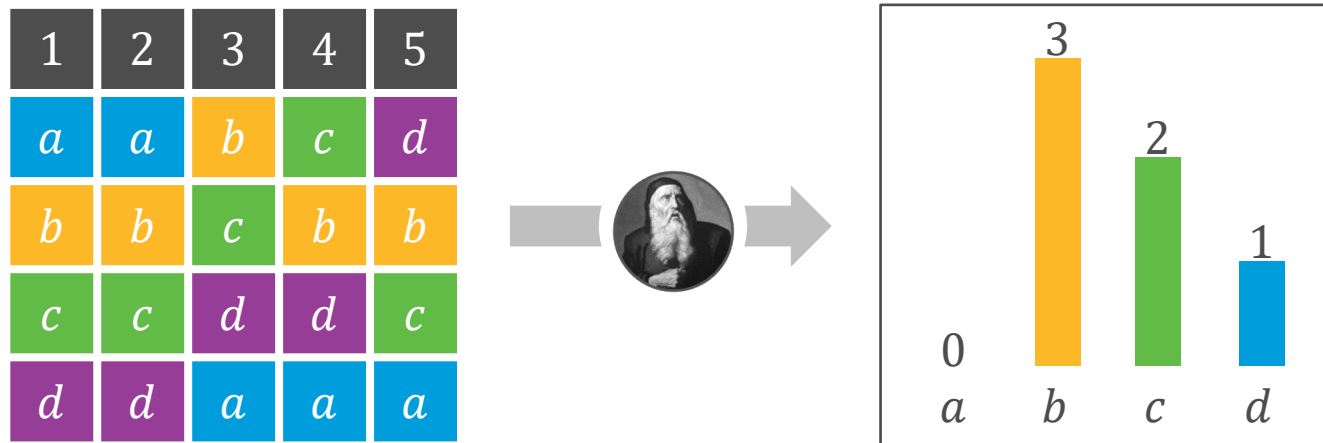
*c.* 1232–1315

Monk, missionary, and philosopher; one of the most influential intellectuals of his time. Also remembered for publishing a medieval parenting guide.



# LLULL'S RULE

- Each alternative receives one point for each head-to-head comparison it wins (as well as for tied comparisons)



- Llull's rule is Condorcet consistent



# Charles Lutwidge Dodgson

1832–1898

Professor of mathematics at Oxford,  
pioneer photographer, and beloved  
author. Also known for not plagiarizing  
Condorcet's work.

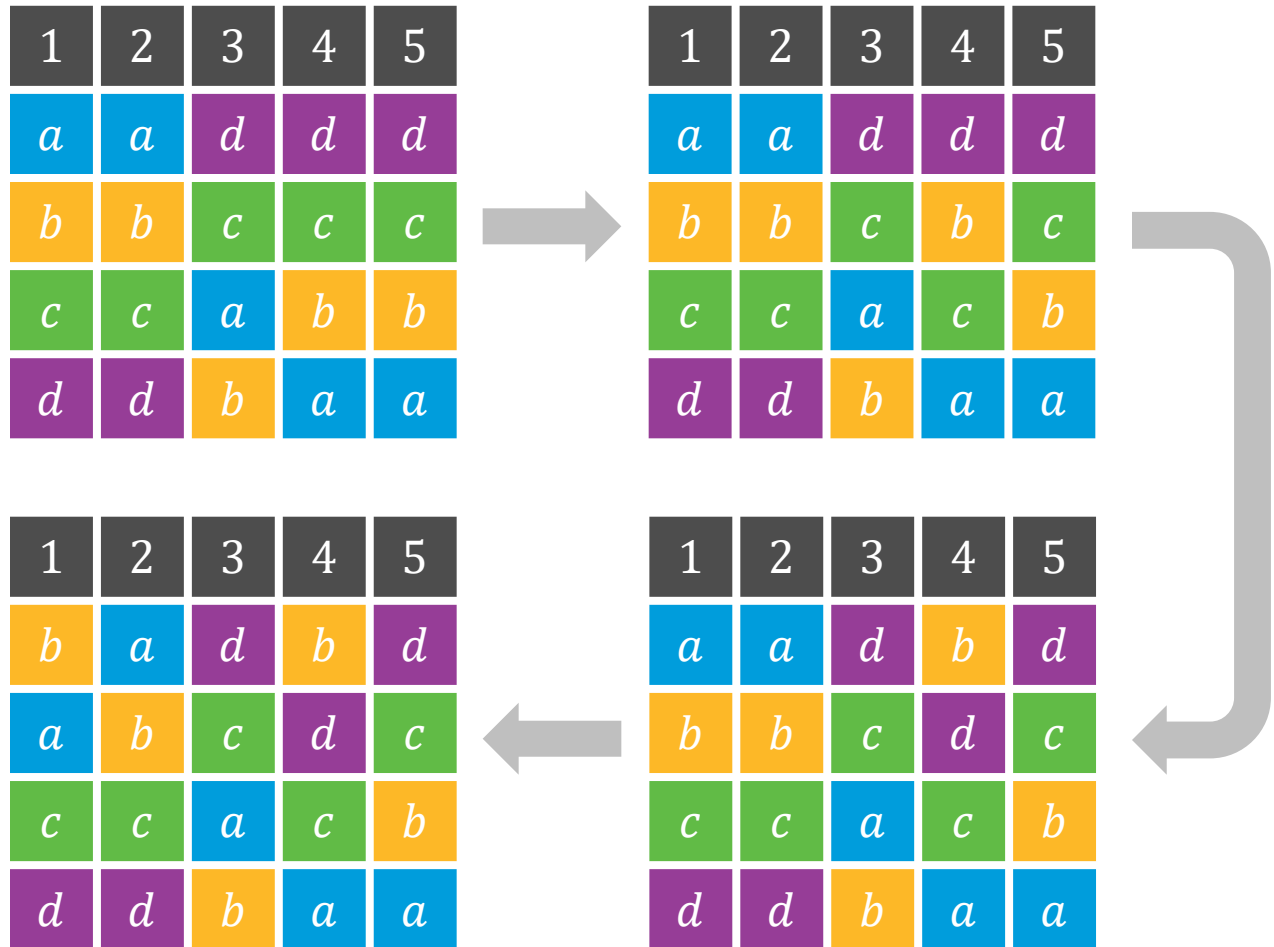


# DODGSON'S RULE

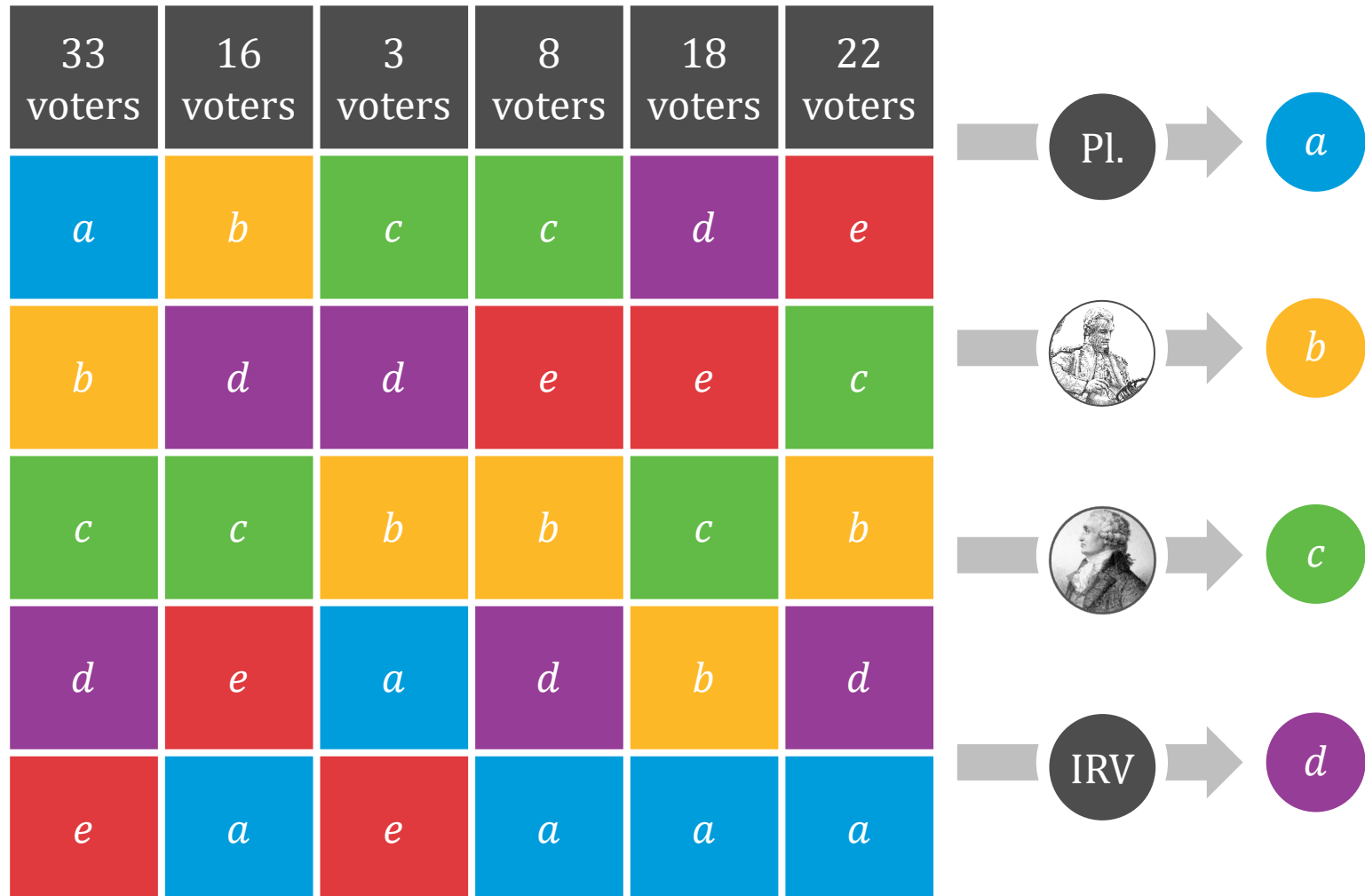
- The **Dodgson score** of an alternative  $x$  is the minimum number of swaps between adjacent alternatives needed to make  $x$  a Condorcet winner; select an alternative with minimum score
- Dodgson's rule is Condorcet consistent
- Dodgson's rule is NP-hard to compute [Bartholdi et al. 1989]

# DODGSON'S RULE

What is the Dodgson score of  $b$ ?



# AWESOME EXAMPLE



# One rule to rule them all?