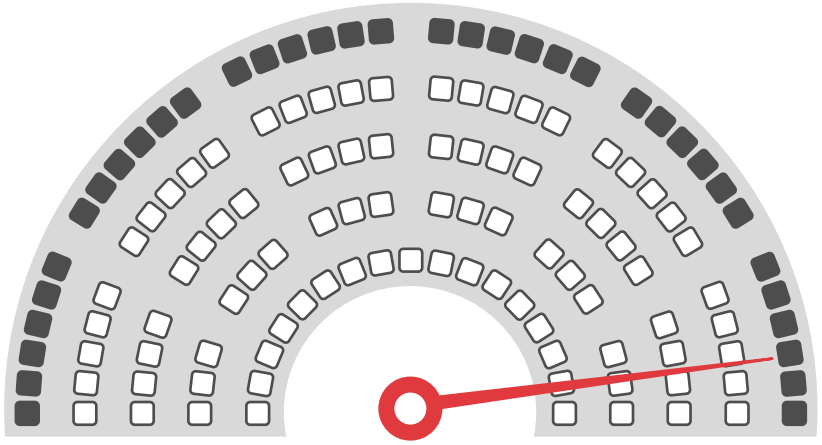


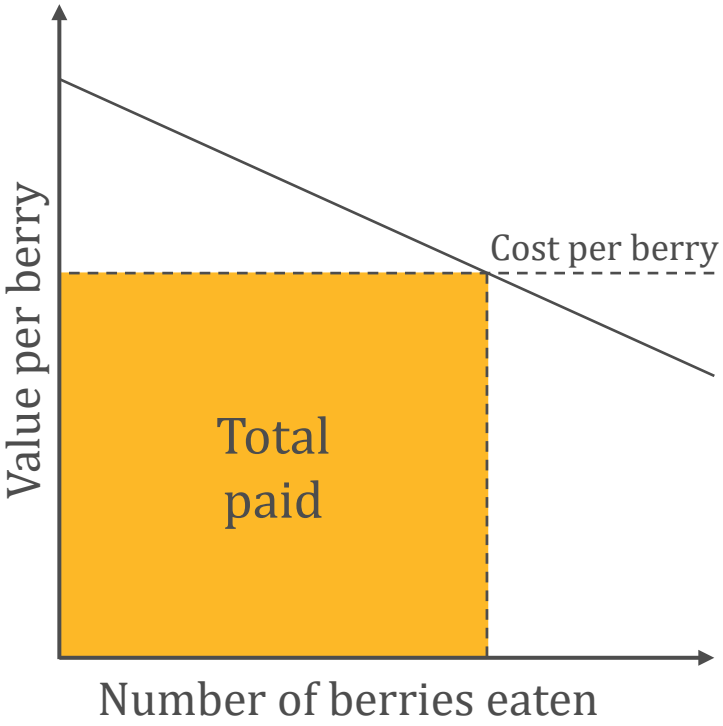
Optimized Democracy

Fall 2025

Quadratic Voting and Funding

Ariel Procaccia | Harvard University

PRIVATE GOODS

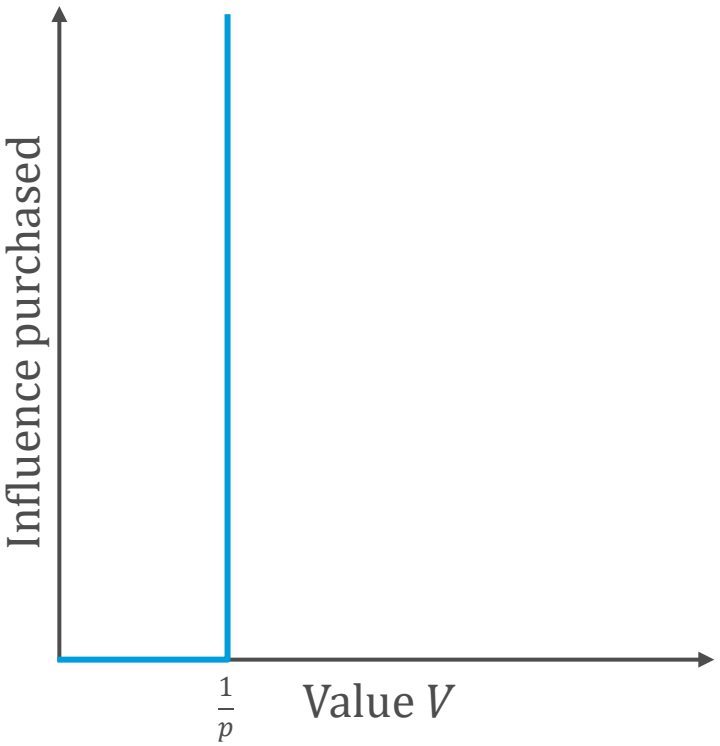


In a setting with private goods and money, the question of how to aggregate preferences is easy, because it's separable across individuals: I will buy berries until my marginal value for the next berry is equal to its cost

PUBLIC GOODS

- In the setting of public goods or decisions, that is no longer the case
- Given the funding cuts, Harvard will buy a ping pong table only if faculty members contribute $400
- Let's say (although this technically doesn't make sense) that every dollar I donate increases the probability the table is funded by p, and my own value is V
- If $pV \geq 1$, I contribute (a lot), and if $pV < 1$, I don't contribute

ONE DOLLAR, ONE VOTE

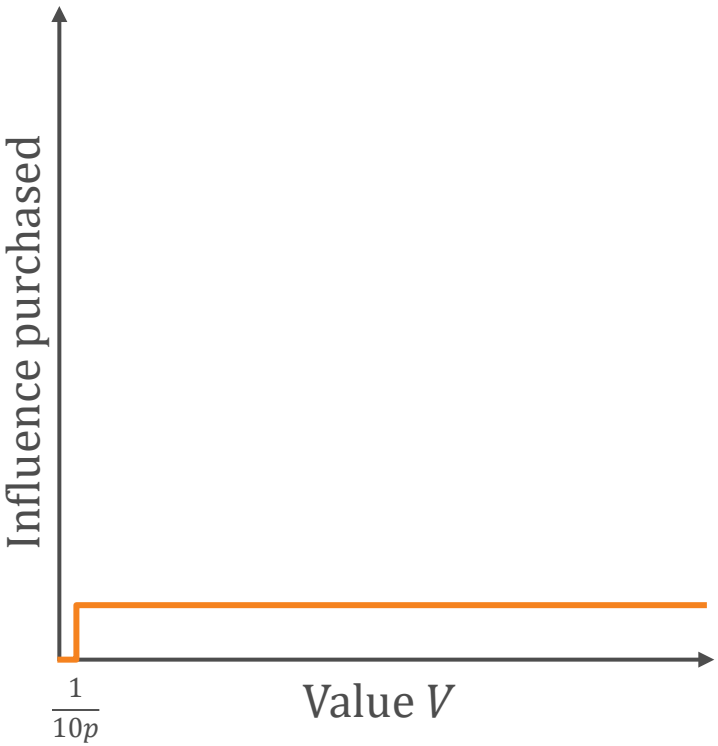


Under “one dollar, one vote,” I would want to purchase as much influence as possible when $pV \geq 1$, and the table is funded if someone really wants it

ONE PERSON, ONE VOTE

- Now imagine that Harvard will buy the ping pong table only if 400 faculty members vote in favor
- It costs me \$0.1 (in effort) to cast a vote
- I'll cast a vote if $pV \geq 0.1$, but I can't purchase more influence (or the cost of purchasing more is ∞)

ONE PERSON, ONE VOTE

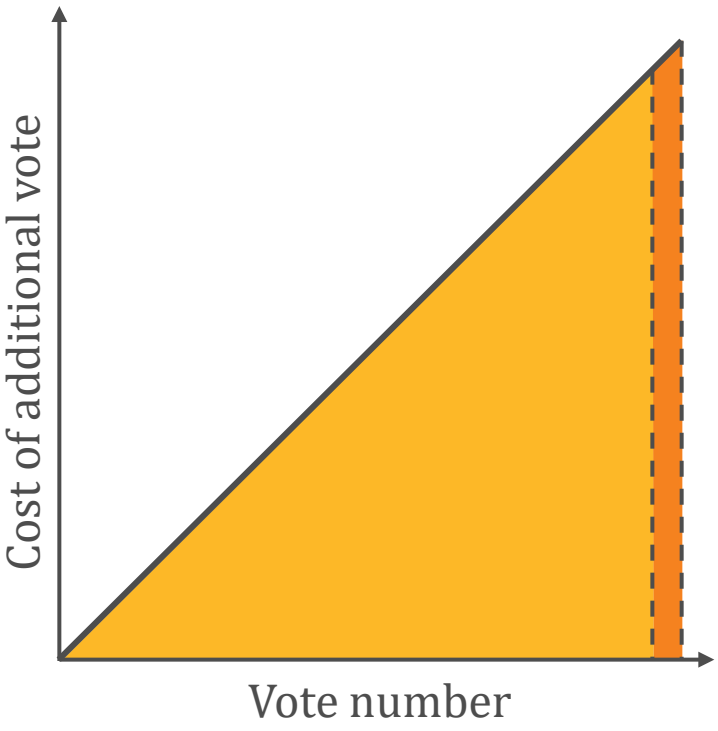


Under “one person, one vote,” I would want to purchase one unit of influence when $pV \geq 0.1$, and the table is funded if many people want it at least a little

QUADRATIC PAYMENTS

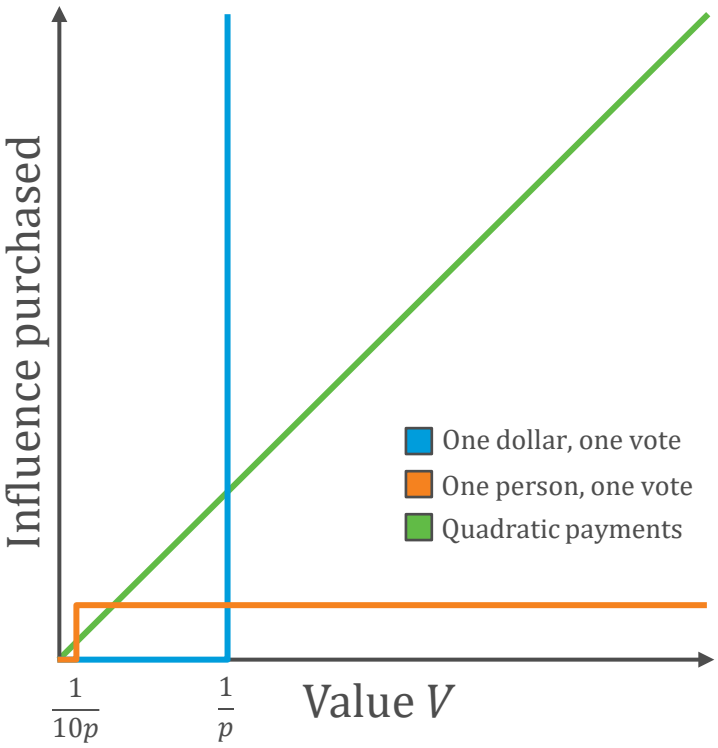
- If my value is V, I'm willing to pay up to $V/100$ for an additional 1% chance of funding the table
- If your value is $V/2$, you're willing to pay $V/200$ for the same unit of influence
- What's a scheme that would ensure that you would buy half as many votes as me?
- Key idea: My kth vote (or unit of influence) costs $\$k$

QUADRATIC PAYMENTS



Cost of kth vote (orange) is k, but overall cost (yellow and orange) is $k(k+1)/2 = \Theta(k^2)$

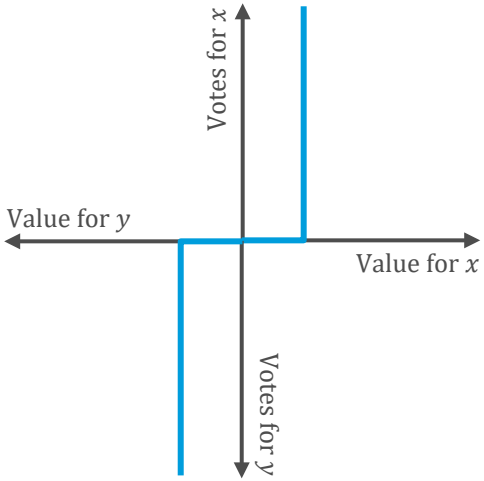
QUADRATIC PAYMENTS



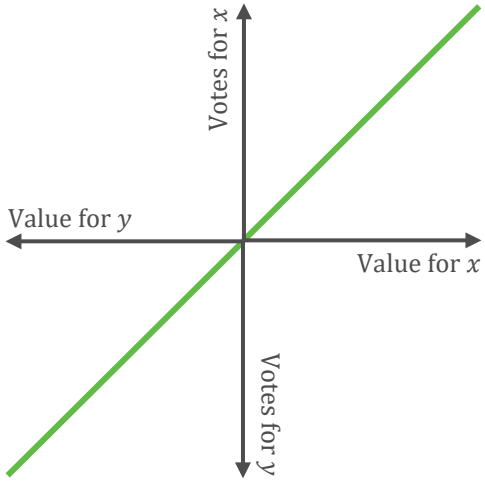
Under quadratic payments, the influence I purchase is proportional to my value

QUADRATIC VOTING: BINARY CASE

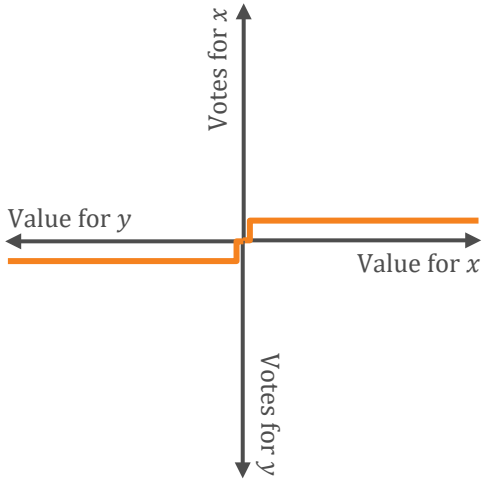
- Suppose there are two alternatives, x and y
- As before, my kth vote in favor of an alternative costs k
- Typically, the quadratic formula is simplified: casting k votes costs k^2



One dollar, one vote



Quadratic voting



One person, one vote

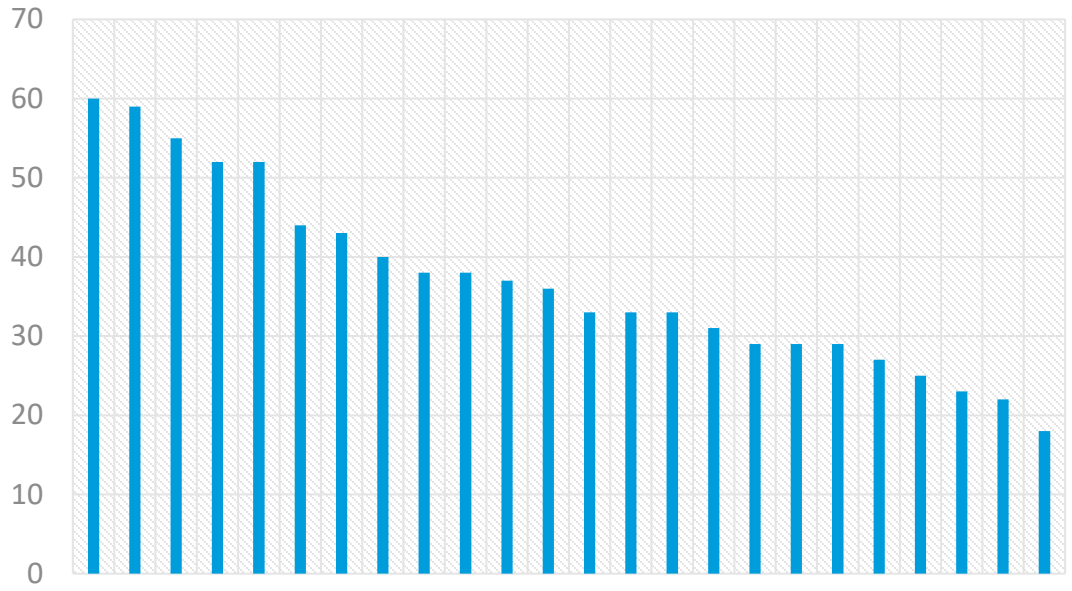
QUADRATIC VOTING: GENERAL CASE

- The idea behind quadratic voting easily extends to the case of m alternatives
- If a voter casts k votes for an alternative, they pay k^2
- Notably, it is not a voter's total number of votes that's squared, but their votes for each alternative

DEMO OF QUADRATIC VOTING

https://qv.geek.sg/vote?election=2a828d37-5738-458b-b49e-cc3cf5cb08a0

QUADRATIC VOTING IN COLORADO



60	Equal Pay For Equal Work Act
59	Demographic Notes For Certain Legislative Bills
55	CDPHE Maternal Mortality Review Committee
52	Youth Mental Health Ed & Suicide Prevention
52	Grants For Property Tax Rent & Heat
44	Mental Health Parity Insurance Medicaid
43	Health Care Cost Savings Act Of 2019
40	Increase Student Aid Application Completion Rates
38	School Nurse Grant Program
38	Comprehensive Human Sexuality Education
37	GreenHouse Gas Pollution Impact In Fiscal Notes
36	School Incentives To Use Colorado Food & Producers
33	Expand Child Nutrition School Lunch Protection Act
33	Sexual Assault While In Custody Or Detained
33	Office Of Public Guardianship Operation Conditions State
31	Court Administrator Reminder Program Increase Tax
29	Credit Allocation Affordable Housing Modify Innovative
29	System Enhancement Regulate Student Education Loan Servicers
29	Motor Vehicle Income Tax Credits Wildfire Mitigation
27	Wildland-Urban Interface Areas Child Care Expenses Tax
25	Credit Low-income Families Investment In Primary Care
23	To Reduce Health Cost CO Child Abuse Response And
22	Evaluation Network Child & Youth Behavioral Health
18	Colorado Resiliency Office Reauthorization Funding

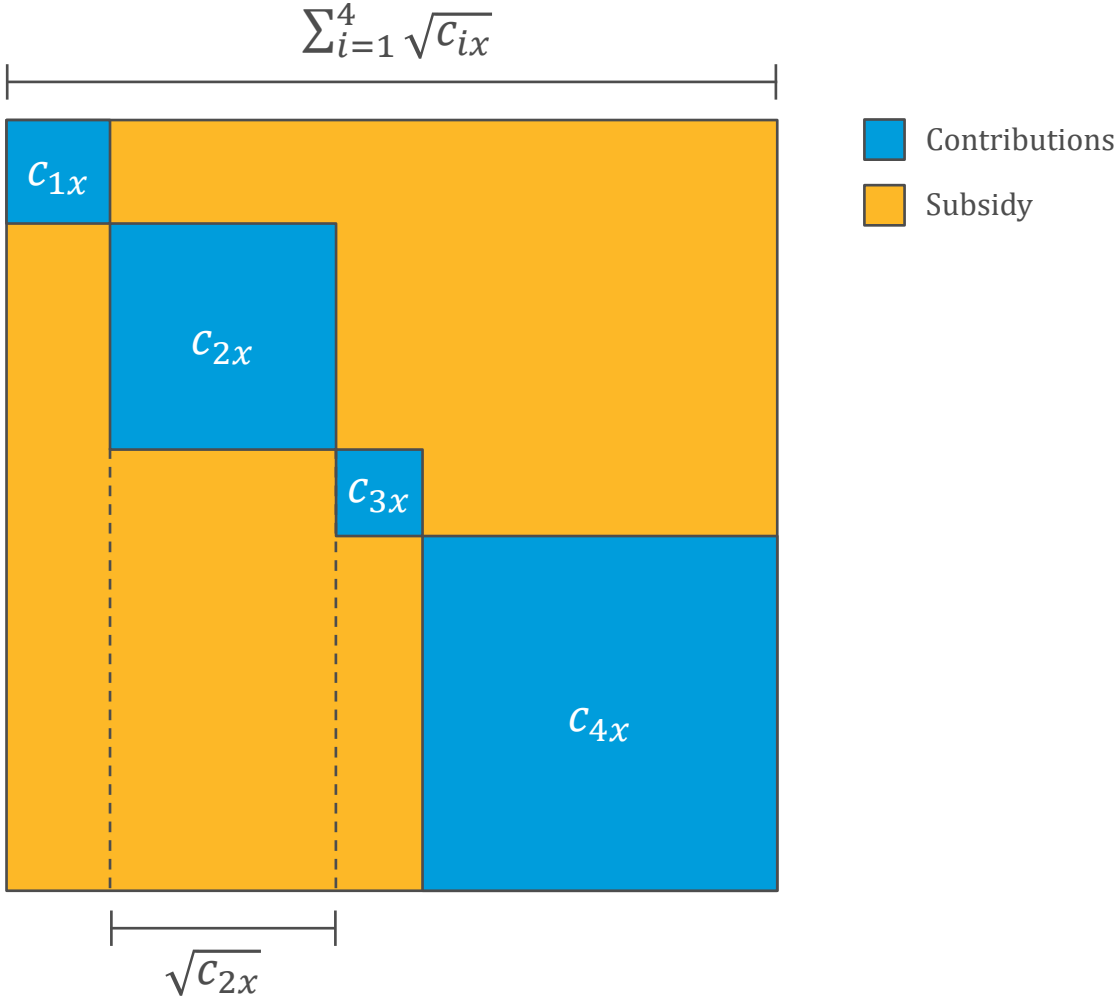
QUADRATIC FUNDING

- Anyone can propose public goods (alternatives)
- Let c_{ix} be the contribution of player $i \in N$ to alternative x, then the payment to x is

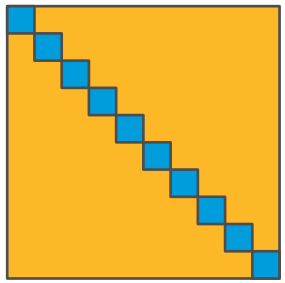
$$\left(\sum_{i=1}^{n} \sqrt{c_{ix}}\right)^2$$

- If $n > 1$, this payment is greater than $\sum_i c_{ix}$ and the difference comes from a subsidy pool

QUADRATIC FUNDING



QUADRATIC FUNDING



$c_{ix} = 1$ for $i = 1, \ldots, 10$
Subsidy $= 90$

$c_{1x} = 10$
Subsidy $= 0$

Many small contributions receive a much larger subsidy than few large contributions!

A MODEL FOR PUBLIC FUNDING

- As usual, the players are $N = \{1, \dots, n\}$ and the alternatives are A
- Let $v_{ix}(f_x)$ be the utility player i receives from alternative x being funded at level f_x
- Each v_{ix} is assumed to be smooth, strictly concave, and strictly increasing
- Each player $i \in N$ contributes c_{ix} to x
- The utility of i is $\sum_{x \in A}(v_{ix}(f_x) - c_{ix})$
- A mechanism takes as input c_{ix} for all $i \in N$ and $x \in A$, and outputs f_x for all $x \in A$

WELFARE AND OPTIMALITY

- Denote $u_x = \sum_{i \in N} v_{ix}$
- The social welfare is $\sum_{x \in A}(u_x(f_x) - f_x)$
- Note that the social welfare includes the deficit $\sum_{x \in A}(f_x - \sum_{i \in N} c_{ix})$
- Social welfare is maximized when for all $x \in A$, $f_x = 0$ if $u_x'(0) \leq 1$, and otherwise f_x is the unique value satisfying $u_x'(f_x) = 1$
- A mechanism is welfare-maximizing if it satisfies these conditions

PRIVATE CONTRIBUTIONS

- Under the private contributions mechanism, $f_x = \sum_{i \in N} c_{ix}$
- For all $i \in N$ and $x \in A$, i seeks to maximize $v_{ix}(f_x) - c_{ix}$
- If $c_{ix} > 0$ then $v'_{ix}(f_x) = 1$
- However, when there is more than one contribution to x, it could be the case that $u'_x(f_x) > 1$
- Therefore, the mechanism is suboptimal

PRIVATE CONTRIBUTIONS: EXAMPLE

- One alternative x
- $n = 10$ players
- $v_{ix}(f_x) = \sqrt{f_x}$ for all $i \in N$

Poll

Under private contributions, what is the value of f_x that maximizes social welfare?

- 1
- 5
- 10
- 25

QUADRATIC FUNDING IS OPTIMAL

- Recall that under quadratic funding, $f_x = \left(\sum_{i \in N} \sqrt{c_{ix}}\right)^2$
- Theorem: Quadratic funding is optimal
- Proof:
  - Player i maximizes $v_{ix}(f_x) - c_{ix}$
  - If $c_{ix} > 0$ then

$$\frac{2v'_{ix}(f_x) \cdot \left(\sum_{j \in N} \sqrt{c_{jx}}\right)}{2\sqrt{c_{ix}}} = 1 \Rightarrow v'_{ix}(f_x) = \frac{\sqrt{c_{ix}}}{\sum_{j \in N} \sqrt{c_{jx}}}$$

  - Therefore,

$$u'_x(f_x) = \sum_{i \in N} v'_{ix}(f_x) = 1 \quad \blacksquare$$

QUADRATIC FUNDING VIA GITCOIN

$67 million distributed to more than 3715 projects through quadratic funding

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