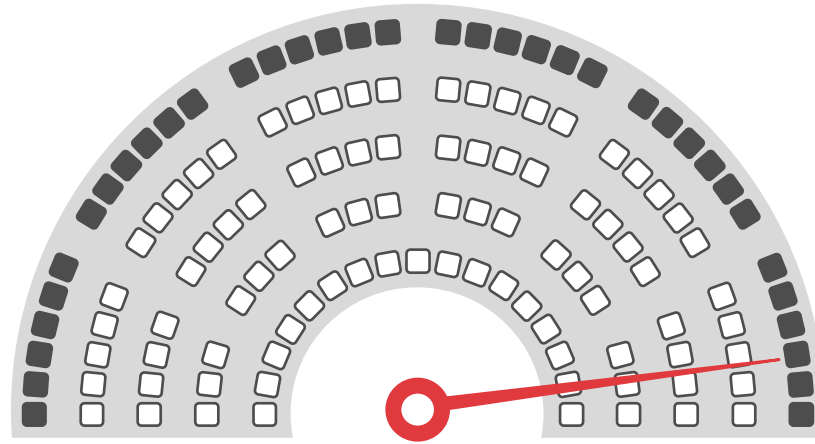




Optimized Democracy

Spring 2024 | Lecture 9

Participatory Budgeting

Ariel Procaccia | Harvard University

PARTICIPATORY BUDGETING



Allocation of a city's budget based
on the votes of residents

PARTICIPATORY BUDGETING



Porto Alegre

Brazil

Since 1989



Paris

France

€76M (2023)



Madrid

Spain

€50M (2022)



Boston

USA

Office of PB (2021)



EXAMPLE BALLOT: BOSTON

You have selected **2 / 4** projects.

Water Bottle Refill Stations at Parks

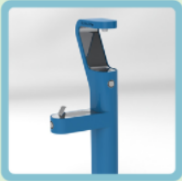
Multiple installations of the water fountain and bottle refill designs at different parks in Boston.

Estimated Cost: \$260,000

Location: Parks around Boston, West roxbury, Dorchester, South Boston, Roslindale, Downtown, Fenway

✓ Selected

Remove



Click image for slide show

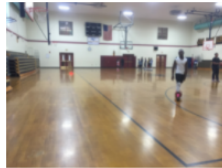
BLA Gym Renovations

No more net/less hoops and broken bleachers--the BLA gym will be revamped to include newly painted hoops and floors, and repaired bleachers.

Estimated Cost: \$475,000

Location: 205 Townsend Street, Roxbury

Select



Click image for slide show

Ringer Park Renovation

Renovate the main path in Ringer Playground so it is no longer cracked and safer to walk on.

Estimated Cost: \$280,000

Location: Ringer Park, Brighton

✓ Selected

Remove



Click image for slide show

Bike Lane Installation

After a study, bike lanes will begin to be installed around Charlestown Navy Yard, Bunker Hill housing, and Charlestown High.

Estimated Cost: \$200,000

Location: Charlestown

Select



Click image for slide show

Wicked Free Wifi 2.0

Wicked free Wi-Fi 2.0 provides Wi-fi at locations with young people.

Estimated Cost: \$119,000

Location: Various High Schools and Community Centers, Dorchester, Roxbury, East Boston, Charlestown

Select



Click image for slide show

Track at Walker Park

Build a track that goes around the field that is behind Walker Playground.

Estimated Cost: \$240,000

Location: Walker Playground on Norfolk St. and Astoria St., Mattapan

Select

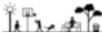


Click image for slide show

EXAMPLE BALLOT: NYC

Selected \$400,000 of \$2,000,000 total budget.
You still have \$1,600,000 left.

Exit Help **Español** 中文



Parks & Recreation

La Isla Gardens Renovation of green spaces including installation of water zone device in community gardens, wood for constructing benches and tables, and materials for a garden. Estimated Cost: \$300,000 Location: 96 West 163rd St. (Bronx - Highbridge) ✓ Selected Remove	Pier 107 Renovations to Pier 107: Refinishing walking areas to allow for public access. Estimated Cost: \$350,000 Location: 107th St. on E.River Esplanade (El Barrio/East Harlem) Select
Blake Hobbs Playground: Court Renovation Renovate the playground, basketball court and handball court. Estimated Cost: \$300,000 Location: E.102nd-104th Sts. & 2nd Ave. (El Barrio/East Harlem) Select	Improvements to Diamante Garden Installation of an irrigation system; construction of a brick path for safety and accessibility; reconstruction of the stage flooring, roof and planting bench; upgrade picnic area. Estimated Cost: \$100,000 Location: 306-310 E.118th St. & 2nd Ave. (El Barrio/East Harlem) ✓ Selected Remove
Thomas Jefferson Park: Dog Run Upgrades Upgrade Tom's Dog Run Park's water fountain and irrigation system; install new fences and gates; create section for smaller dogs. Estimated Cost: \$200,000 Location: 1st Ave btw E.111 St.-114 St. (El Barrio/East Harlem) Select	

PARTICIPATORY BUDGETING MODEL

- Each voter $i \in N$ still casts an approval vote $\alpha_i \subseteq A$
- Each $x \in A$ has a cost $c(x)$, and there is a budget B
- The outcome is a budget-feasible subset $W \subseteq A$ such that $c(W) = \sum_{x \in W} c(x) \leq B$
- For now, we still assume that the utility of voter $i \in N$ for $W \subseteq A$ is $u_i(W) = |\alpha_i \cap W|$

APPROVAL VOTING, REVISITED

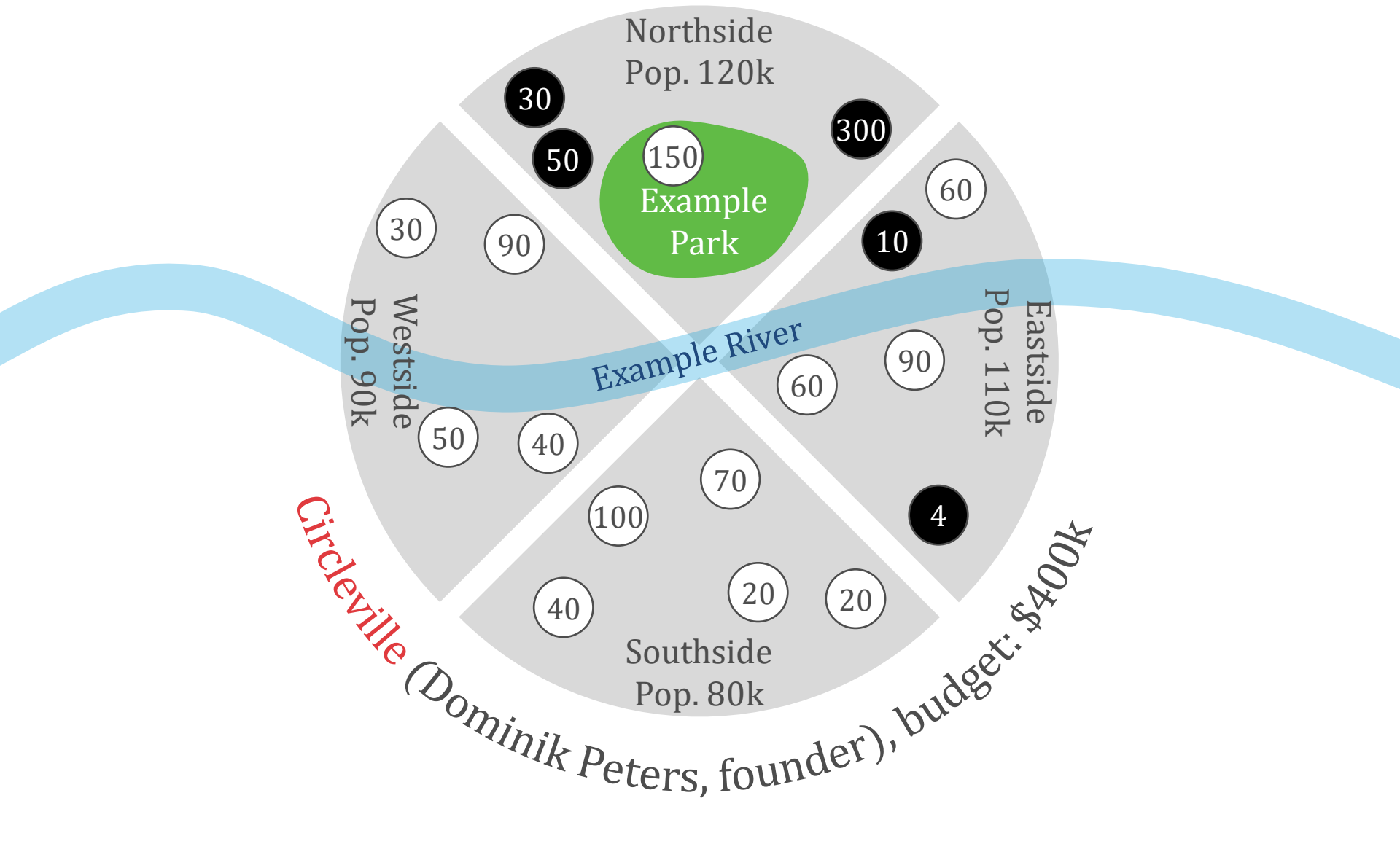
- A natural interpretation of approval voting is to maximize $\sum_{i \in N} u_i(W)$ subject to the budget constraint, which amounts to a knapsack problem
- In practice, a greedy algorithm is often used, which adds alternatives in order of approval score, skipping those that are unaffordable

GREEDY AV: EXAMPLE

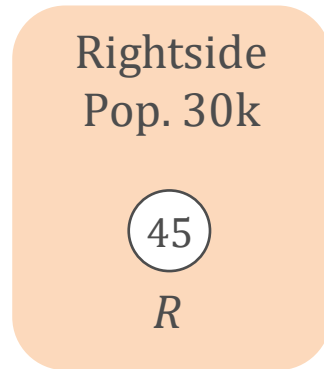
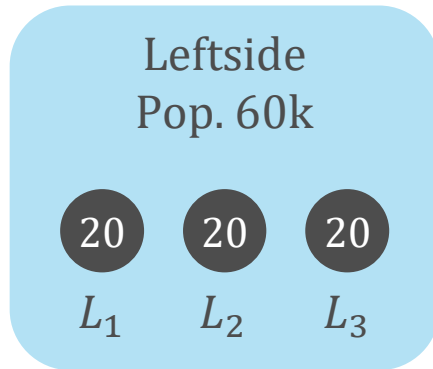
Paris, 4th District, 2019

Votes	Cost (€)	Greedy	Optimal
788	30,000	✓	✓
706	15,000	✓	✓
702	300,000	✓	✓
655	1,000,000	✓	
649	120,000	✓	✓
630	200,000		✓
528	20,000		✓
491	15,000		✓
473	20,000		✓
453	5,000		✓
410	150,000		✓
315	350,000		✓
265	30,000		✓
240	10,000		✓
228	120,000		✓
Total:		1,465,000	1,385,000

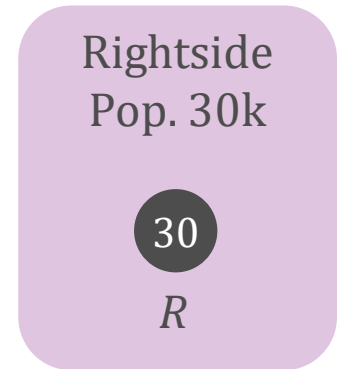
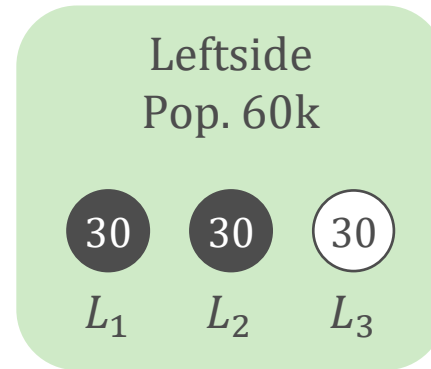
GREEDY AV: EXAMPLE



PAV TO THE RESCUE?



Rectangleville, budget: \$90k



New Rectangleville, budget: \$90k

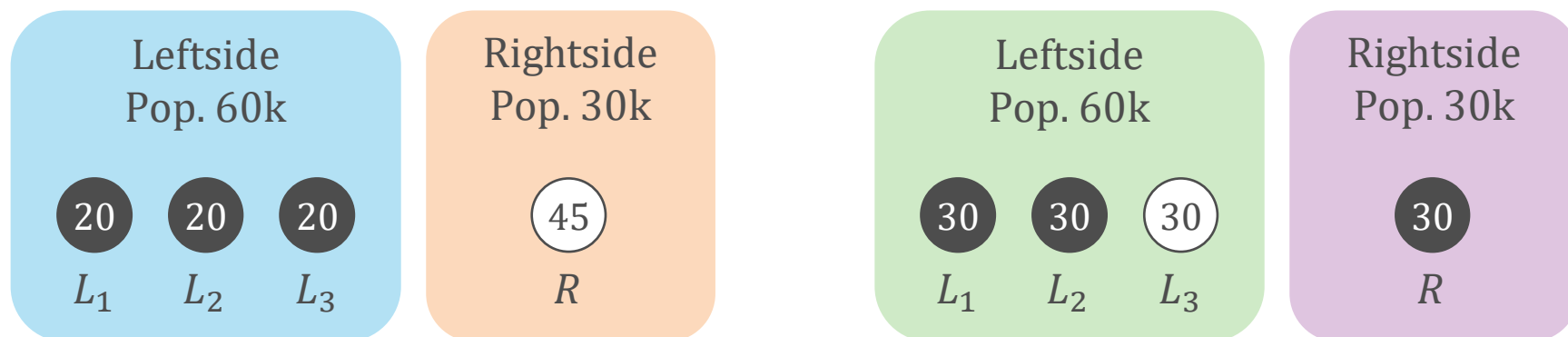
Poll

In which of the two towns (R and NR) does PAV give the proportional outcome shown in black?

- Only R
- Only NR
- Both
- Neither



PAV TO THE RESCUE?



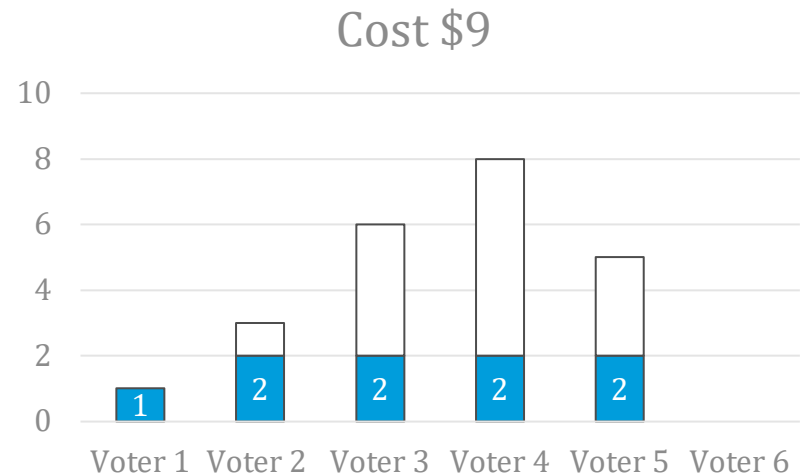
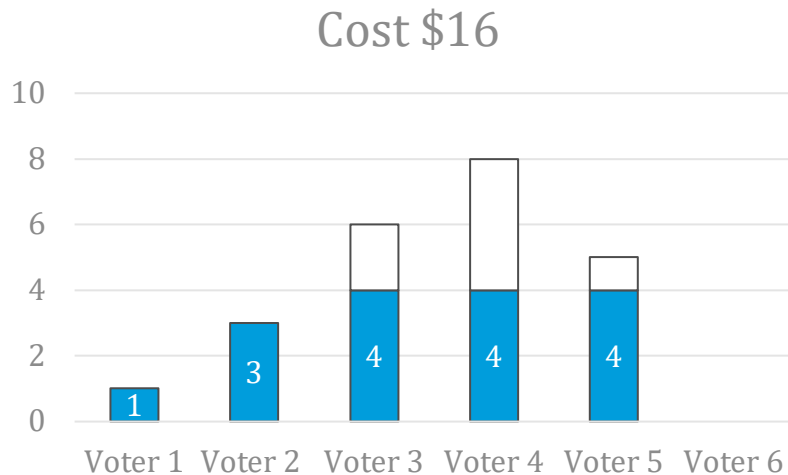
Rectangleville, budget: \$90k

New Rectangleville, budget: \$90k

- PAV can't distinguish between the two examples and therefore cannot identify which outcomes are proportional
- **Theorem:** Every voting rule that only depends on the collection of budget-feasible subsets must fail proportionality, even on instances with a district structure

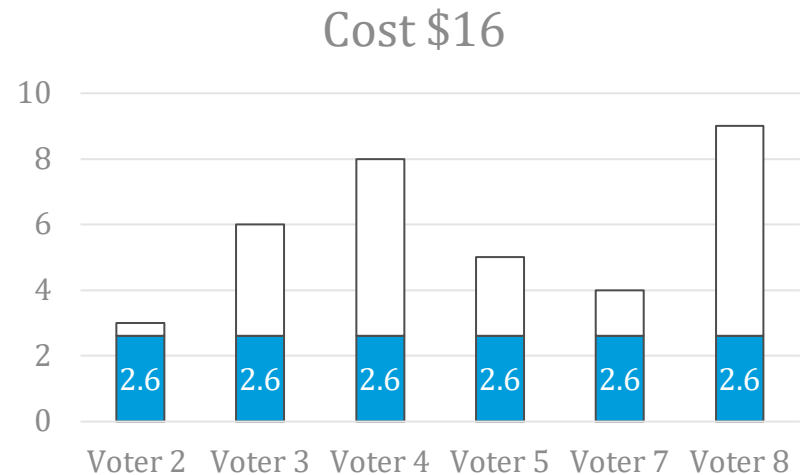
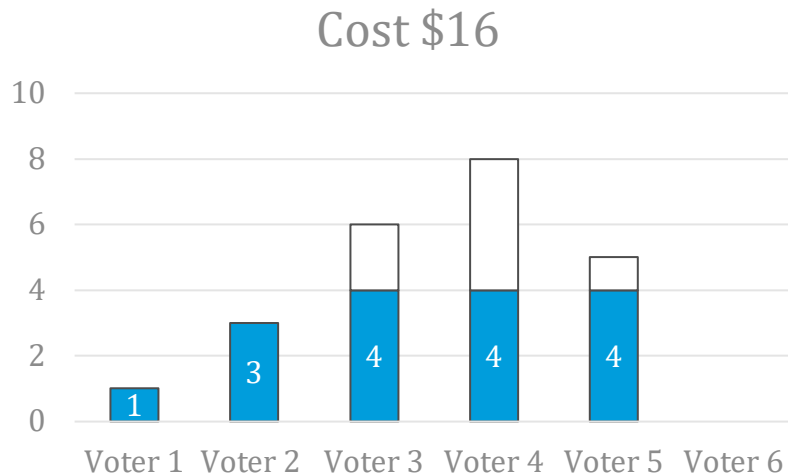
METHOD OF EQUAL SHARES

- Give a budget of B/n to each voter
- Do until the budget runs out:
 - For each alternative, divide its cost as evenly as possible among its supporters
 - Fund an affordable alternative with the lowest max payment



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METHOD OF EQUAL SHARES

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 - For each alternative, divide its cost as evenly as possible among its supporters
 - Fund an affordable alternative with the lowest max payment
- **Extended justified representation** (for participatory budgeting): For all $S \subseteq N$ such that $|S| \geq \gamma \cdot n$ and $T \subseteq \bigcap_{i \in S} \alpha_i$ such that $c(T) \leq \gamma \cdot B$ there is $i \in S$ such that $u_i(W) \geq u_i(T)$
- **Theorem:** The Method of Equal Shares satisfies EJR
- Used in 2023 in Aarau (Switzerland), Wieliczka (Poland) and Świecie

A DIFFERENT PERSPECTIVE

- In the 2019 PB election of Paris' 16th District, a refurbishment of a sports facility received 775 votes and cost €560k, and materials for a school project received 670 votes and cost €3k
- Let us consider general additive utilities, i.e., $u_i(W) = \sum_{x \in W} u_i(x)$, where $u_i(x) \in \mathbb{R}^+$
- The goal is to find $W \subseteq A$ that maximizes the social welfare $sw(W, \mathbf{u}) = \sum_{i \in N} u_i(W)$ subject to the budget constraint $c(W) \leq B$
- But we don't necessarily want to ask voters to explicitly report utilities
- Instead, we'll ask voters to cast votes in some **input format**

INPUT FORMATS

Ranking
by value



Knapsack
voting



Budget
9

Ranking
by VFM



Threshold
approval



Threshold
5



Utility 6
Cost 6



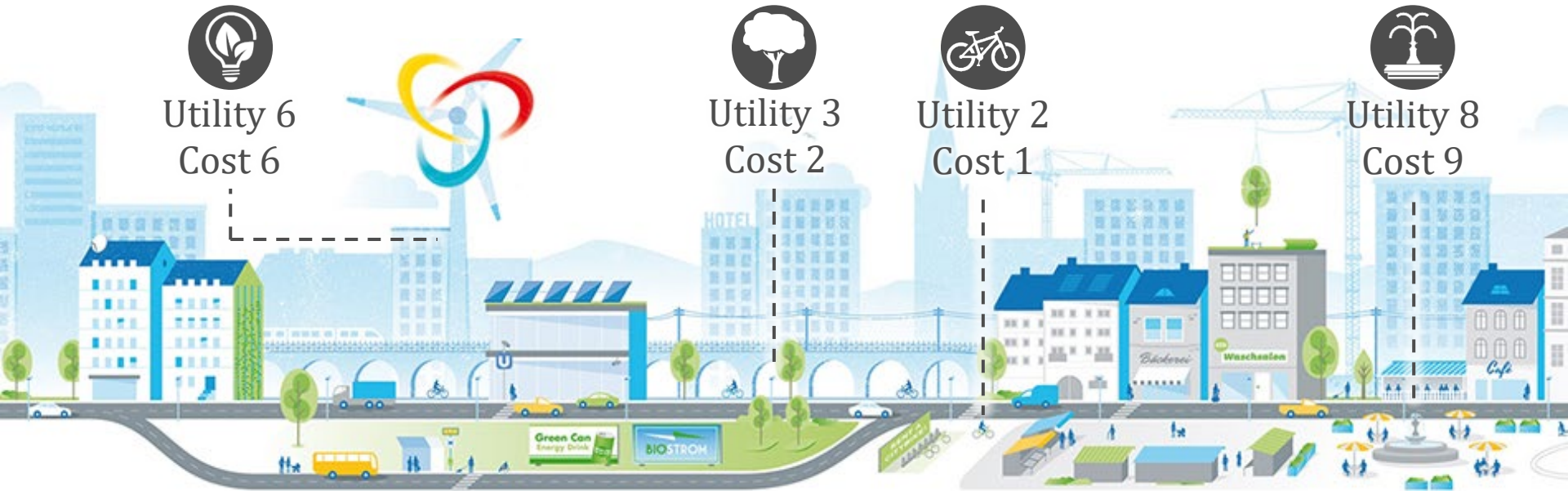
Utility 3
Cost 2



Utility 2
Cost 1



Utility 8
Cost 9



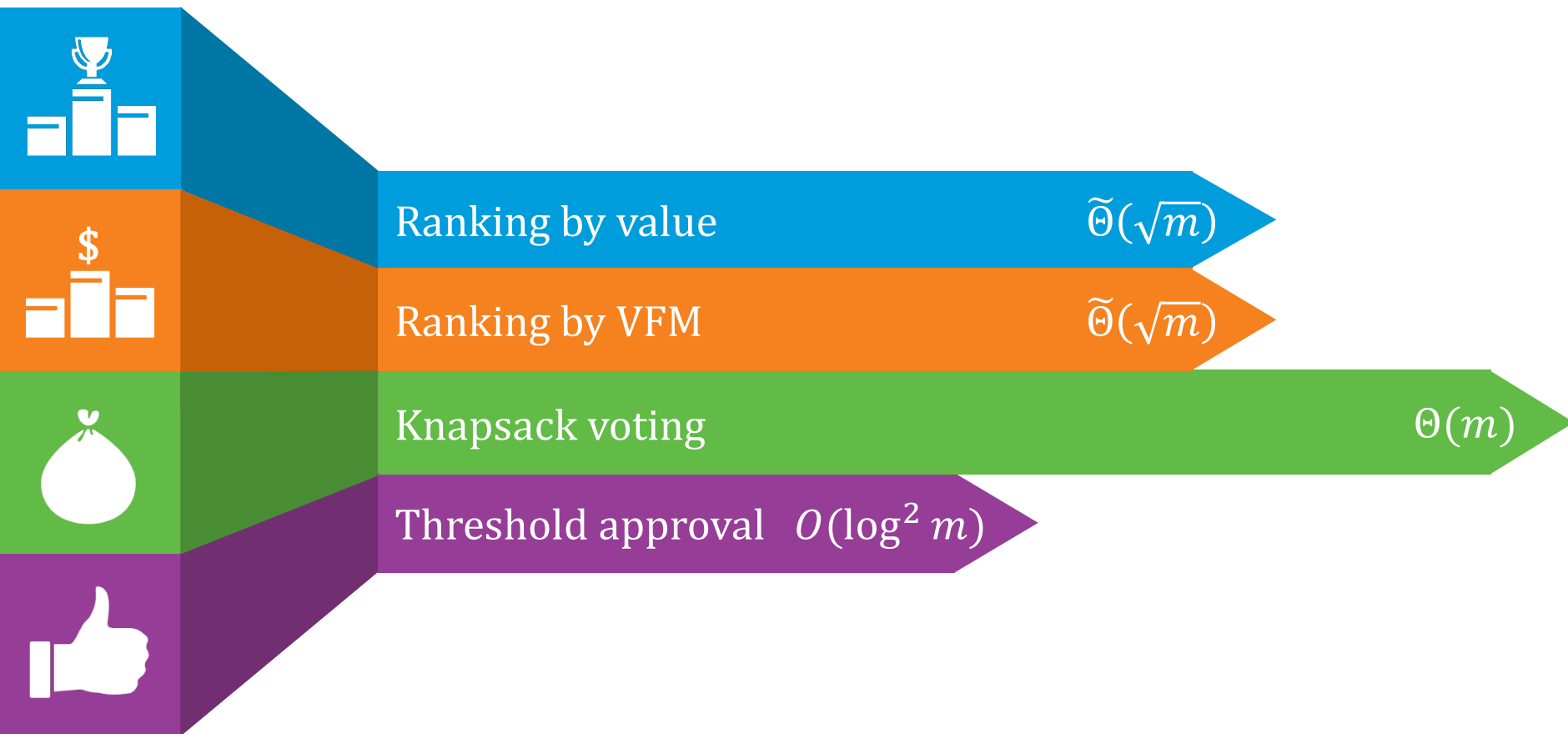
IMPLICIT UTILITARIAN VOTING

- Voter i reports a vote σ_i that is **consistent** with u_i ; denote $u_i \triangleright \sigma_i$
- A randomized voting rule f maps an input profile σ to a distribution over budget-feasible subsets of alternatives
- The **distortion** of f on σ is

$$\max_{u \triangleright \sigma} \frac{\max_{W \subseteq A: c(W) \leq B} \text{SW}(W, u)}{\mathbb{E}[\text{SW}(f(\sigma), u)]}$$

- Associate an input format with the **worst-case** distortion of the **best** voting rule

THEORETICAL DISTORTION



BIBLIOGRAPHY

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Proportional Participatory Budgeting with Cardinal Utilities. NeurIPS 2021.

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and N. Shah. **Preference Elicitation for Participatory Budgeting.** Management Science, 2021.

