

Optimized Democracy

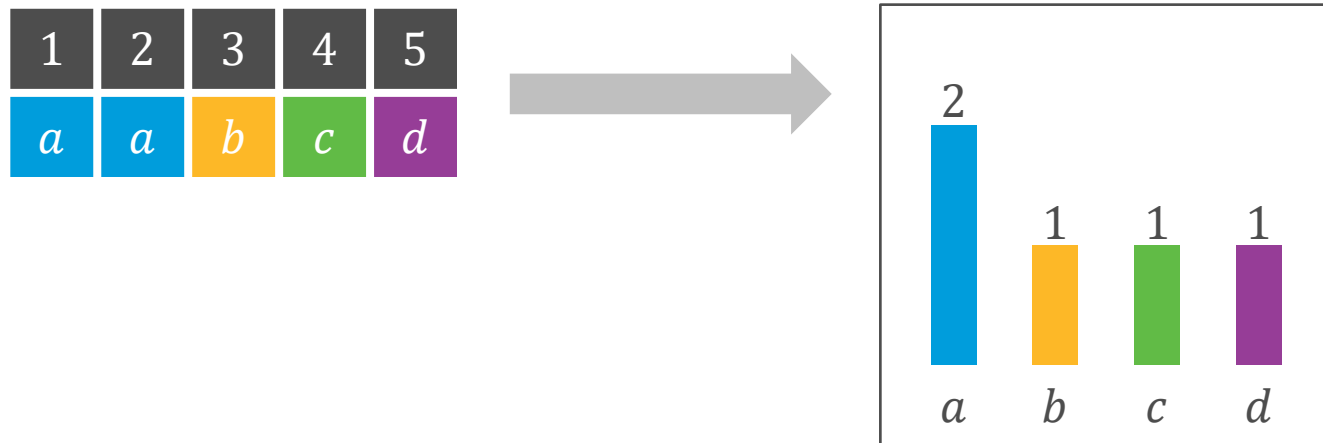
Spring 2024 | Lecture 1

Voting Rules

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PLURALITY

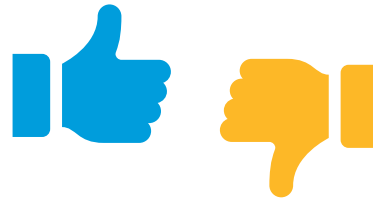
- Each person votes for a single alternative, and the alternative with most points wins
- A highly problematic voting rule!



SOME BALLOT TYPES



Rankings



Approvals



Scores/stars

We will focus on rankings!



Jean-Charles de Borda

1733–1799

Mathematician, engineer, and naval officer. Also remembered as an instigator of the metric system.



BORDA COUNT

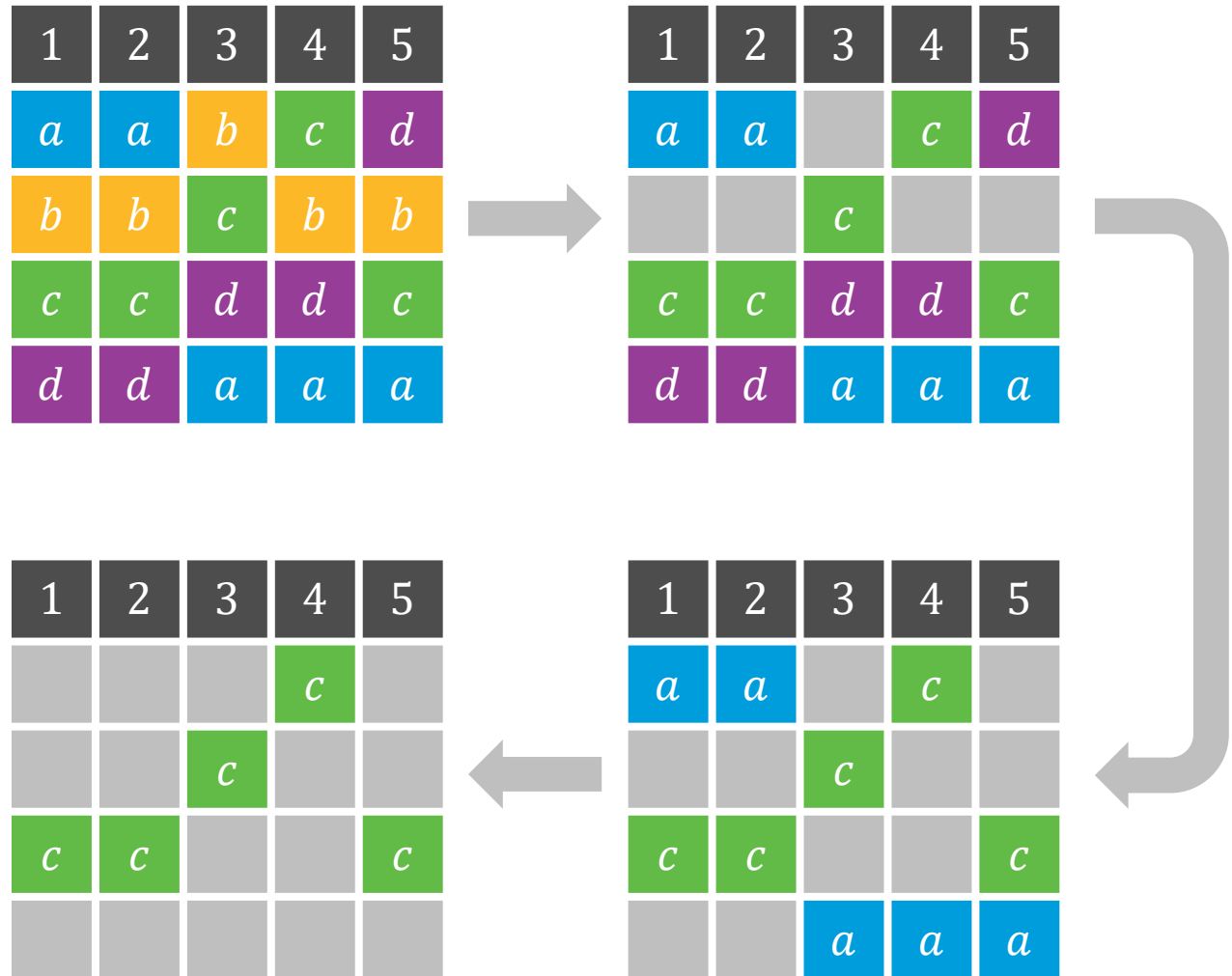
- Each voter awards $m - k$ points to the alternative placed in the k 'th position, where m is the number of alternatives



SINGLE TRANSFERABLE VOTE

- Also known as “alternative vote,” “instant-runoff voting” and (misleadingly) “ranked-choice voting”
- Votes are tabulated in rounds, where in each round the alternative with the lowest plurality score is eliminated; last alternative left standing is the winner

SINGLE-TRANSFERABLE VOTE



STV AROUND THE WORLD



■ Ireland

Used for all public elections

■ Canada

Used in Ontario for municipal elections

■ Australia

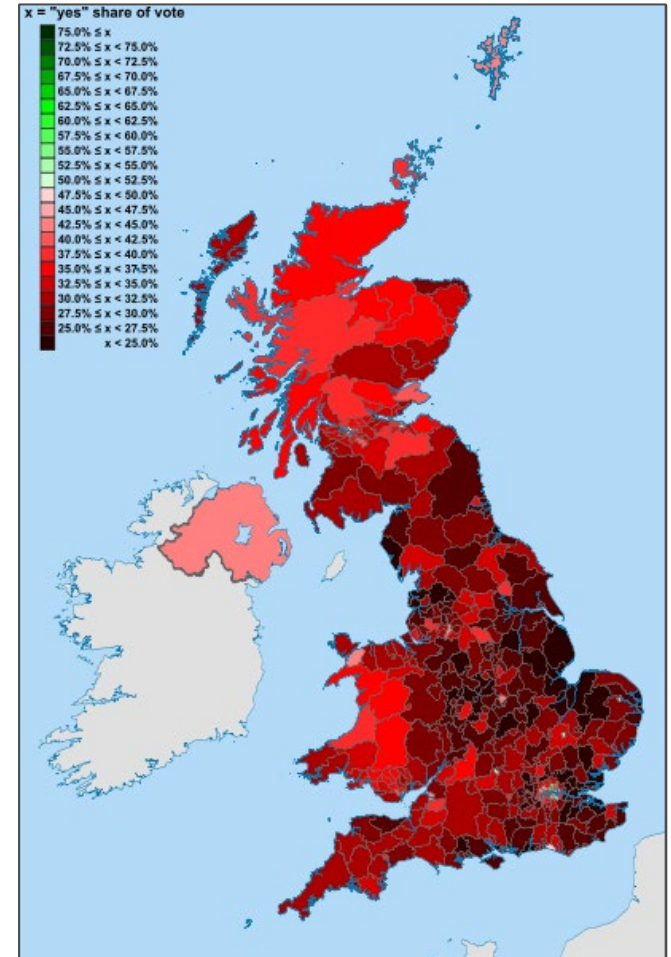
Used for parliamentary elections

■ USA

Used for statewide elections in ME and AK, and in cities like NYC and Cambridge

BARRIERS TO ADOPTION

- UK referendum (2011): Choose between plurality and STV as a method for electing MPs
- Academics agreed STV is better
- But STV was seen as beneficial to a particular politician





Marquis de Condorcet

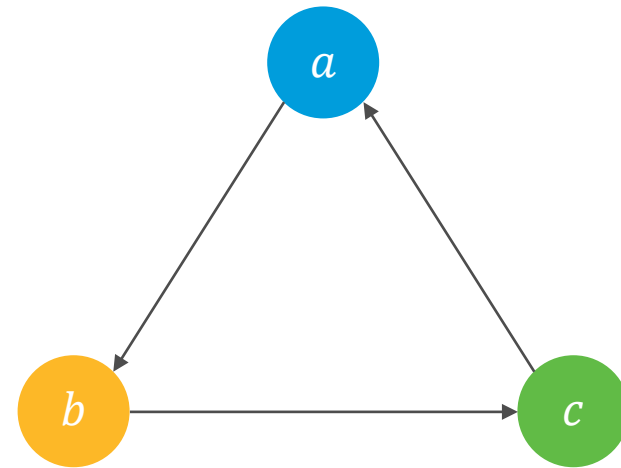
1743–1794

Philosopher, mathematician,
enlightened nobleman. Also known
for dying mysteriously in prison.



THE CONDORCET PARADOX

1	2	3
<i>a</i>	<i>c</i>	<i>b</i>
<i>b</i>	<i>a</i>	<i>c</i>
<i>c</i>	<i>b</i>	<i>a</i>



The preferences of the majority may be cyclical!

CONDORCET CONSISTENT RULES

- A **Condorcet winner** is an alternative that defeats every other alternative in a head-to-head comparison
- A rule is **Condorcet consistent** if it always selects a Condorcet winner whenever it is presented with a profile that contains one

Poll

Which rule is Condorcet consistent?

- Plurality
- Borda Count
- Both rules
- Neither one





Ramon Llull

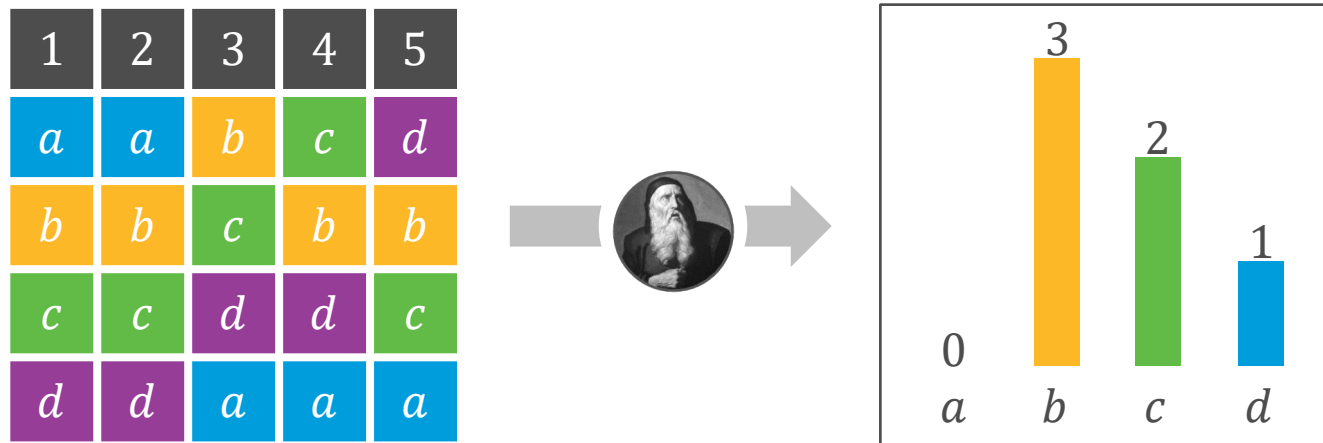
c. 1232–1315

Monk, missionary, and philosopher; one of the most influential intellectuals of his time. Also remembered for publishing a medieval parenting guide.



LLULL'S RULE

- Each alternative receives one point for each head-to-head comparison it wins (as well as for tied comparisons)



- Llull's rule is Condorcet consistent



Charles Lutwidge Dodgson

1832–1898

Professor of mathematics at Oxford,
pioneer photographer, and beloved
author. Also known for not plagiarizing
Condorcet's work.

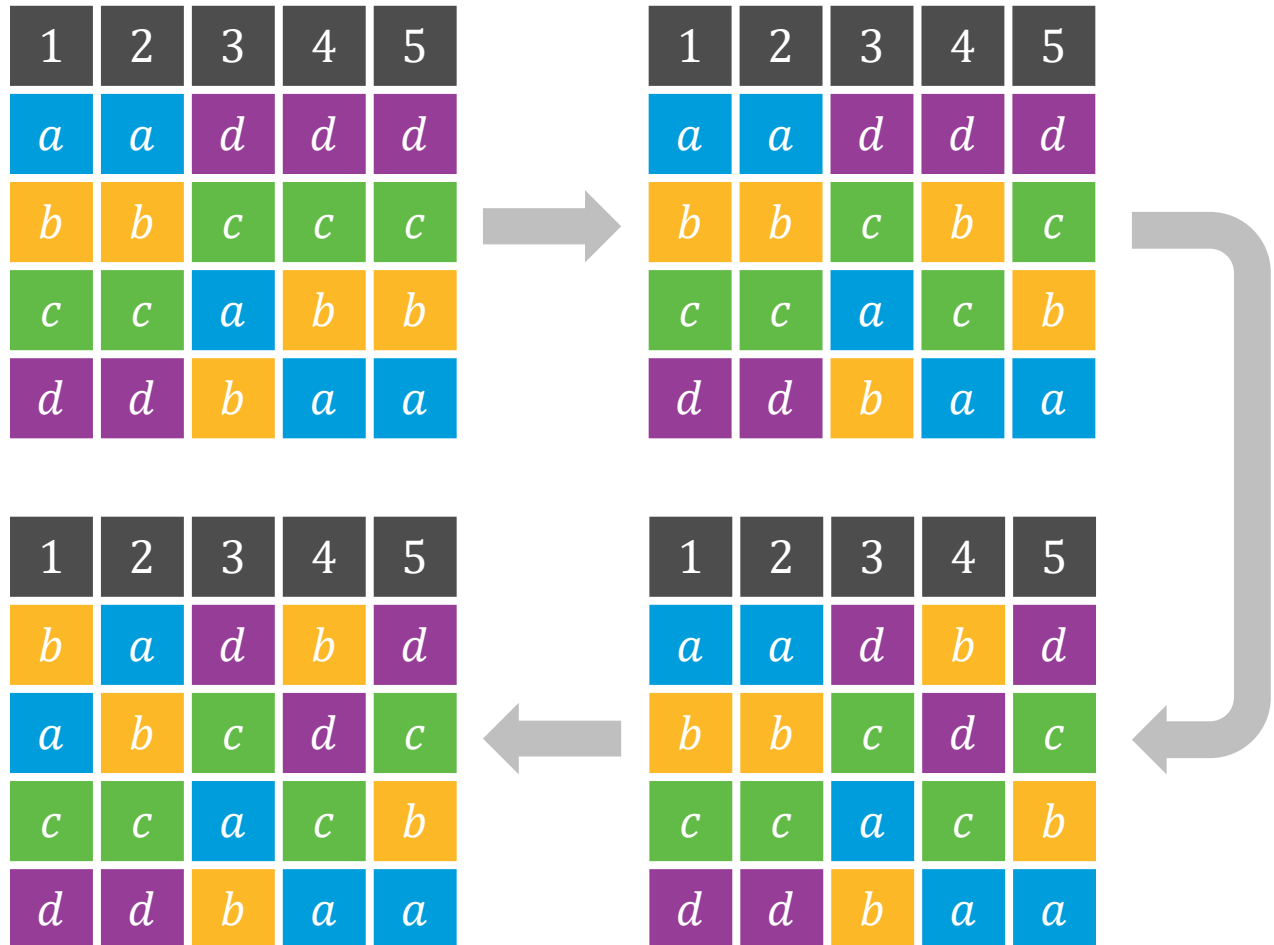


DODGSON'S RULE

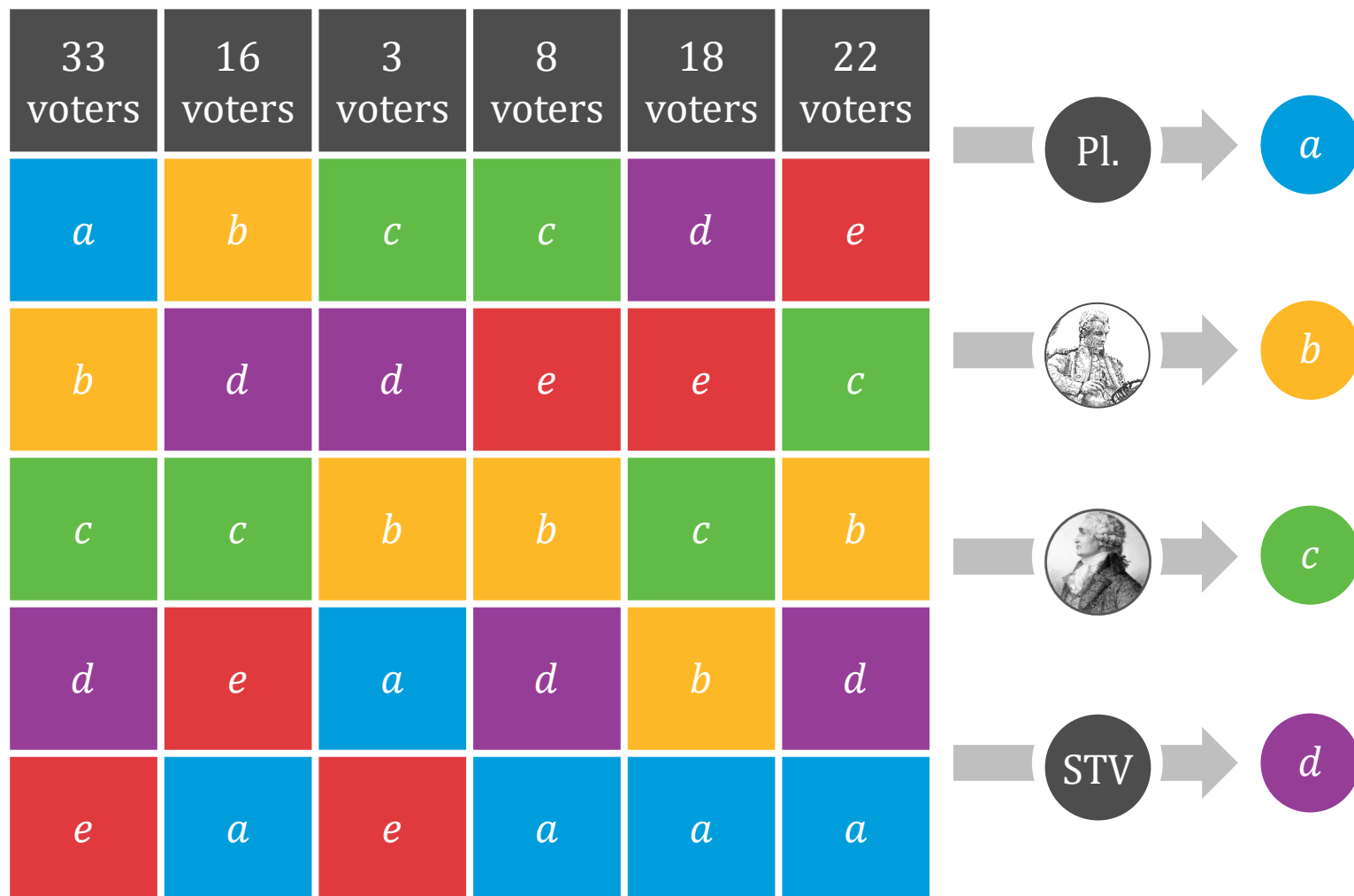
- The **Dodgson score** of an alternative x is the minimum number of swaps between adjacent alternatives needed to make x a Condorcet winner; select an alternative with minimum score
- Dodgson's rule is Condorcet consistent
- Dodgson's rule is NP-hard to compute [Bartholdi et al. 1989]

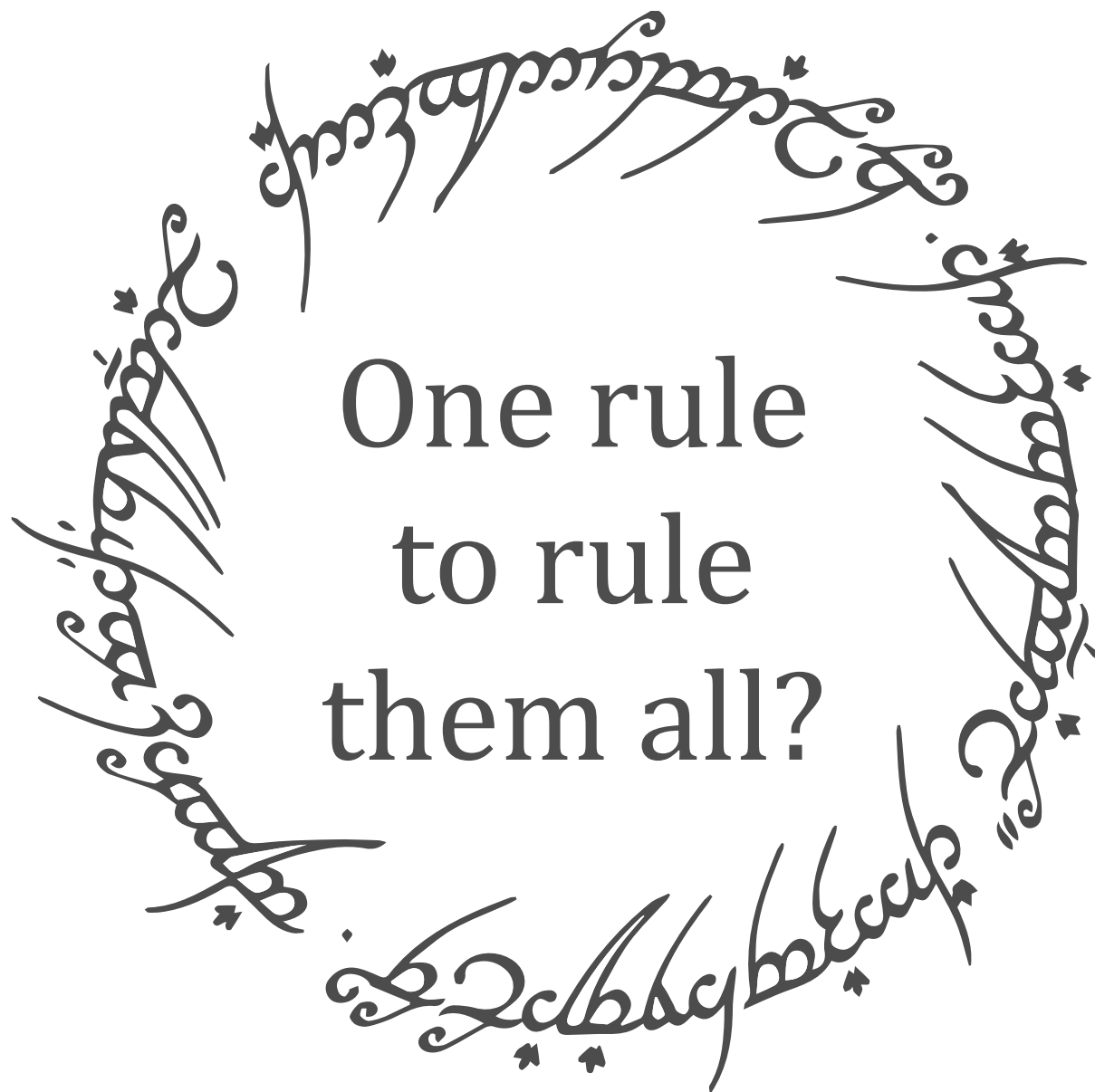
DODGSON'S RULE

What is the Dodgson score of b ?



AWESOME EXAMPLE





One rule
to rule
them all?