Optimized Democracy

Spring 2023 | Lecture 7

Liquid Democracy

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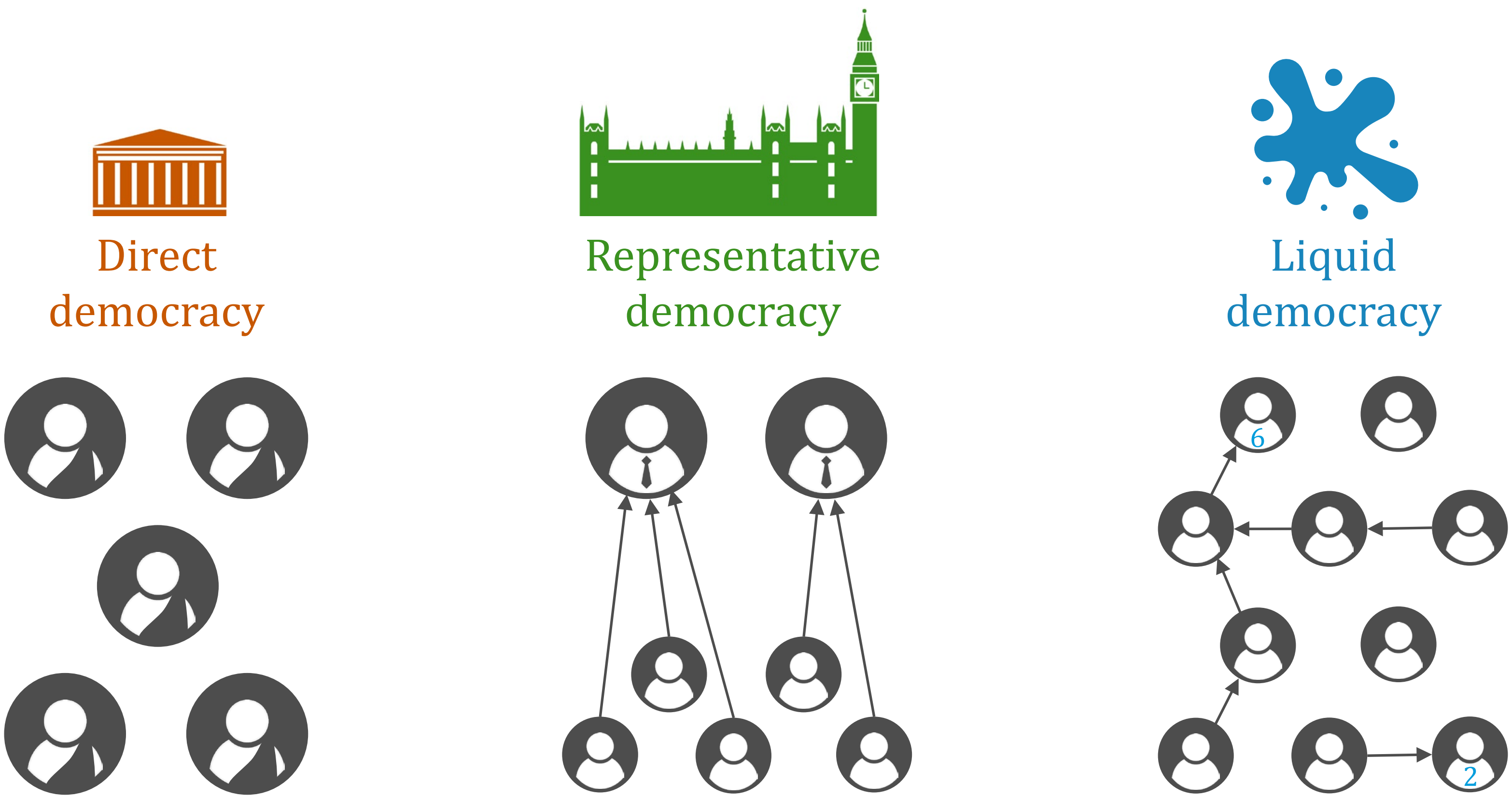
FORMS OF DEMOCRACY

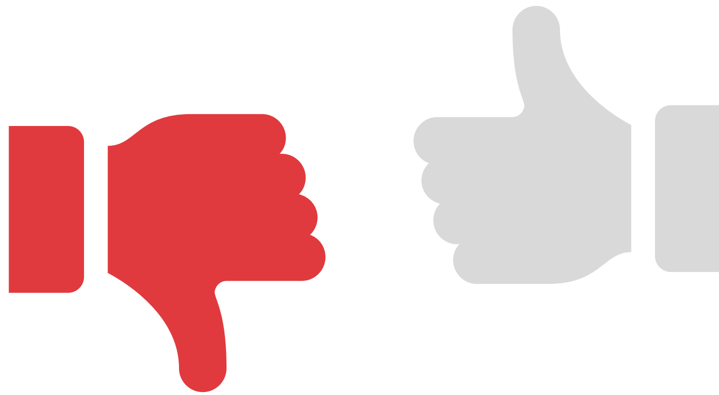


LIQUID DEMOCRACY SYSTEMS

LiquidFeedback
Germany
Since 2010

DemocracyOS
Argentina
Since 2012

Flux
Australia
Since 2016



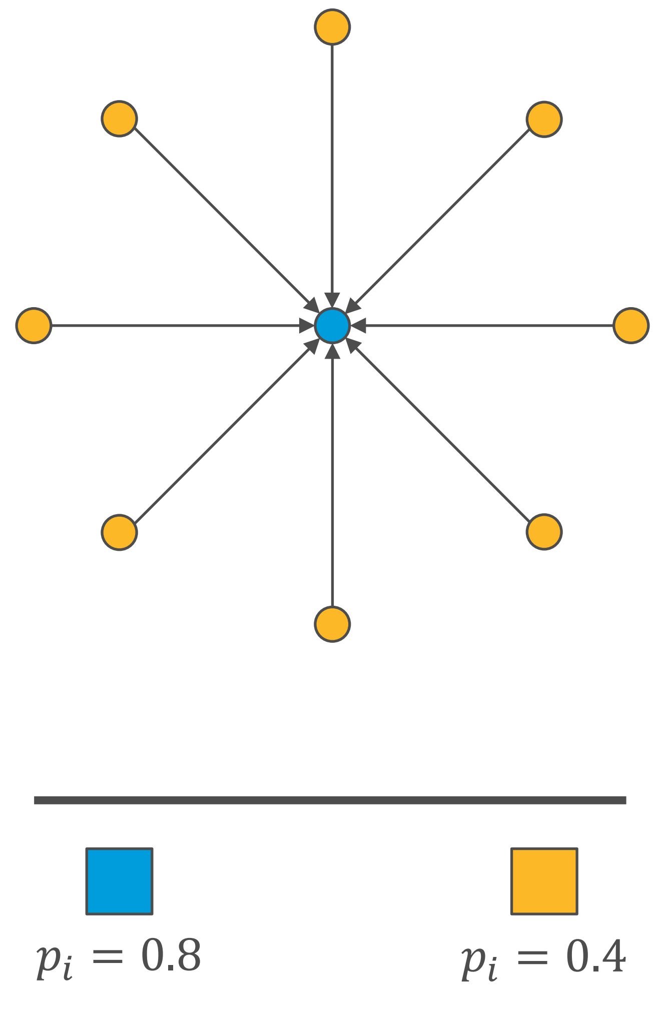
Part I:
Bad news in an objective model

THE MODEL

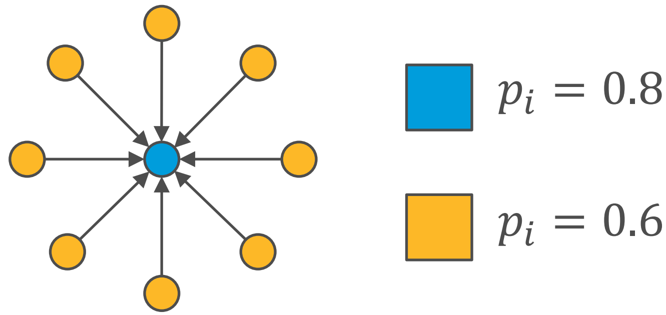
- Underlying labeled directed graph $G = (V, E, \boldsymbol{p})$ on n vertices, where V is the set of voters, and $(i, j) \in E$ if i knows j
- There are two alternatives, correct and incorrect
- Decisions are made based on majority vote
- Each voter i has a competence level p_i, which is their probability of voting correctly
- i approves j if $(i, j) \in E$ and $p_j > p_i + \alpha$
- Denote i's approved neighbors by $A_G(i)$

LIQUID VS. DIRECT DEMOCRACY

- Consider a star with n vertices; leaves have $p_i = 0.4$, center has $p_i = 0.8$, and $\alpha < 0.4$
- Direct democracy: By the Condorcet Jury Theorem, probability that majority is correct $\rightarrow 0$ as $n \rightarrow \infty$
- Under liquid democracy, all leaves delegate, and the probability of correctness is 0.8



LIQUID VS. DIRECT DEMOCRACY



Poll 1

Which system would be more accurate if we raised the competence levels of the leaves to 0.6 and set $\alpha < 0.2$?

- Liquid Democracy
- Direct Democracy
- It’s a tie!

DELEGATION MECHANISMS

- Can we give liquid democracy an edge via smarter delegation?
- A delegation mechanism observes G and the approval relation, and outputs for each $i \in V$ a probability distribution over $A_G(i) \cup \{i\}$ that represents the probability that i delegates their vote to each approved neighbor or votes directly
- Denote the probability that delegation mechanism M makes a correct decision on G by $P_M(G)$

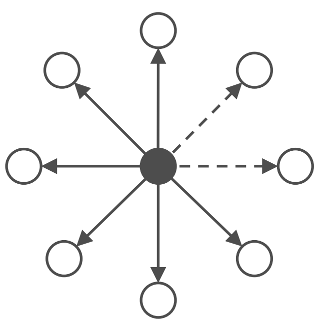
DELEGATION MECHANISMS

- $P_M(G)$ is defined via the following process:
  1. Apply M to G
  2. Sample the probability distribution for each vertex to obtain an acyclic delegation graph, where each sink i of the delegation graph has weight equal to the number of vertices with directed paths to i, including i
  3. Each sink i votes for the correct alternative with probability p_i
  4. A decision is made based on weighted majority

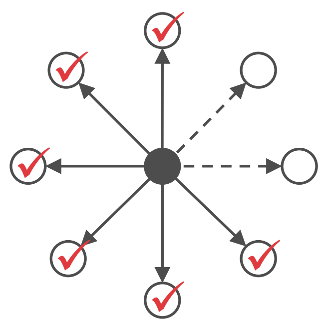
LOCAL DELEGATION MECHANISMS

In a local delegation mechanism, the distribution of each vertex i depends only on $\{j \in V : (i, j) \in E\}$ and $A_G(i)$

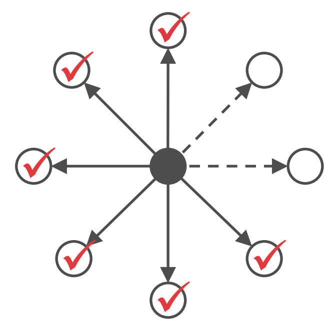
Examples:



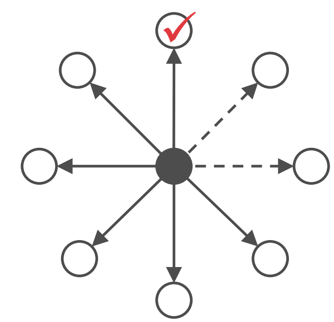
Direct voting: no delegation



Delegate to a random approved neighbor



Delegate to a random approved neighbor if most neighbors are approved

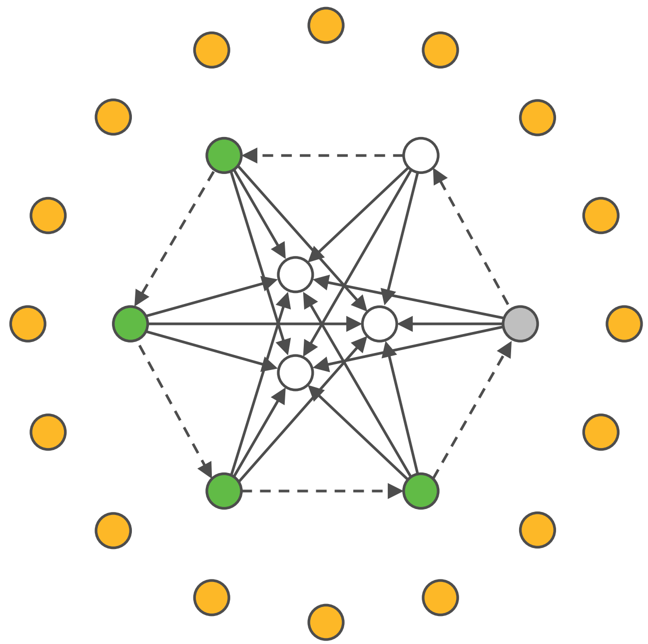


Delegate to a specific approved neighbor

FIRST, DO NO HARM

- Define $\text{gain}(M, G) = P_M(G) - P_D(G)$, where D is direct voting
- Mechanism M satisfies the do no harm (DNH) property if for every $\epsilon > 0$ there exists $n_0 \in \mathbb{N}$ such that on all graphs G_n on $n \geq n_0$ vertices, $\text{gain}(M, G_n) \geq -\epsilon$
- Mechanism M satisfies the positive gain (PG) property if there exist $\gamma > 0$ and graph G such that $\text{gain}(M, G) \geq \gamma$
- Theorem: For any $\alpha \in [0,1)$, there is no local delegation mechanism that satisfies the DNH and PG properties

PROOF BY ILLUSTRATION



High competence	Medium competence	Low competence
$p_i = \frac{1+\alpha'}{2}$	$p_i = \text{mess}$	$p_i = \frac{1-\alpha'}{2}$

Foto: dapd

Fotostrecke

Photo Gallery: Germany's Pirates Promote Digital Democracy

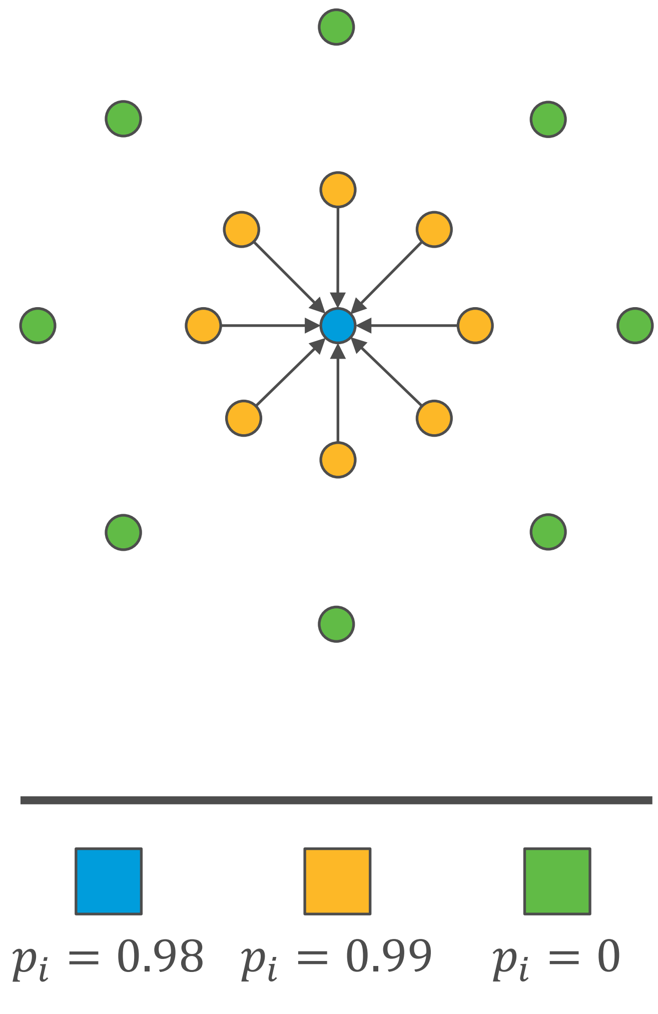
3 Bilder

Liquid Democracy

Web Platform Makes Professor Most Powerful Pirate

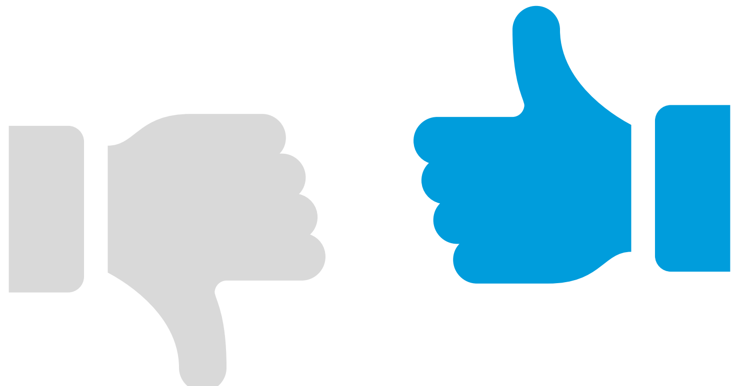
EXTENSIONS

- Delegating to less competent voters can be highly beneficial
- Consider a star with k leaves where the center has $p_i = 0.98$ and the leaves have $p_i = 0.99$, add k isolated vertices with $p_i = 0$
- When all vertices vote independently the probability of success $\rightarrow 0$ as $k \rightarrow \infty$, but when the center votes for the entire star, the probability of success is 0.98



EXTENSIONS

- Is there a recipe for detecting the best possible delegations?
- In the OPTIMAL DELEGATION problem, we are given a labeled graph (including competence levels), and asked to coordinate delegations to maximize the probability of selecting the correct alternative
- Theorem: Approximating the optimal value of OPTIMAL DELEGATION within an additive term of 1/16 is NP-hard



Part II:

Generally good news in a subjective model with optional participation

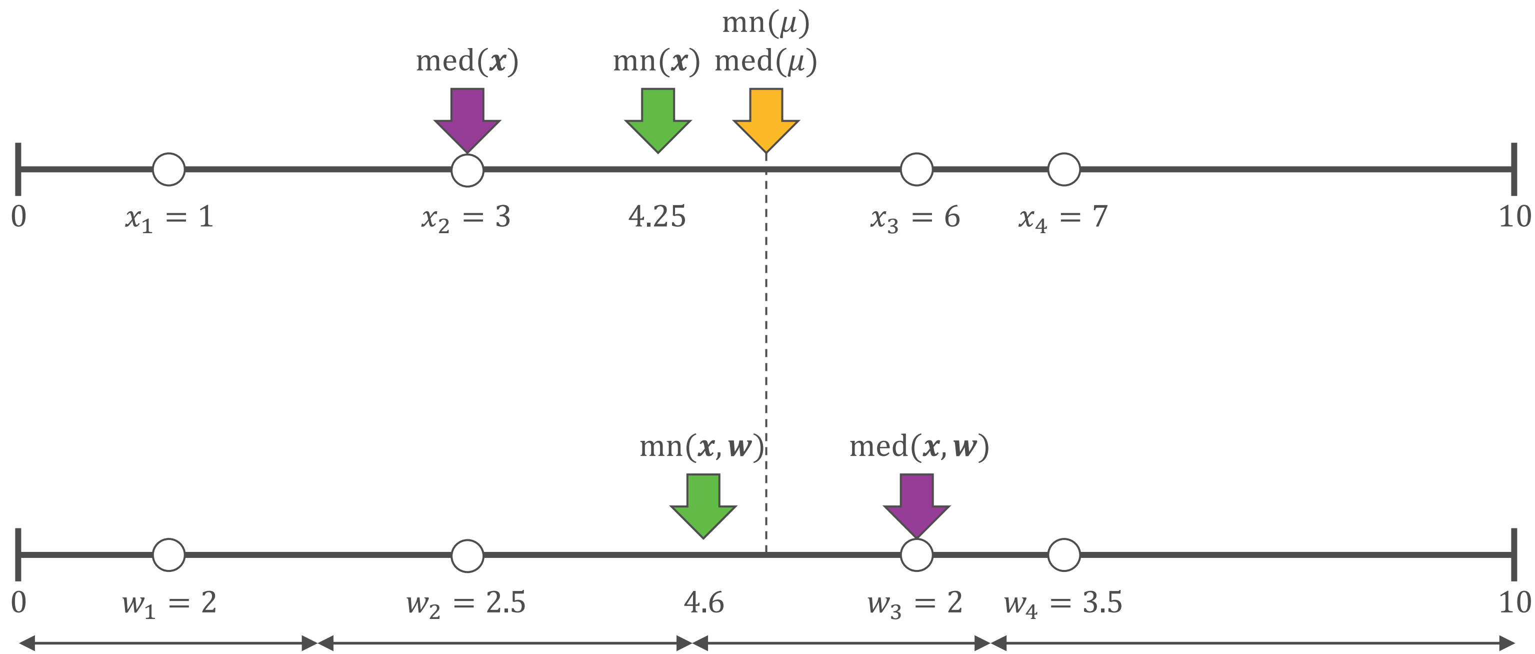
THE MODEL

- Infinite population of voters given by a distribution μ over the interval $[a, b]$
- Set N of n proxies with locations $\boldsymbol{x} \in [a, b]^n$
- Under direct democracy, only the voters in N vote and we compute the median $\text{med}(\boldsymbol{x})$ or the mean $\text{mn}(\boldsymbol{x})$
- Under liquid democracy, each voter in the population delegates to the closest proxy, leading to weights $\boldsymbol{w}$, and we compute the median $\text{med}(\boldsymbol{x}, \boldsymbol{w})$ or the mean $\text{mn}(\boldsymbol{x}, \boldsymbol{w})$

LIQUID VS. DIRECT REDUX

- We are interested in the median of the population $\mathrm{med}(\mu)$ or the mean of the population $\mathrm{mn}(\mu)$
- Direct democracy is evaluated via $|\mathrm{med}(\mu) - \mathrm{med}(\boldsymbol{x})|$ or $|\mathrm{mn}(\mu) - \mathrm{mn}(\boldsymbol{x})|$
- Liquid democracy is evaluated via $|\mathrm{med}(\mu) - \mathrm{med}(\boldsymbol{x}, \boldsymbol{w})|$ or $|\mathrm{mn}(\mu) - \mathrm{mn}(\boldsymbol{x}, \boldsymbol{w})|$

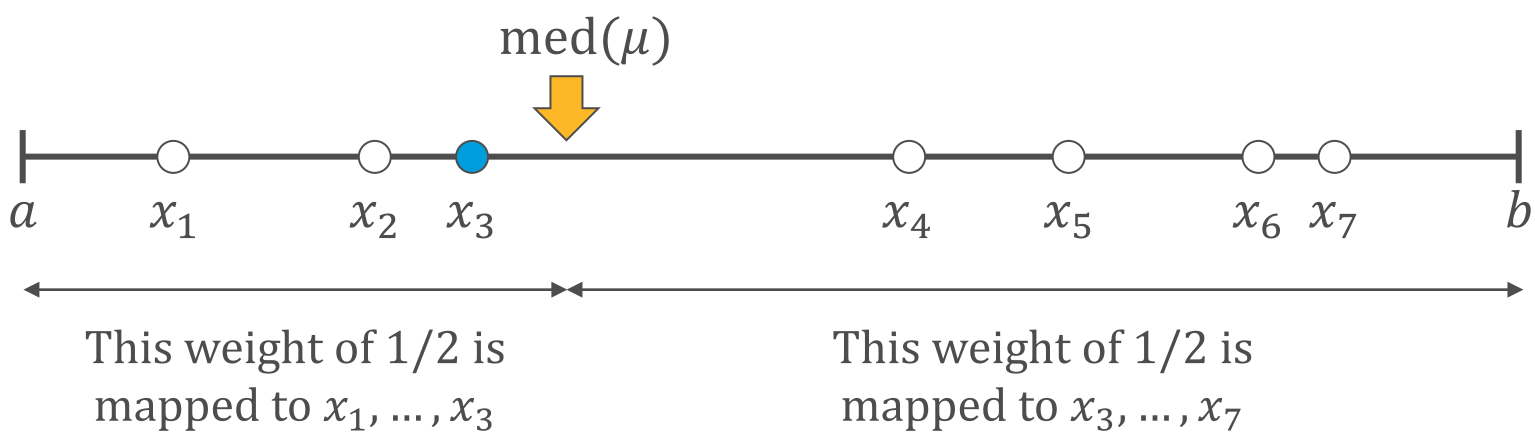
LIQUID VS. DIRECT REDUX



μ is the uniform distribution over [0,10]

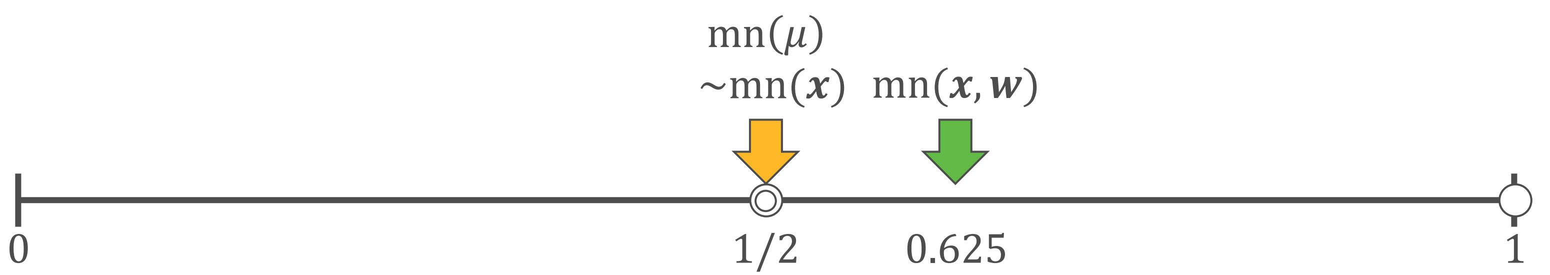
VOTING FOR THE MEDIAN

- **Theorem:** For any $n \in \mathbb{N}$, $\boldsymbol{x} \in [a,b]^n$ and distribution μ,
  $|\text{med}(\mu) - \text{med}(\boldsymbol{x}, \boldsymbol{w})| \leq |\text{med}(\mu) - \text{med}(\boldsymbol{x})|$
- **Proof:** $\text{med}(\boldsymbol{x}, \boldsymbol{w})$ is always the x_i that is closest to $\text{med}(\mu)$, as shown below ■



VOTING FOR THE MEAN

- Theorem: Let $n = 2$, then for any $\boldsymbol{x} \in [a, b]^n$ and distribution μ (conditions apply), $|\mathrm{mn}(\mu) - \mathrm{mn}(\boldsymbol{x}, \boldsymbol{w})| \leq |\mathrm{mn}(\mu) - \mathrm{mn}(\boldsymbol{x})|$
- This result doesn't hold for $n \geq 3$: consider the uniform distribution over $[0,1]$ and $x_1, \ldots, x_{1000} = 1/2$ while $x_{1001} = 1$



SAMPLING TO THE RESCUE?

- This counterexample wouldn't arise if $x_1, \ldots, x_n$ were sampled independently from the distribution μ

Poll 2

Suppose μ is the uniform distribution over $[a, b]$ and $x_1, \ldots, x_n$ are sampled independently from μ. Which of $\mathrm{mn}(\boldsymbol{x})$ and $\mathrm{mn}(\boldsymbol{x}, \boldsymbol{w})$ approaches $\mathrm{mn}(\mu)$ as $n \to \infty$?

- Only $\mathrm{mn}(\boldsymbol{x})$
- Only $\mathrm{mn}(\boldsymbol{x}, \boldsymbol{w})$
- Both
- Neither

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