Optimized Democracy

Spring 2023 | Lecture 5

Electoral Competition

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NORMAL-FORM GAME

- To model competition we will need basic terminology in game theory
- A game in normal form consists of:
  - Set of players $N = \{1, \dots, n\}$
  - Strategy set S
  - For each $i \in N$, utility function $u_i: S^n \to \mathbb{R}$, which gives the utility of player i, $u_i(s_1, \dots, s_n)$, when each $j \in N$ plays the strategy $s_j \in S$

THE PRISONER'S DILEMMA

	Cooperate	Defect
Cooperate	-1,-1	-9,0
Defect	0,-9	-6,-6

What would you do?

THE PROFESSOR'S DILEMMA

Professor \ Class	Listen	Sleep
Make effort	$10^6, 10^6$	-10,0
Slack off	0,-10	0,0

Dominant strategies?

John Forbes Nash

1928–2015

Mathematician and Nobel laureate in economics. Also remembered as the protagonist in “A Beautiful Mind.”

NASH EQUILIBRIUM

- In a Nash equilibrium, no player wants to unilaterally deviate
- Each player's strategy is a best response to strategies of others
- Formally, a Nash equilibrium is a vector of strategies $\boldsymbol{s} = (s_1 \dots, s_n) \in S^n$ such that for all $i \in N, s_i' \in S$,

$$u_i(\boldsymbol{s}) \geq u_i(s_i', \boldsymbol{s}_{-i})$$

THE PROFESSOR'S DILEMMA

		Class	
		Listen	Sleep
Professor	Make effort	$10^6, 10^6$	-10,0
	Slack off	0,-10	0,0

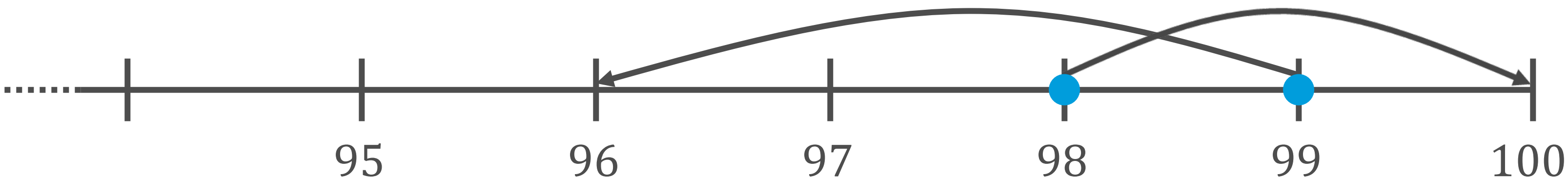
Nash equilibria?

ROCK-PAPER-SCISSORS

			
	0,0	-1,1	1,-1
	1,-1	0,0	-1,1
	-1,1	1,-1	0,0

Nash equilibria?

CAVEAT: NE PREDICTS OUTCOMES?



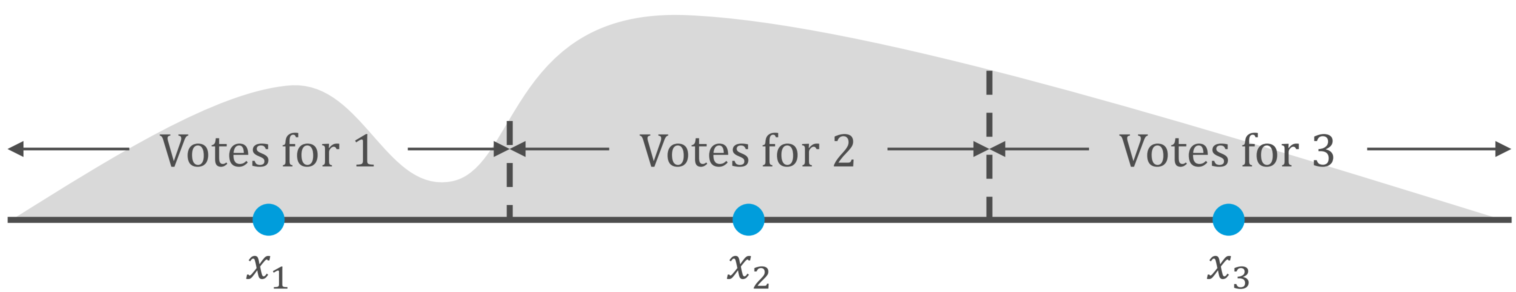
Two players, strategies are $\{2, \dots, 100\}$. If both choose the same number, that is what they get. If one chooses s, the other t, and $s < t$, the former player gets $s + 2$, and the latter gets $s - 2$.

Poll 1

Suppose you are paired with another random student, and you must play this game with them (for real money) without communicating. What would you choose?

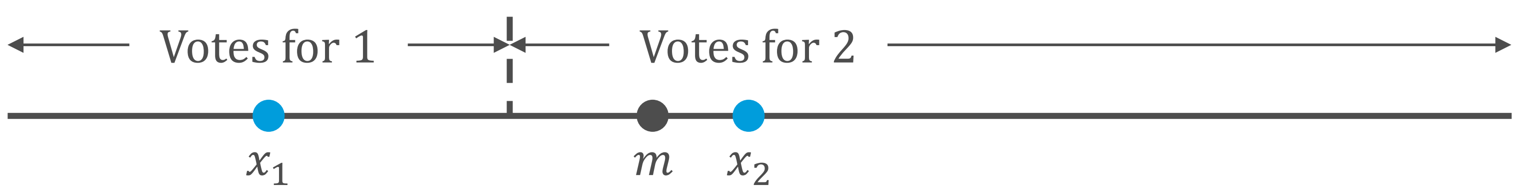
THE HOTELLING MODEL

- Political spectrum is $\mathbb{R}$
- There is a nonatomic distribution of voters, each with a peak in $\mathbb{R}$
- Players are candidates, who strategically choose positions $x_1, \dots, x_n$
- Each candidate attracts the votes of voters who are closest to them, with votes being split equally in case of a tie



THE HOTELLING MODEL

- Two candidates seek to win a plurality of votes
- The utility of each candidate is 1 if they win, 1/2 if they tie, and 0 if they lose
- Denote the median peak by m (assume for simplicity that it's unique)



Who wins?

NE FOR TWO CANDIDATES

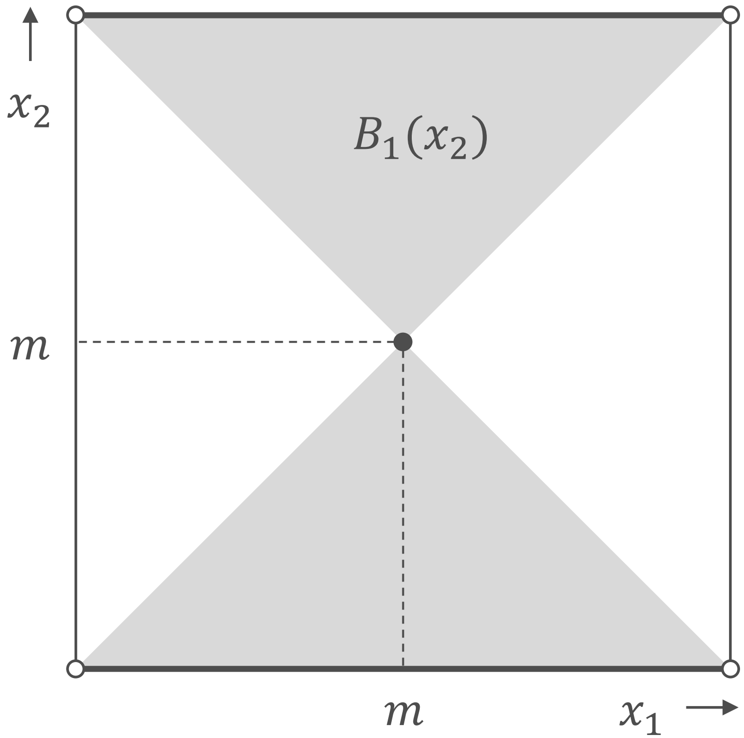
- If $x_2 < m$, the best response for 1 is all positions x_1 such that

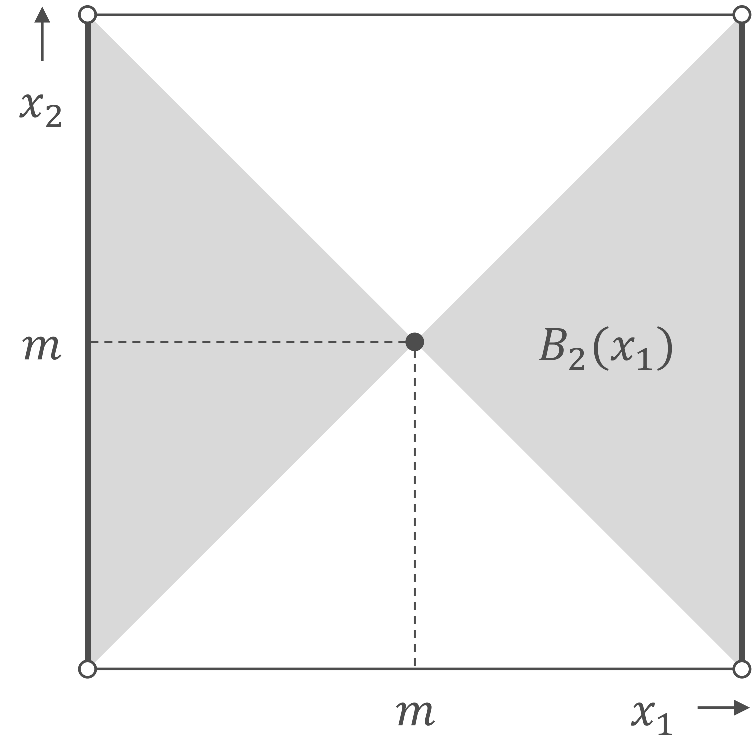
$$x_1 > x_2 \quad \text{and} \quad \frac{x_1 + x_2}{2} < m$$

- A symmetric argument holds if $x_2 > m$
- If $x_2 = m$, the best response for 1 is m
- Therefore, it holds that

$$B_1(x_2) = \begin{cases} \{x_1 : x_2 < x_1 < 2m - x_2\} & x_2 < m \\ \{m\} & x_2 = m \\ \{x_1 : 2m - x_2 < x_1 < x_2\} & x_2 > m \end{cases}$$

NE FOR TWO CANDIDATES





The unique Nash equilibrium is at (m, m)

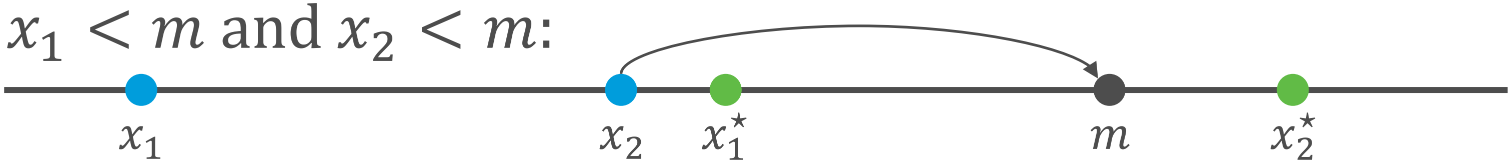
POLICY-MOTIVATED CANDIDATES

- What if candidates care about policy and not just about winning?
- Suppose i has a preferred position $x_i^\star$, and their utility depends on the distance between $x_i^\star$ and the position of the winner
- If there's a tie then candidates evaluate the induced lottery over winning positions
- Theorem: If $x_1^\star < m < x_2^\star$ then (m, m) is the unique Nash equilibrium

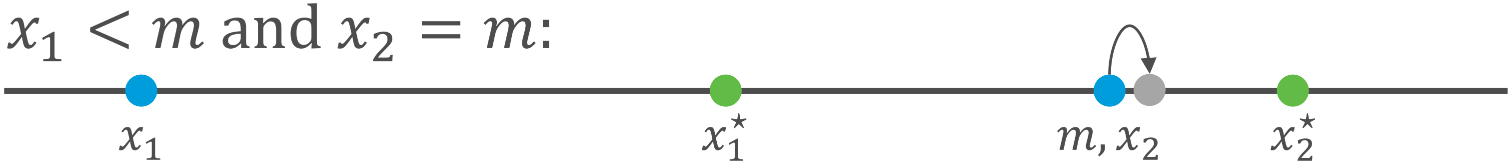
PROOF SKETCH

Rule out cases for which $(x_1, x_2) \neq (m, m)$:

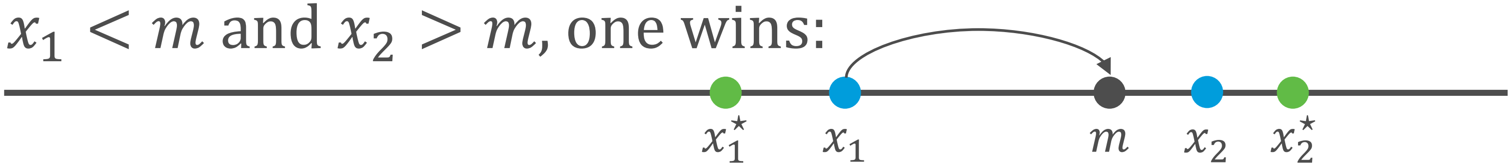
$x_1 < m$ and $x_2 < m$:



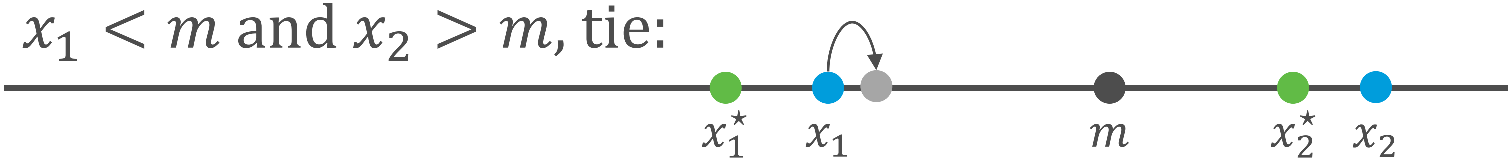
$x_1 < m$ and $x_2 = m$:



$x_1 < m$ and $x_2 > m$, one wins:



$x_1 < m$ and $x_2 > m$, tie:



Harold Hotelling

1895–1973

“The competition for votes between the Republican and Democratic parties does not lead to a clear drawing of issues, an adoption of two strongly contrasted positions between which the voter may choose. Instead, each party strives to make its platform as much like the other's as possible.”

(m, m) USED TO MAKE SENSE

TOWARD A MORE RESPONSIBLE
TWO-PARTY SYSTEM

A Report of the Committee on Political Parties
American Political Science Association

Vol. XLIV September, 1950 Number 3, Part 2

INTRODUCING UNCERTAINTY

- Both candidates believe that the median peak m is distributed according to a distribution μ with strictly positive density over an interval I
- For $x_1 < x_2$, the probability that 1 wins is

$$\pi_1(x_1, x_2) = \Pr_{m \sim \mu}\left[m < \frac{x_1 + x_2}{2}\right]$$

- Candidate i's utility for (x_1, x_2) is

$$\pi_1(x_1, x_2)U_i(x_1) + \pi_2(x_1, x_2)U_i(x_2)$$

where the maximizer of U_i is $x_i^\star$

INTRODUCING UNCERTAINTY

- **Theorem:** Assume $x_1^\star, x_2^\star \in I$ and $x_1^\star \neq x_2^\star$, then in any Nash equilibrium (x_1, x_2) it holds that $x_1 \neq x_2$
- **Proof:**
  - If $x_1 = x_2 = x^\star$ then $x^\star$ is enacted with probability 1
  - Without loss of generality $x^\star < x_1^\star$
  - If 1 moves to $x_1' \in (x^\star, x_1^\star)$, then $\pi_1(x_1', x_2) > 0$ and they are better off ■

EXTENSIONS

- We introduced policy-motivation and uncertainty into the original Hotelling model, but there are other gaps from reality

Poll 2

What are some other ingredients that are missing from the model?

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M. J. Osborne. An Introduction to Game Theory (Chapter 3). Oxford University Press, 2003.

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