CS 1360 Spring 2025
Midterm Exam (Practice)

Problem 1: Game Theory

1. **[10 pts]** Provide a definition (using mathematical notation) of *best response* in a normal-form game.

2. **[15 pts]** Consider a 2-player game in normal form, and denote the strategy set of each player by S. Let $B_1(x_2)$ denote the set of (possibly mixed) best response strategies of player 1 to the (possibly mixed) strategy x_2 of player 2. For convenience, let us fix some mixed strategy $x_2^\star$ for player 2, and denote $\alpha = u_1(x_1, x_2^\star)$ for all $x_1 \in B_1(x_2^\star)$, that is, α is the maximum utility player 1 can achieve against $x_2^\star$.

   Show that $x_1 \in B_1(x_2^\star)$ *if and only if* every pure strategy $s \in S$ in the support of x_1 (i.e., every pure strategy s such that $x_1(s) > 0$) is itself in $B_1(x_2^\star)$ (i.e., $u_1(s, x_2^\star) = \alpha$).

   Note: Do not forget to show both directions.

Problem 2: Equilibrium Computation

1. **[10 pts]** Define (using mathematical notation) the notion of *strong Stackelberg equilibrium (SSE).*

2. **[15 pts]** Describe a polynomial-time algorithm that computes an SSE in a given Stackelberg game.

   Note: This was done in class.

Problem 3: The Price of Anarchy

1. **[10 pts]** Describe in words the concept of *price of anarchy* of a class of games.

2. **[15 pts]** Assignment 2 introduced scheduling games on *related machines.* Here we are interested in scheduling games on *unrelated machines.* The players $N = \{1, \ldots, n\}$ are associated with tasks and there is a set M of m machines. Each player chooses a machine to place their task on, that is, the strategy space of each player is M. The weights of players (or tasks) are now machine-dependent: player i has weight $w_{i\mu}$ on

machine μ. A strategy profile induces an assignment $A : N \to M$ of players (or tasks) to machines. The total load on machine μ is $\ell_\mu = \sum_{i \in N:\ A(i)=\mu} w_{i\mu}$. The *cost* of player i is $\ell_{A(i)}$. Our objective function is the *makespan*, which is the maximum load on any machine: $\text{cost}(A) = \max_{\mu \in M} \ell_\mu$.

Show that for any number of players $n \geq 2$, scheduling games on unrelated machines have an unbounded (i.e., arbitrarily high) price of anarchy.

Problem 4: The Epistemic Approach to Voting

1. **[10 pts]** Define (using mathematical notation) the *Mallows noise model*.

2. **[15 pts]** Construct a preference profile such that, under the Mallows model with any value of the parameter $\phi \in (0, 1)$, the ranking given by Borda count (i.e., ranking the alternatives by Borda score, breaking ties as you wish) is *not* a maximum likelihood estimator for the ground-truth ranking.

   Hint: What would Kemeny do?