CS 1360 Spring 2025
Final Exam (Practice)

Problem 1: Social Choice

1. **[5 pts]** Define (using mathematical notation) the concept of a *neutral* social choice function.

2. **[15 pts]** A social choice function f is *anonymous* if for any permutation $\pi : N \to N$ and any preference profile $\boldsymbol{\sigma}$, $f(\sigma_1, \sigma_2, \ldots, \sigma_n) = f(\sigma_{\pi(1)}, \sigma_{\pi(2)}, \ldots, \sigma_{\pi(n)})$. Informally, changing the order (or "names") of voters does not change the outcome. Prove that, for the case of two voters and two alternatives, no social choice function is both anonymous and neutral.

Problem 2: Indivisible Goods

1. **[5 pts]** Define (using mathematical notation) the notion of *envy freeness up to one good* (EF1).

2. **[15 pts]** Let there be two players with additive, strictly positive valuations over a set of goods. Consider the following algorithm: player 1 divides the goods into two bundles X_1 and X_2 in a way that maximizes $\min\{V_1(X_1), V_1(X_2)\}$ (intuitively, player 1 divides the goods as evenly as possible according to their own valuation). Then, player 2 chooses their favorite bundle, and player 1 receives the remaining bundle. Prove that this algorithm produces an EF1 allocation.

Problem 3: Online Matching Algorithms

1. **[5 pts]** Define (using mathematical notation) the competitive ratio of an online (bipartite) matching algorithm.

2. **[15 pts]** Consider the following online matching algorithm. When an online vertex $v \in V$ arrives, if it has an offline neighbor u that has already been matched (that is, there is $u \in U$ such that $(u, v) \in E$ and u has been matched with $v' \in V$ that arrived before v), don't match v. Otherwise, match v with an arbitrary unmatched neighbor.

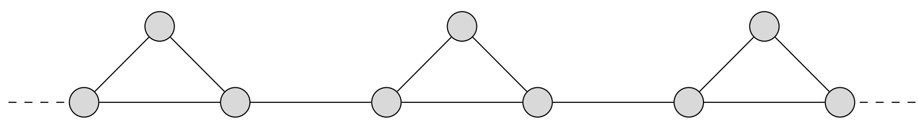
What is the competitive ratio of this algorithm, as a function of n (the number of vertices on each side)?

Note: Establish an upper bound and a lower bound. The two bounds should ideally be equal.

Problem 4: Cascade Models

1. [**5 pts**] Define the *contagion threshold* of an infinite graph (with bounded degrees).

2. [**15 pts**] Consider a graph G which is composed of an infinite sequence of triangles, where adjacent triangles are connected by a single edge, as shown below:



   What is the contagion threshold of G?

   Note: Establish an upper bound and a lower bound. The two bounds should ideally be equal.

Problem 5: Feature Attribution

1. [**5 pts**] Define (using mathematical notation) the *Shapley value* $\sigma_i(N, v)$ of player $i \in N$ in a cooperative game (N, v).

2. [**15 pts**] Prove that $\sum_{i \in N} \sigma_i(N, v) = v(N)$.

   Note: This was done in class.