Economics and Computation (Spring 2025)
Assignment #5

Due: 4/30/2025 11:59pm ET

Problem 1: Random assignment

[15 points] A random assignment P is envy free if for all $i, j \in N$ and $x \in G$, $\sum_{y \succeq_{\sigma_i} x} p_{iy} \geq \sum_{y \succeq_{\sigma_i} x} p_{jy}$.

Prove that the Probabilistic Serial Mechanism produces an envy-free random assignment.

Problem 2: Cascade models

[10 points] In Lecture 17 we discussed the coordination game. Consider a similar game, called the *local public goods game*, which is defined using the notation used in Slide 3. The possible actions are again $a_i \in \{0, 1\}$, but here 1 corresponds to investing in a good that is useful to the neighbors of i, and 0 corresponds to not investing. The utility of i is

$$u_i(\mathbf{a}) = \begin{cases} 1 - c & a_i = 1 \\ 1 & a_i = 0 \text{ and } n_{i,1}(\mathbf{a}_{-i}) \geq 1 \\ 0 & \text{otherwise} \end{cases}$$

That is, i gets a payoff of 1 if at least one player in their neighborhood (including themselves) invests, but investment has a cost of $c \in (0, 1)$.

Design a polynomial-time algorithm that computes a pure Nash equilibrium in a given local public goods game.

Problem 3: Influence maximization

[15 pts] Given an undirected graph $G = (V, E)$, define the following set function over subsets $S \subseteq V$:

$$f(S) = |\{(u, v) \in E : \; u \in S, v \notin S\}|.$$

Is f monotone? Is it submodular? Prove or disprove each property.

Problem 4: No-regret learning

[15 pts] Consider the analysis of (deterministic) weighted majority in Lecture 19, Slides 10–13. Assume that there is a perfect expert that never makes mistakes. Show that there is a value of

ϵ such that the modified weighted majority algorithm (Slide 13) makes at most $\log_2 n$ mistakes, where n is the number of experts.

Problem 5: Feature attribution

[15 points] The Shapley value is hard to compute, but it is easy to estimate accurately using a Monte Carlo algorithm. Specifically, given a player i whose Shapley value we wish to estimate, consider the following algorithm: For $t = 1, \ldots, m$, sample a random permutation π_t and compute $v(S^i_{\pi_t} \cup \{i\}) - v(S^i_{\pi_t})$; then return the marginal contribution of i averaged across the m samples.

Assume that $v(S) \in [0,1]$ for all $S \subseteq N$ and v is monotonic. Show that, given $\epsilon, \delta > 0$ and $m = O(\ln(1/\delta)/\epsilon^2)$, the above algorithm outputs an estimate $\hat{\sigma}_i$ of the Shapley value of i such that $|\sigma_i - \hat{\sigma}_i| < \epsilon$ with probability at least $1 - \delta$.

Guidance: Each sample is a random variable; what can you say about their expectations? Plug these random variables into Hoeffding's Inequality: Let $X_1, \ldots, X_m$ be i.i.d. random variables bounded in $[0,1]$ with $\mathbb{E}[X_j] = \mu$ for $j = 1, \ldots, m$, then

$$\Pr\left[\left|\frac{1}{m}\sum_{t=1}^{m} X_t - \mu\right| \geq \epsilon\right] \leq 2 \cdot e^{-2m\epsilon^2}.$$

Problem 6: Formulate a research question

[30 points] Formulate a research question that is relevant to one of the topics covered in this assignment: random assignment, cascade models, influence maximization, no-regret learning, and cooperative game theory. Refer to this document for guidelines.

During the process of formulating your question, keep track of your findings in a "research journal." At a minimum, it should include brainstorming ideas for questions and notes on relevant papers that you have identified.

Please submit the following deliverables:

1. Your research question.
2. A *brief* explanation of why it satisfies each of the following criteria:
   (a) Relevant: Which course topics is the question related to?
   (b) Nontrivial: What is an immediate way of attempting to answer the question and why does it fail?
   (c) Feasible: How would you tackle the question if you had the entire semester?
   (d) Novel: List the 1–3 most closely related papers that you have identified in your literature review and explain how your question differs.

   Note: *Your writeup of all four parts of Item 2 must be at most two pages long overall.*
3. Append your research journal to the PDF that contains your solutions. The research journal will not be graded; it is there to show your work.