

Economics and Computation (Spring 2025)

Due: 3/24/2025 11:59pm ET

Problem 1: The VCG Mechanism

[15 points] Under the VCG Mechanism, the allocation rule maximizes (utilitarian) social welfare. Consider, instead, a *weighted social welfare* objective, defined as $\max_{x \in A} \sum_{i \in N} \alpha_i v_i(x)$, for given multipliers $\alpha_i \geq 0$, $i = 1, \dots, n$.

For an allocation rule optimizing weighted social welfare, design a payment rule such that the mechanism is strategyproof.

Problem 2: Strategyproof approximation algorithms

[15 points] In Lecture 9, we discussed the greedy mechanism for single-minded bidders. Let us modify the mechanism by tweaking the allocation rule: instead of ordering bids by decreasing w_i , sort the bids by decreasing $w_i / \sqrt{|T_i|}$. It turns out that this gives a \sqrt{m} -approximation.

Prove that the modified greedy algorithm is strategyproof.

Guidance: Adapt the lemma on Slide 7 by showing that the critical value of i is $\min\{w'_i : w'_i / \sqrt{|T_i|} \geq \max_{j \in N_i^c(T_i)} w_j / \sqrt{|T_j|}\}$. Next, adapt the proof on Slide 8.

Problem 3: Cake cutting

[20 points] In class we discussed the Even-Paz Algorithm, which guarantees a proportional allocation of the cake with $O(n \log n)$ queries in the Robertson-Webb Model. However, in the analysis we made the simplifying assumption that $n = 2^k$ for some $k \in \mathbb{N}$.

Generalize the algorithm to an arbitrary number of players n , and prove that your generalized algorithm is proportional and that it requires $O(n \log n)$ queries.

Problem 4: Rent division

[20 points] In class we stated two lemmas in the context of the rent division with quasi-linear utilities. The second lemma can actually be strengthened as follows:

Lemma: If (π, \mathbf{p}) is an EF solution and σ is a welfare-maximizing assignment, then for all $i \in N$, $v_{i\pi(i)} - p_{\pi(i)} = v_{i\sigma(i)} - p_{\sigma(i)}$.

Using this lemma (without proof), describe a polynomial-time algorithm for computing the maximin solution and prove its correctness.

Guidance: Follow the algorithm sketch on Slide 19 of Lecture 11 but replace the system of linear inequalities with an appropriate linear program. You may assume that a linear program can be solved in polynomial time. Note that a linear program can maximize the minimum of linear functions, similarly to our linear program for computing a maximin strategy in a two-player zero-sum game.

Problem 5: Formulate a research question

[30 points] Formulate a research question that is relevant to one of the topics covered in this assignment: the VCG Mechanism, strategyproof approximation algorithms, cake cutting, and rent division. Refer to [this document](#) for guidelines.

During the process of formulating your question, keep track of your findings in a “research journal.” At a minimum, it should include brainstorming ideas for questions and notes on relevant papers that you have identified.

Please submit the following deliverables:

1. Your research question.
2. A *brief* explanation of why it satisfies each of the following criteria:
 - (a) Relevant: Which course topics is the question related to?
 - (b) Nontrivial: What is an immediate way of attempting to answer the question and why does it fail?
 - (c) Feasible: How would you tackle the question if you had the entire semester?
 - (d) Novel: List the 1–3 most closely related papers that you have identified in your literature review and explain how your question differs.

Note: *Your writeup of all four parts of Item 2 must be at most two pages long overall.*

3. Append your research journal to the PDF that contains your solutions. The research journal will not be graded; it is there to show your work.