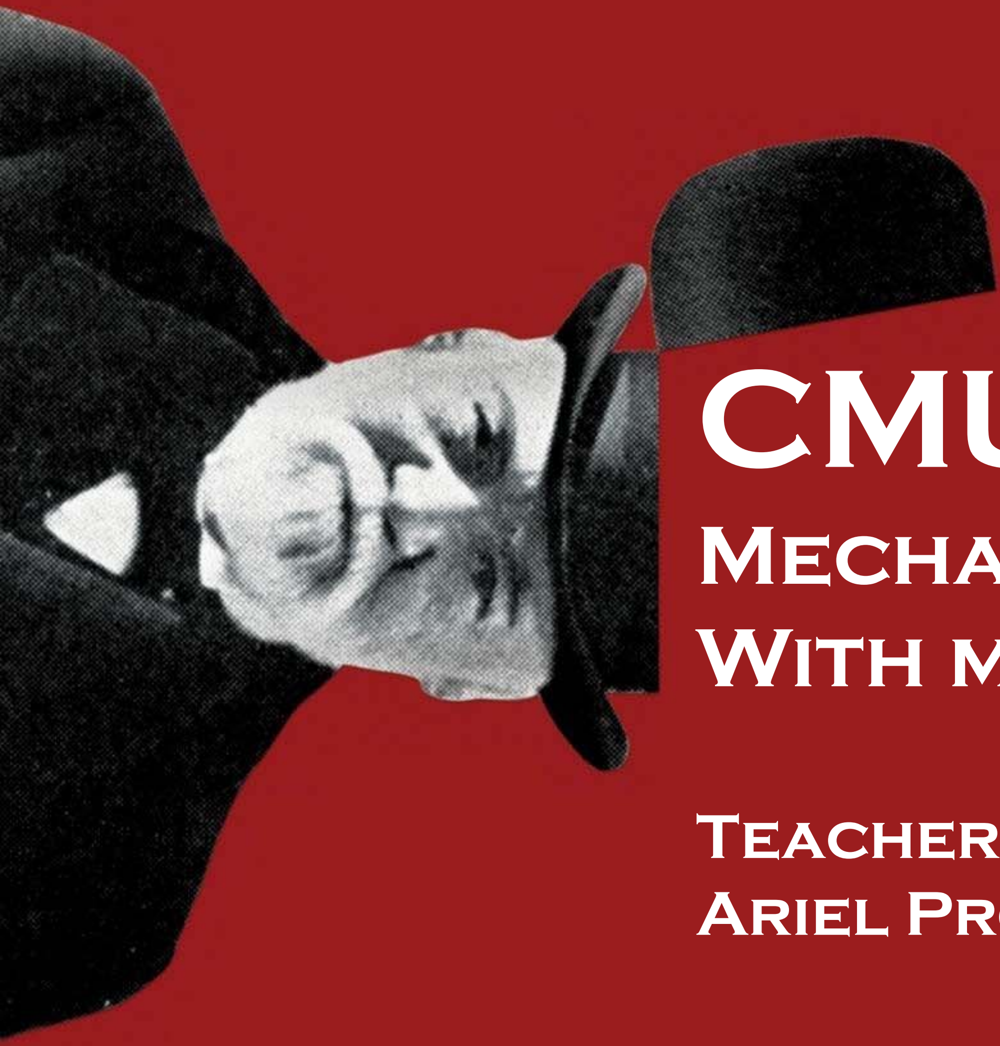


# CMU 15-896

## Mechanism design 2: With money

Teacher:
Ariel Procaccia

# MD WITH MONEY

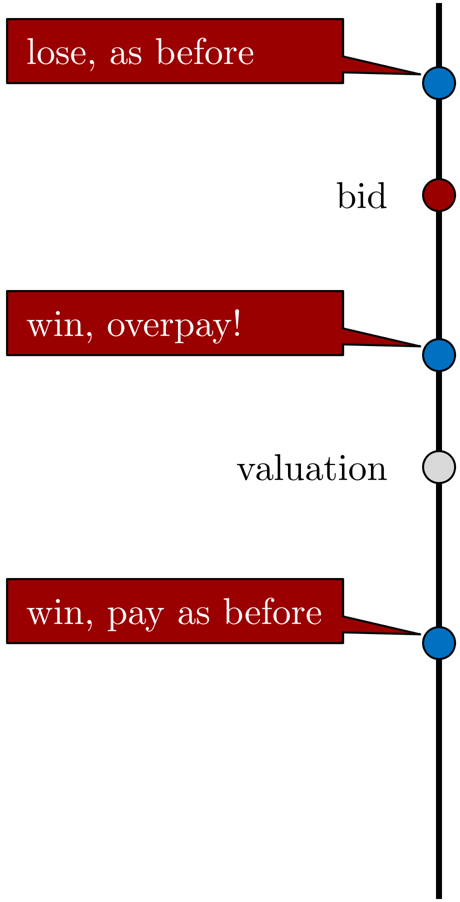
- Money gives us a powerful tool to align the incentives of players with the designer's objectives
- We will only cover a tiny fraction of the very basics of auction theory and algorithmic mechanism design

# Second-Price Auction

- Bidders submit sealed bids
- One good allocated to highest bidder
- Winner pays price of second highest bid!!
- Bidder's utility = value minus payment when winning, zero when losing
- Amazing observation: Second-price auction is strategyproof; bidding true valuation is a dominant strategy!!

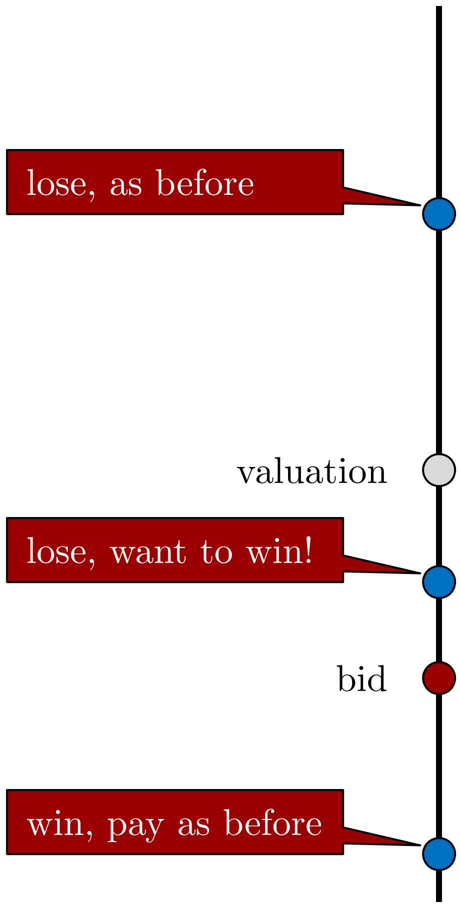
# Strategyproofness: bidding high

- Three cases based on highest other bid (blue dot)
- Higher than bid: lose before and after
- Lower than valuation: win before and after, pay same
- Between bid and valuation: lose before, win after but overpay



# Strategyproofness: bidding low

- Three cases based on highest other bid (blue dot)
- Higher than valuation: lose before and after
- Lower than bid: win before and after, pay the same
- Between valuation and bid: win before with profit, lose after ∎



# Vickrey-Clarke-Groves Mechanism

- $N$ = set of bidders, $M$ = set of $m$ items
- Each bidder has a combinatorial valuation function $v_i: 2^M \to \mathbb{R}^+$
- Choose an allocation $\boldsymbol{A} = (A_1, \dots, A_n)$ to maximize social welfare: $\sum_{i \in N} v_i(A_i)$
- If the outcome is $\boldsymbol{A}$, bidder $i$ pays

$$\max_{\boldsymbol{A}'} \sum_{j \neq i} v_j(A'_j) - \sum_{j \neq i} v_j(A_j)$$

# VCG Mechanism

- Suppose we run VCG and there are:
  - 1 item, denoted $a$
  - 2 bidders
  - $v_1(\{a\}) = 7,\ v_2(\{a\}) = 3$

Poll: What is the payment of player 1 in this example?

# VCG Mechanism

- Theorem: VCG is strategyproof
- Proof: When the outcome is $A$, the utility of bidder $i$ is

$$v_i(A_i) - \left[\max_{A'} \sum_{j \neq i} v_j(A'_j) - \sum_{j \neq i} v_j(A_j)\right]$$

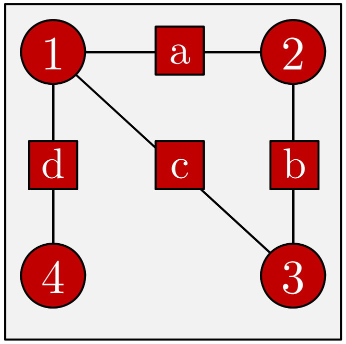
$$= \underbrace{\sum_{j \in N} v_j(A_j)}_{\text{Aligned with social welfare}} - \underbrace{\max_{A'} \sum_{j \neq i} v_j(A'_j)}_{\text{Independent of the bid of } i}$$

# Single minded bidders

- Allocate to maximize social welfare
- Consider the special case of single minded bidders: each bidder $i$ values a subset $S_i$ of items at $t_i$ and any subset that does not contain $S_i$ at 0
- Theorem (folk): optimal winner determination is NP-complete, even with single minded bidders

# Winner determination is hard

- Independent set (IS): given a graph, is there a set of vertices of size $k$ such that no two are connected?
- Given an instance of IS:
  - The set of items is $E$
  - Player for each vertex
  - Desired bundle is adjacent edges, value is 1
- A set of winners $W$ satisfies $S_i \cap S_j = \emptyset$ for every $i \neq j \in W$ iff the vertices in $W$ are an IS ■




1: {a,c,d}
2: {a,b}
3: {b,c}
4: {d}

# SP approximation

- In fact, optimal winner determination in combinatorial auctions with single-minded bidders is NP-hard to approximate to a factor better than $m^{1/2-\epsilon}$
- If we want computational efficiency, can't run VCG
- Need to design a new strategyproof, computationally efficient approx algorithm

# The greedy mechanism:

- Initialization:
  - Reorder the bids such that $\frac{v_1^*}{\sqrt{|S_1^*|}} \geq \frac{v_2^*}{\sqrt{|S_2^*|}} \geq \cdots \geq \frac{v_n^*}{\sqrt{|S_n^*|}}$
  - $W \leftarrow \emptyset$
- For $i = 1, \ldots, n$: if $S_i^* \cap (\bigcup_{j \in W} S_j^*) = \emptyset$ then $W \leftarrow W \cup \{i\}$
- Output:
  - Allocation: The set of winners is $W$
  - Payments: For each $i \in W$, $p_i = v_j^* \cdot \sqrt{|S_i^*|} / \sqrt{|S_j^*|}$, where $j$ is the smallest index such that $S_i^* \cap S_j^* \neq \emptyset$, and for all $k < j, k \neq i, S_k^* \cap S_i^* = \emptyset$ (if no such $j$ exists then $p_i = 0$)

# SP approximation

- Theorem [Lehmann et al. 2001]: The greedy mechanism is strategyproof, poly time, and gives a $\sqrt{m}$-approximation
- Note that the mechanism satisfies the following two properties:
  - Monotonicity: If $i$ wins with $(S_i^*, v_i^*)$, he will win with $v_i' > v_i^*$ and $S_i' \subset S_i^*$
  - Critical payment: A bidder who wins pays the minimum value needed to win

# Proof of SP

- We will show that bidder $i$ cannot gain by reporting $(S_i', v_i')$ instead of truthful $(S_i, v_i)$
- Can assume that $(S_i', v_i')$ is a winning bid and $S_i \subseteq S_i'$
- $(S_i, v_i')$ with payment $p$ is at least as good as $(S_i', v_i')$ with payment $p'$ because $p \leq p'$
- $(S_i, v_i)$ is at least as good as $(S_i, v_i')$ by similar reasoning to Vickrey auction ■

# Proof of approximation

- For $i \in W$, let
$$\mathrm{OPT}_i = \{j \in \mathrm{OPT}, j \geq i : S_i^* \cap S_j^* \neq \emptyset\}$$
- $\mathrm{OPT} \subseteq \bigcup_{i \in W} \mathrm{OPT}_i$, so enough that for $i \in W$,
$$\sum_{j \in \mathrm{OPT}_i} v_j^* \leq \sqrt{m}\, v_i^* \tag{1}$$
- For each $j \in \mathrm{OPT}_i$, $v_j^* \leq \dfrac{v_i^* \sqrt{|S_j^*|}}{\sqrt{|S_i^*|}}$

# Proof of approximation

- Summing over all $j \in \mathrm{OPT}_i$,

$$\sum_{j \in \mathrm{OPT}_i} v_j^* \le \frac{v_i^*}{\sqrt{|S_i^*|}} \sum_{j \in \mathrm{OPT}_i} \sqrt{|S_j^*|} \qquad (2)$$

- Using Cauchy-Schwarz $\left(\sum x_i y_i \le \sqrt{\sum_i x_i^2}\sqrt{\sum_i y_i^2}\right)$,

$$\sum_{j \in \mathrm{OPT}_i} \sqrt{|S_j^*|} \le \sqrt{|\mathrm{OPT}_i|} \sqrt{\sum_{j \in \mathrm{OPT}_i} |S_j^*|} \qquad (3)$$

# Proof of approximation

- $\sum_{j \in \mathrm{OPT}_i} |S_j^*| \leq m$
- $|\mathrm{OPT}_i| \leq |S_i^*|$
- Plugging into (3),

$$\sum_{j \in \mathrm{OPT}_i} \sqrt{|S_j^*|} \leq \sqrt{|S_i^*|} \cdot \sqrt{m}$$

- Plugging into (2), we get (1) ■

# Why MD? Olympic Badminton!


http://youtu.be/hdK4vPz0qaI

# Turing's Invisible Hand

## Computation, Economics, and Game Theory

« AGT lecture videos and notes    AGT course (Weeks 3 and 4) »

### Olympic badminton is not incentive compatible — revisited

October 18, 2013 by Ariel Procaccia | Edit

More than a year ago a badminton scandal during the London 2012 Olympics rocked the really-care-about-mechanism-design corner of the blogosphere. Bobby Kleinberg reported:

> The competition is structured in two stages: a round-robin stage in which the teams compete to earn placement in a second-stage single-elimination tournament. The second-seeded team in the competition, Tian Qing and Zhao Yunlei of China, suffered an upset loss to a Danish team in the first stage. As a result, they were placed as one of the lowest-seeded teams in the second-stage tournament. At that point it became advantageous for the remaining teams that had already secured a second-stage berth to lose their final first-stage match in order to avoid being paired against Qing and Yunlei in the first round of the second stage. Spectators watched in frustration and disbelief as the athletes deliberately served into the net and hit shots out of bounds in an effort to lose the match. Olympic officials afterward decided to disqualify the offending teams from the competition.

Bobby asked whether it's time for the International Olympic Committee to start reading research papers on mechanism design. A year ago the answer was "probably not", but a recent paper may soon have members of the IOC scrambling to get their math degrees. There are two interesting things about this paper, written by Marc Pauly and published in Social Choice and Welfare.

The first interesting thing is, well, the results. Pauly considers competition formats in which the competing teams (or players) are partitioned into two subsets. In the

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Soc Choice Welf (2014) 43:29–46
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ORIGINAL PAPER

# Can strategizing in round-robin subtournaments be avoided?

**Marc Pauly**

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© Springer-Verlag Berlin Heidelberg 2013

**Abstract** This paper develops a mathematical model of strategic manipulation in complex sports competition formats such as the soccer world cup or the Olympic games. Strategic manipulation refers here to the possibility that a team may lose a match on purpose in order to increase its prospects of winning the competition. In particular, the paper looks at round-robin tournaments where both first- and second-ranked players proceed to the next round. This standard format used in many sports gives rise to the possibility of strategic manipulation, as exhibited recently in the 2012 Olympic games. An impossibility theorem is proved which demonstrates that under a number of reasonable side-constraints, strategy-proofness is impossible to obtain.