

CMU 15-896

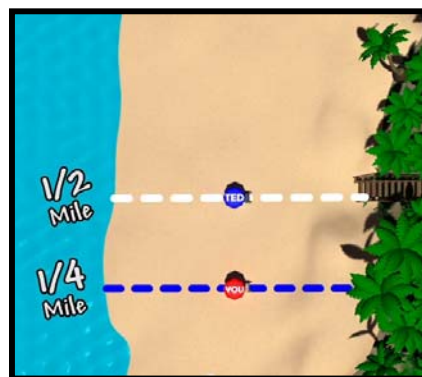
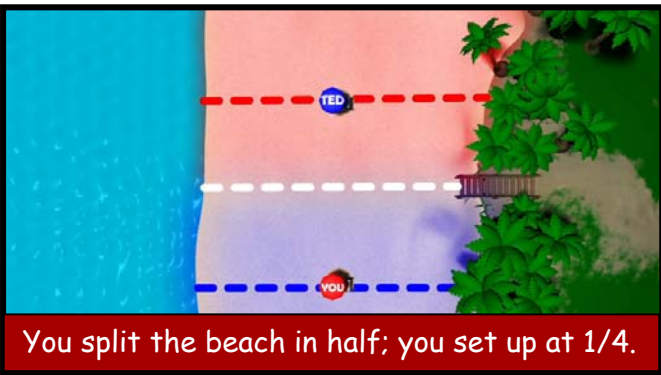
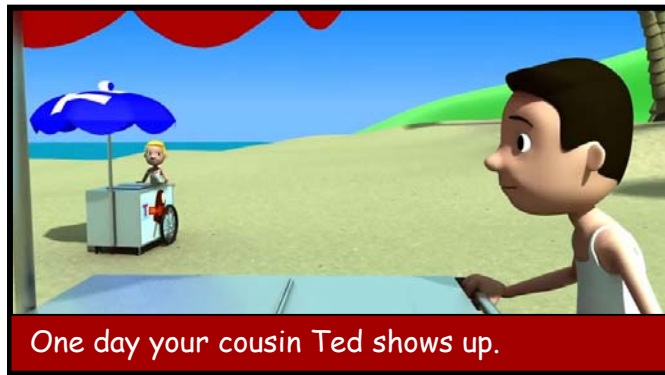
**NONCOOPERATIVE GAMES 1:
BASIC CONCEPTS**

**TEACHER:
ARIEL PROCACCIA**

NORMAL-FORM GAME

- A **game in normal form** consists of:
 - Set of players $N = \{1, \dots, n\}$
 - Strategy set S
 - For each $i \in N$, utility function $u_i: S^n \rightarrow \mathbb{R}$: if each $j \in N$ plays the strategy $s_j \in S$, the utility of player i is $u_i(s_1, \dots, s_n)$
- Next example created by taking screenshots of
http://youtu.be/jILgxeNBK_8





THE ICE CREAM WARS

- $N = \{1,2\}$
- $S = [0,1]$
- $u_i(s_i, s_j) = \begin{cases} \frac{s_i + s_j}{2}, & s_i < s_j \\ 1 - \frac{s_i + s_j}{2}, & s_i > s_j \\ \frac{1}{2}, & s_i = s_j \end{cases}$
- To be continued...



THE PRISONER'S DILEMMA

- Two men are charged with a crime
- They are told that:
 - If one rats out and the other does not, the rat will be freed, other jailed for nine years
 - If both rat out, both will be jailed for six years
- They also know that if neither rats out, both will be jailed for one year



THE PRISONER'S DILEMMA

	Cooperate	Defect
Cooperate	-1,-1	-9,0
Defect	0,-9	-6,-6

What would you do?

PRISONER'S DILEMMA ON TV



<http://youtu.be/S0qjK3TWZE8>

THE PROFESSOR'S DILEMMA

		Class	
		Listen	Sleep
Professor	Make effort	$10^6, 10^6$	$-10, 0$
	Slack off	$0, -10$	$0, 0$

Dominant strategies?

NASH EQUILIBRIUM

- Each player's strategy is a **best response** to strategies of others
- Formally, a **Nash equilibrium** is a vector of strategies $s = (s_1 \dots, s_n) \in S^n$ such that

$$\forall i \in N, \forall s'_i \in S, u_i(s) \geq u_i(s'_i, s_{-i})$$



NASH EQUILIBRIUM



<http://youtu.be/CemLiSI5ox8>

RUSSEL CROWE WAS WRONG

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Computation, Economics, and Game Theory

« STOC Submissions: message from the PC Chair

Russell Crowe was wrong

October 30, 2012 by Ariel Procaccia | Edit

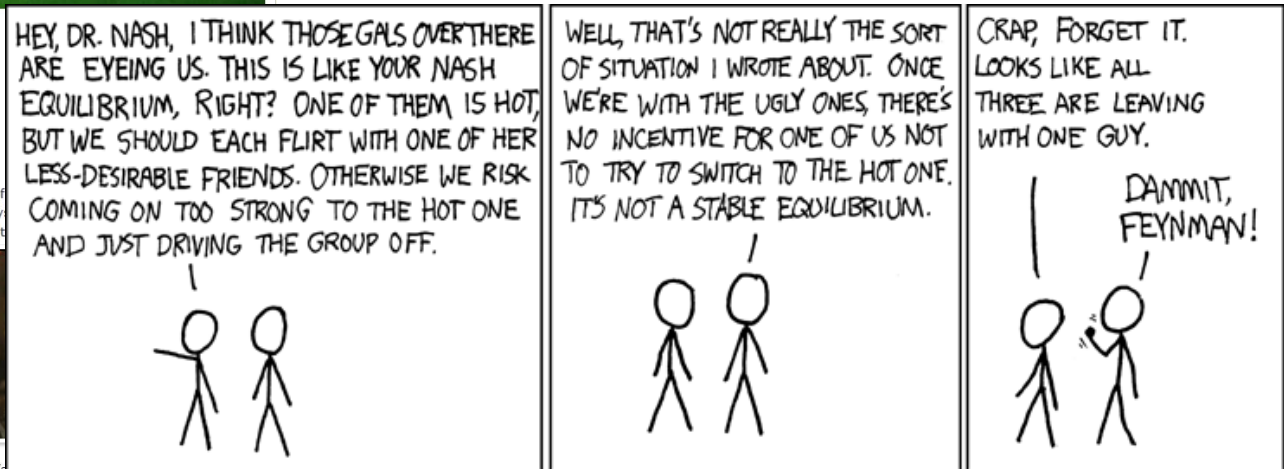
Yesterday I taught the first of five algorithmic economics lectures in my undergraduate AI course. This lecture just introduced the basic concepts of game theory, focusing on Nash equilibria. I was contemplating various ways of making the lecture more lively, and it occurred to me that I could stand on the shoulders of giants. Indeed, didn't Russell Crowe already explain Nash's ideas in *A Beautiful Mind*, complete with a 1940's-style male chauvinistic example?



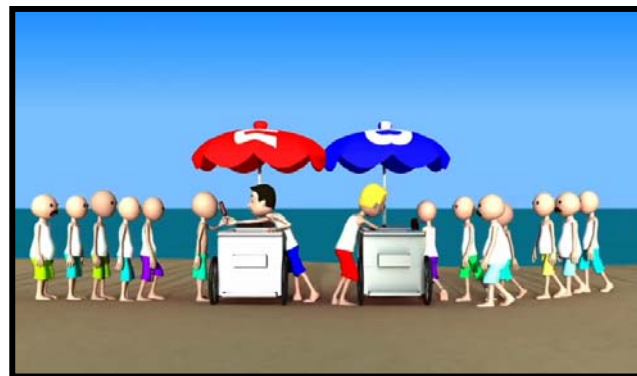
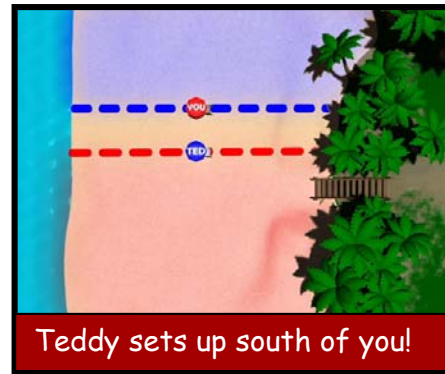
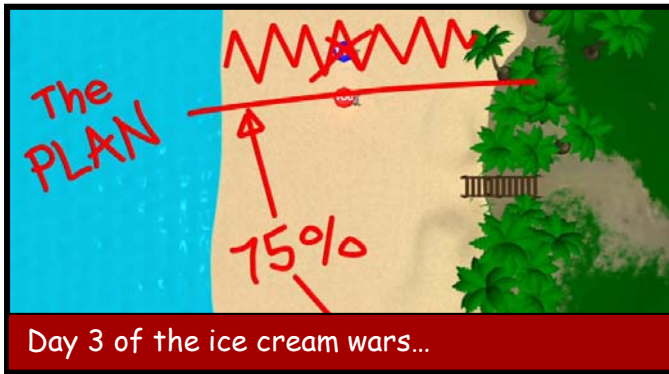
The first and last time I watched the movie was when it was released in 2001. Back then I was an undergrad freshman, working for 20+ hours a week on the programming exercises of Hebrew U's Intro to CS course, which was taught by some guy called Noam Nisan. I didn't know anything about game theory, and Crowe's explanation made a lot of sense at the time.

I easily found the relevant scene on youtube. In the scene, Nash's friends are trying to figure out how to seduce a beautiful blonde and her less beautiful friends. Then Nash/Crowe has an epiphany. The hubbub of the seedy Princeton bar is drowned by inspirational music, as Nash announces:

January 2012
December 2011
November 2011
October 2011
September 2011
August 2011
July 2011
June 2011



END OF THE ICE CREAM WARS



ROCK-PAPER-SCISSORS

	R	P	S
R	0,0	-1,1	1,-1
P	1,-1	0,0	-1,1
S	-1,1	1,-1	0,0

Nash equilibrium?

MIXED STRATEGIES

- A **mixed strategy** is a probability distribution over (pure) strategies
- The mixed strategy of player $i \in N$ is x_i , where

$$x_i(s_i) = \Pr[i \text{ plays } s_i]$$

- The utility of player $i \in N$ is

$$u_i(x_1, \dots, x_n) = \sum_{(s_1, \dots, s_n) \in S^n} u_i(s_1, \dots, s_n) \cdot \prod_{j=1}^n x_j(s_j)$$



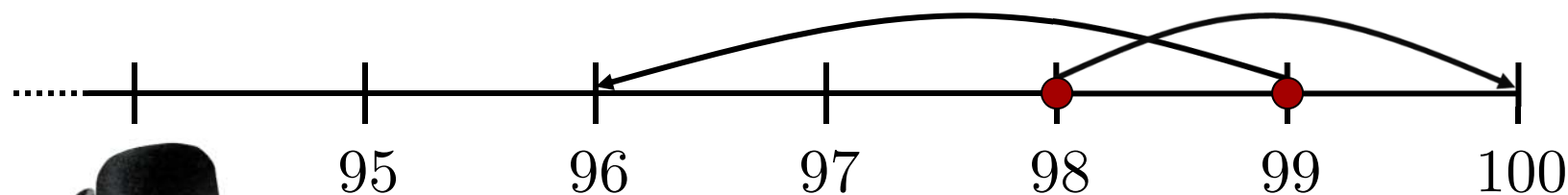
NASH'S THEOREM

- **Theorem [Nash, 1950]:** if everything is finite then there exists at least one (possibly mixed) Nash equilibrium
- We'll talk about computation some other time



DOES NE MAKE SENSE?

- Two players, strategies are $\{2, \dots, 100\}$
- If both choose the same number, that is what they get
- If one chooses s , the other t , and $s < t$, the former player gets $s + 2$, and the latter gets $s - 2$
- Poll 1: what would you choose?



CORRELATED EQUILIBRIUM

- Let $N = \{1,2\}$ for simplicity
- A mediator chooses a pair of strategies (s_1, s_2) according to a distribution p over S^2
- Reveals s_1 to player 1 and s_2 to player 2
- When player 1 gets $s_1 \in S$, he knows that the distribution over strategies of 2 is

$$\Pr[s_2 | s_1] = \frac{\Pr[s_1 \wedge s_2]}{\Pr[s_1]} = \frac{p(s_1, s_2)}{\sum_{s'_2 \in S} p(s_1, s'_2)}$$

CORRELATED EQUILIBRIUM

- Player 1 is best responding if for all $s'_1 \in S$

$$\sum_{s_2 \in S} \Pr[s_2 | s_1] u_1(s_1, s_2) \geq \sum_{s_2 \in S} \Pr[s_2 | s_1] u_1(s'_1, s_2)$$

- Equivalently,

$$\sum_{s_2 \in S} p(s_1, s_2) u_1(s_1, s_2) \geq \sum_{s_2 \in S} p(s_1, s_2) u_1(s'_1, s_2)$$

- p is a **correlated equilibrium (CE)** if both players are best responding



GAME OF CHICKEN



<http://youtu.be/u7hZ9jKrwvo>

GAME OF CHICKEN

- **Social welfare** is the sum of utilities
- Pure NE: (C,D) and (D,C), social welfare = 5
- Mixed NE: both $(1/2, 1/2)$, social welfare = 4
- Optimal social welfare = 6

	Dare	Chicken
Dare	0,0	4,1
Chicken	1,4	3,3

GAME OF CHICKEN

- Correlated equilibrium:

- $(D,D): 0$

- $(D,C): \frac{1}{3}$

- $(C,D): \frac{1}{3}$

- $(C,C): \frac{1}{3}$

	Dare	Chicken
Dare	0,0	4,1
Chicken	1,4	3,3

- Social welfare of CE = $\frac{16}{3}$



IMPLEMENTATION OF CE

- Instead of a mediator, use a hat!
- Balls in hat are labeled with “chicken” or “dare”, each blindfolded player takes a ball



Which balls implement the distribution of the previous slide?



CE vs. NE

- **Poll 2:** What is the relation between CE and NE?
 1. $CE \Rightarrow NE$
 2. $NE \Rightarrow CE$
 3. $NE \Leftrightarrow CE$
 4. $NE \parallel CE$



CE As LP

- Can compute CE via linear programming in polynomial time!

find $\forall s_1, s_2 \in S, p(s_1, s_2)$

s.t. $\forall s_1, s'_1, s_2 \in S, \sum_{s_2 \in A} p(s_1, s_2) u_1(s_1, s_2) \geq \sum_{s_2 \in A} p(s_1, s_2) u_1(s'_1, s_2)$

$\forall s_1, s_2, s'_2 \in S, \sum_{s_1 \in A} p(s_1, s_2) u_2(s_1, s_2) \geq \sum_{s_1 \in A} p(s_1, s_2) u_2(s_1, s'_2)$

$\sum_{s_1, s_2 \in S} p(s_1, s_2) = 1$

$\forall s_1, s_2 \in S, p(s_1, s_2) \in [0, 1]$

