TRUTH JUSTICE ALGOS

Social Choice I:
Basic Concepts

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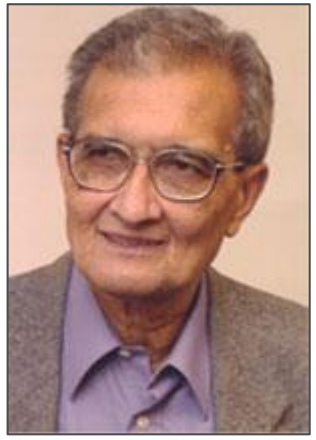
SOCIAL CHOICE THEORY

- A mathematical theory that deals with aggregation of individual preferences
- Origins in ancient Greece
- Formal foundations: 18th Century (Condorcet and Borda)
- 19th Century: Charles Dodgson
- 20th Century: Nobel prizes to Arrow and Sen



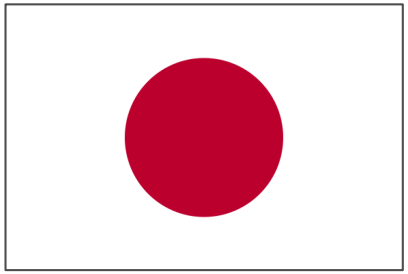



THE VOTING MODEL

- Set of voters $N = \{1, \dots, n\}$
- Set of alternatives A; denote $|A| = m$
- Each voter has a ranking $\sigma_i \in \mathcal{L}$ over the alternatives; $x \succ_i y$ means that voter i prefers x to y
- A preference profile $\boldsymbol{\sigma} \in \mathcal{L}^n$ is a collection of all voters' rankings
- A voting rule is a function $f: \mathcal{L}^n \to A$

VOTE OVER CUISINES

Indian
(In)



Japanese
(J)

Chinese
(C)

Italian
(It)

Mexican
(M)

VOTING RULES

- A positional scoring rule is defined by a score vector $(s_1, \dots, s_m)$
- Each voter gives s_k points to the alternative ranked in position k
- Alternative with most points wins
- Examples:
  - Plurality: $(1,0, \dots, 0)$
  - Borda: $(m-1, m-2, \dots, 0)$

Lordi
Eurovision 2006 winners

MORE VOTING RULES

- x beats y in a pairwise election if the majority of voters prefer x to y, i.e., $|\{i \in N : x \succ_i y\}| > n/2$
- Plurality with runoff
  - First round: two alternatives with highest plurality scores survive
  - Second round: pairwise election between these two alternatives

MORE VOTING RULES

- Single Transferable vote (STV)
  - $m - 1$ rounds
  - In each round, alternative with least plurality votes is eliminated
  - Alternative left standing is the winner
  - Used in:
    - Ireland, Malta, Australia, and New Zealand
    - US: Maine (governor, US congress), cities like San Francisco and Cambridge

STV: EXAMPLE

2 voters	2 voters	1 voter
a	*b*	*c*
b	*a*	*d*
c	*d*	*b*
d	*c*	*a*

2 voters	2 voters	1 voter
a	*b*	*c*
b	*a*	*b*
c	*c*	*a*

2 voters	2 voters	1 voter
a	*b*	*b*
b	*a*	*a*

2 voters	2 voters	1 voter
b	*b*	*b*

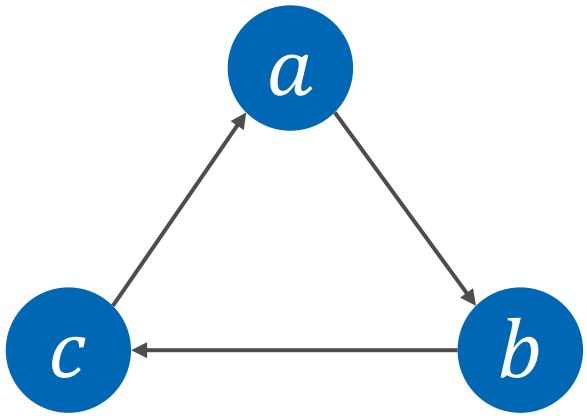
MARQUIS DE CONDORCET

- 18th Century French Mathematician, philosopher, political scientist
- One of the leaders of the French revolution
- After the revolution became a fugitive
- His cover was blown and he died mysteriously in prison

CONDORCET WINNER

- Recall: x beats y in a pairwise election if a majority of voters rank x above y
- Condorcet winner beats every other alternative in pairwise election
- The Condorcet Paradox: There may be a cycle in the majority preference relation

1	2	3
a	c	b
b	a	c
c	b	a



CONDORCET CONSISTENCY

- A voting rule is Condorcet consistent if it selects a Condorcet winner whenever one exists

Poll 1

Which rule is Condorcet consistent?

- Plurality
- Borda count
- Both rules
- Neither one

CONDORCET CONSISTENCY

- **Theorem:** No positional scoring rule is Condorcet consistent
- **Proof:**
  - Assume for ease of exposition that $s_i > s_{i+1}$ for all i
  - Consider the profile on the right
  - a is a Condorcet winner
  - Scores are $3s_1 + 2s_2 + 2s_3$ for a, $3s_1 + 3s_2 + s_3$ for b, so b is selected ■

3 voters	2 voters	1 voter	1 voter
a	b	b	c
b	c	a	a
c	a	c	b

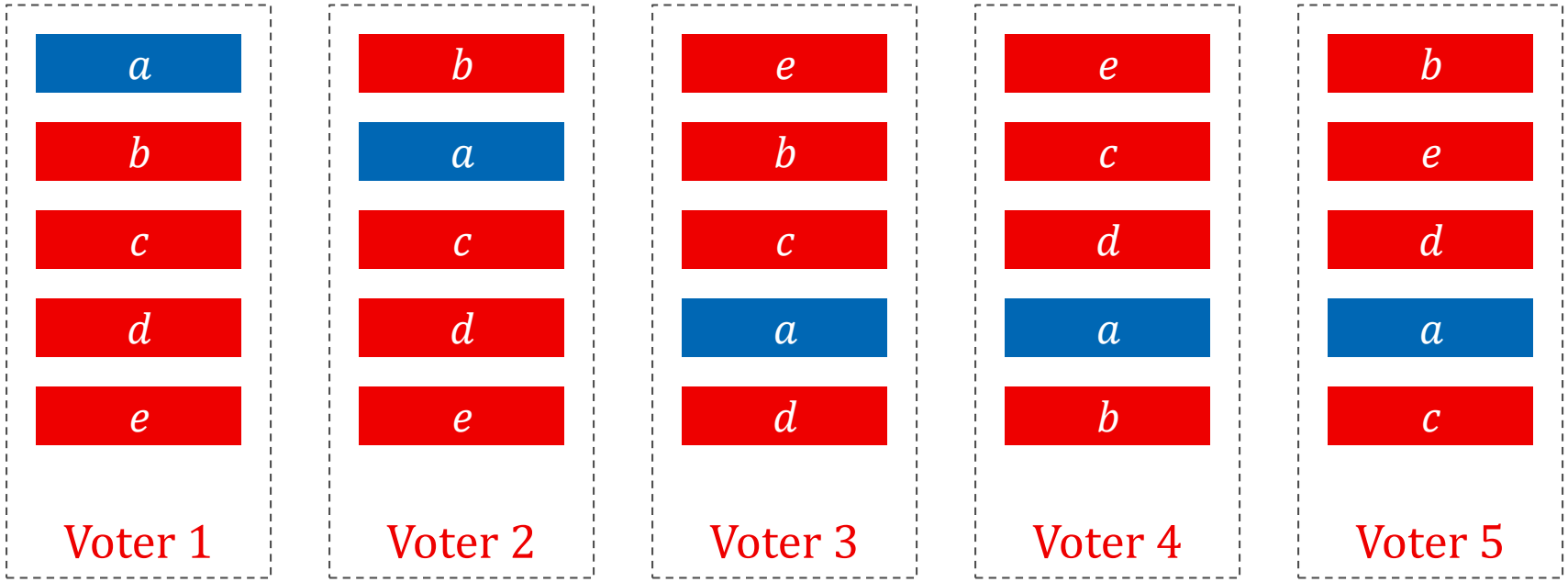
CONDORCET CONSISTENCY

- Copeland
  - Alternative's score is #alternatives it beats in pairwise elections
  - Why does Copeland satisfy the Condorcet criterion?
- Maximin
  - Score of x is $\min_y |\{i \in N: x \succ_i y\}|$
  - Why does Maximin satisfy the Condorcet criterion?

DODGSON'S RULE

- Distance function between profiles: #swaps between adjacent alternatives
- Dodgson score of x is the min distance from a profile where x is a Condorcet winner
- Dodgson's rule: select alternative that minimizes Dodgson score
- The problem of computing the Dodgson score is NP-complete!

DODGSON UNLEASHED



MONOTONICITY

- We say that $\boldsymbol{\sigma}'$ is obtained from $\boldsymbol{\sigma}$ by pushing $x \in A$ upwards if for all $i \in N$ and $y \in A$, $x \succ_i y \Rightarrow x \succ'_i y$, and for all $y, z \neq x$, $y \succ_i z \Leftrightarrow y \succ'_i z$
- A voting rule is monotonic if whenever $f(\boldsymbol{\sigma}) = x$, and $\boldsymbol{\sigma}'$ is obtained from $\boldsymbol{\sigma}$ by pushing x upwards, then $f(\boldsymbol{\sigma}') = x$

Poll 2

Which rule is not monotonic?

- Plurality
- Borda count
- STV
- Copeland

STV IS NOT MONOTONIC

- *c* is the winner in the following profile:

6 voters	2 voters	3 voter	4 voter	2 voters
c	*b*	*b*	*a*	*a*
a	*a*	*c*	*b*	*c*
b	*c*	*a*	*c*	*b*

- But *b* becomes the winner if the rightmost voters push *c* upwards:

6 voters	2 voters	3 voter	4 voter	2 voters
c	*b*	*b*	*a*	*c*
a	*a*	*c*	*b*	*a*
b	*c*	*a*	*c*	*b*

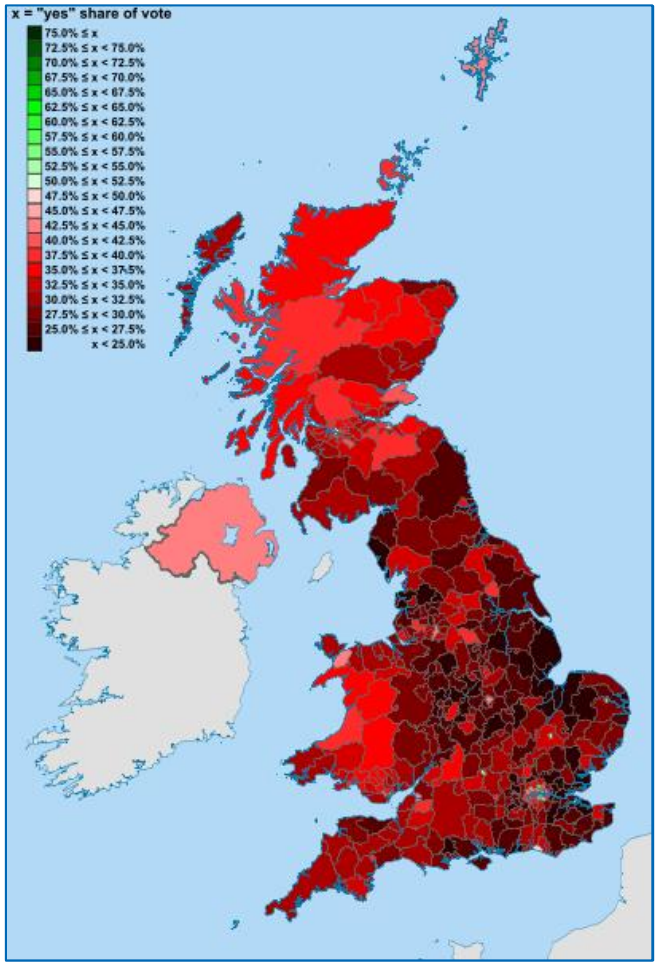
AWESOME EXAMPLE

33 voters	16 voters	3 voters	8 voters	18 voters	22 voters
a	b	c	c	d	e
b	d	d	e	e	c
c	c	b	b	c	b
d	e	a	d	b	d
e	a	e	a	a	a

Different rules select different winners:
Plurality (a), Borda count (b),
Copeland and Maximin (c is a Condorcet winner),
STV (d), and Plurality with runoff (e)

IS SOCIAL CHOICE PRACTICAL?

- UK referendum (2011): Choose between plurality and STV as a method for electing MPs
- Academics agreed STV is better...
- ... but STV seen as beneficial to the hated Nick Clegg
- Hard to change political elections!



COMPUTATIONAL SOCIAL CHOICE

However, in emerging paradigms of democracy and tools for group decision making, the designer is free to choose any voting rule!

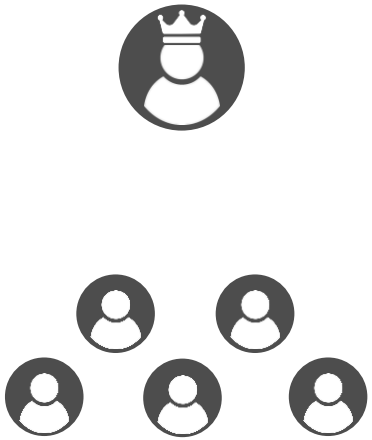
LIQUID DEMOCRACY

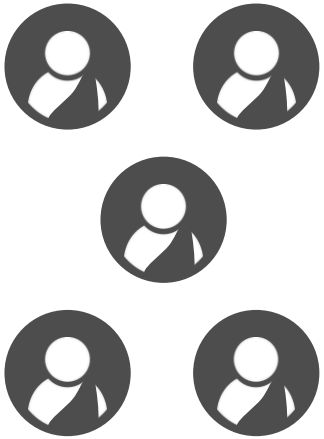
Monarchy or dictatorship

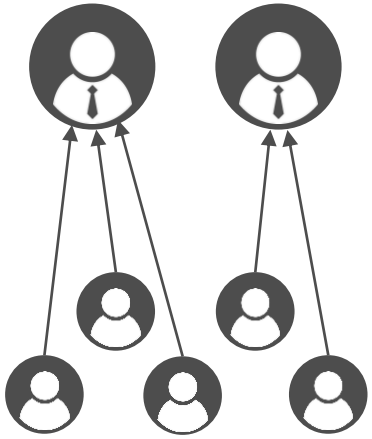
Direct democracy

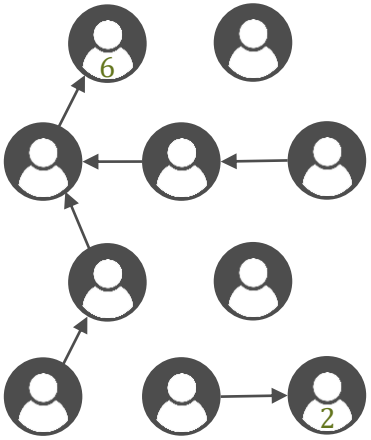
Representative democracy

Liquid democracy









VIRTUAL DEMOCRACY

1. green fire (loc410)

Hours: F Sa 10:00 AM - 5:30 PM
Contact: Admin Phone: 123-456-7890

128.13 /136
Voting Scores

Why this organization?

13 min ▼ (6 min – 64 min)	3+ months ▼ (2 weeks ago – 3 months+)	1.5 ▼ (normal(0) – very low(2))	
~ 20 ▼ (10 – 1500)	53.0% ▼ (3.1% – 53.0%)	$19925 ▼ ($19.9k – $111.6k)	

Total Donations in the last 3 months (0 total, 0 lbs)

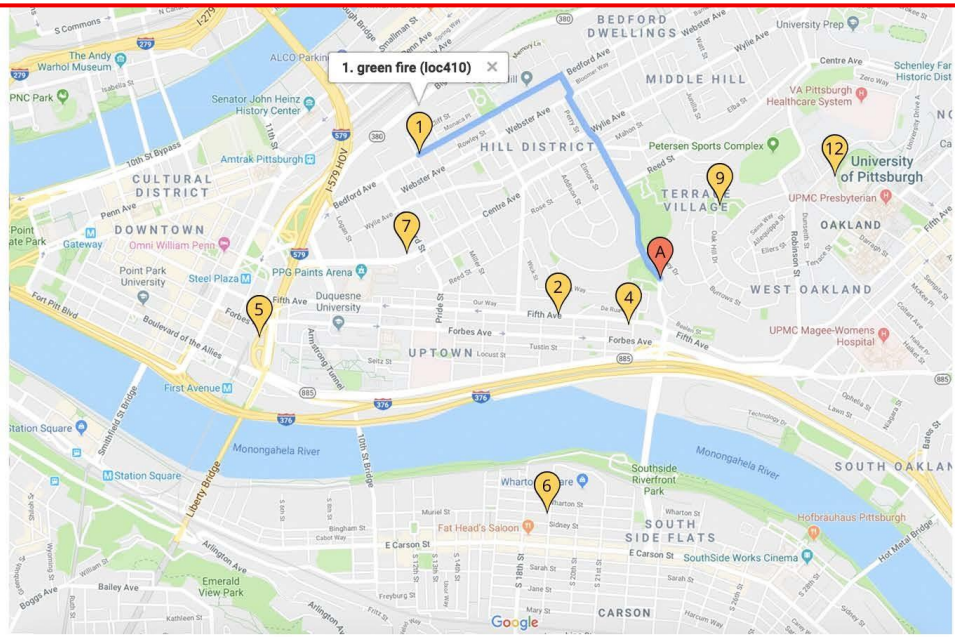
Produce	Baked Goods	Meat & Protein	Prepared Food	Non-Perishable	Dairy	Non-Food	Other
0 lbs	0 lbs	0 lbs	0 lbs	0 lbs	0 lbs	0 lbs	0 lbs

Stakeholder rankings (rank 1-rank 136)

Partners: **3rd** 412 Staff: **5th** Donors: **13th** Volunteers: **17th**

1 20 40 60 80 100 120 136

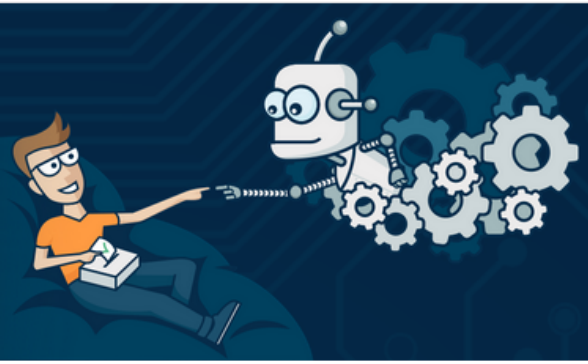
Choose



1. green fire (loc410) 128.13	13 min	06/08/18	20
	very low	53.0%	$19,925
2. hidden river (loc110) 120.58	9 min	05/02/18	30
	low	37.2%	$21,795
3. divine frost (loc376) 120.4	14 min	04/30/18	40
	low	53.0%	$19,925
4. blue fire (loc93) 120.19	9 min	n/a	20
	low	24.7%	$36,649

AI-Driven Decisions

RoboVote is a free service that helps users combine their preferences or opinions into optimal decisions. To do so, RoboVote employs state-of-the-art voting methods developed in artificial intelligence research. Learn More



Poll Types

RoboVote offers two types of polls, which are tailored to different scenarios; it is up to users to indicate to RoboVote which scenario best fits the problem at hand.

Objective Opinions

In this scenario, some alternatives are objectively better than others, and the opinion of a participant reflects an attempt to estimate the correct order. RoboVote's proposed outcome is guaranteed to be as close as possible — based on the available information — to the best outcome. Examples include deciding which product prototype to develop, or which company to invest in, based on a metric such as projected revenue or market share. Try the demo.

Subjective Preferences

In this scenario participants' preferences reflect their subjective taste; RoboVote proposes an outcome that mathematically makes participants as happy as possible overall. Common examples include deciding which restaurant or movie to go to as a group, which destination to choose for a family vacation, or whom to elect as class president. Try the demo.

Ready to get started?

CREATE A POLL