TRUTH JUSTICE ALGOS

Mechanism Design I: Basic Concepts and Myerson's Lemma

Teachers: Ariel Procaccia and Alex Psomas (this time)

MECHANISM DESIGN

- Game Theory: Interaction of rational, competing, strategic agents
- Mechanism Design: "Inverse Game Theory"
  - How do we design systems for rational, competing, strategic agents?
  - We'll be interested in promoting a desired objective
  - In this class we'll focus on auctions, but most of the tools we'll develop are applicable more generally

OLYMPICS 2012: A CAUTIONARY TALE

- 4 groups: A, B, C, D
- 4 teams per group
- Phase 1: Round robin within each group
  - Top two from each group advance in the second phase
- Phase 2: Knockout
  - In the first match , top team from group A is matched with second best of group C. Top team in C with second best from A. Similarly for B and D.
- What does a team want?
  - Maximize probability of winning a gold medal!
- What does the Olympic committee want?

OLYMPICS 2012: A CAUTIONARY TALE

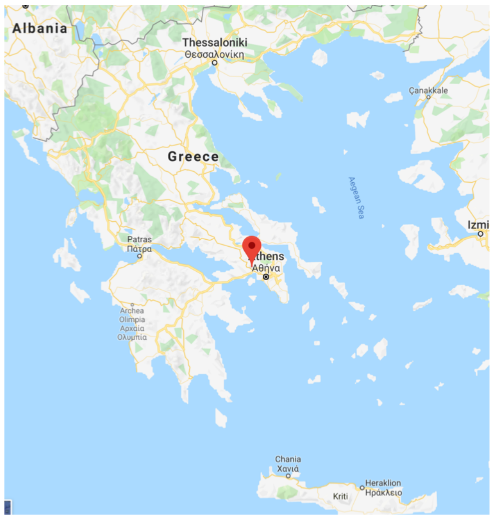
- Phase 1:
  - What if teams A_1 and A_2 have destroyed teams A_3 and A_4, and in the final match are playing each other?
  - No problem! the loser would play the best in C, so A_1 and A_2 are still incentivized to try hard!
  - No problem? What if there's a huge upset in group C, and the (actually) best team ends up in second place?
  - Come on… What are the chances??

OLYMPICS 2012: A CAUTIONARY TALE

Video (17:30) : https://youtu.be/7mq1ioqiWEo

HOT OFF THE PRESS!!!

Mandra:



Floods (Nov 17):

- Greek national exams: Average grade is the only criterion to go to university.
- New law: People from Mandra get a small boost.
- 2018: Huge spike in the number of people that declare Mandra as their primary residence.

THE APPROACH

What’s wrong with these people???

What’s wrong with these rules?

QUESTIONS

- When can we design systems that are robust to strategic manipulation?
- What does computer science bring to the table?
  - How much harder is mechanism design than algorithm design?
- Tradeoffs between simplicity and optimality.

Disclaimer: This is not an economics course

ASSUMPTIONS

- We'll be working in a setting with **money**.
- Agents are **risk neutral:**
  - Value v_i with probability q_i for $i = 1, \dots, n$ is the same as value $\sum_{i=1}^{n} v_i q_i$ deterministically
- Agents have **quasi-linear** utilities:
  - Utility for value v for a price of p equals $v - p$
- We'll focus on **truthfulness**: reporting your true value maximizes your utility (more on this later)
- We'll also ask for **Individual Rationality:** if you say the truth, expected utility (over the randomness of the mechanism) is non-negative.
  - Participating is better than staying home.

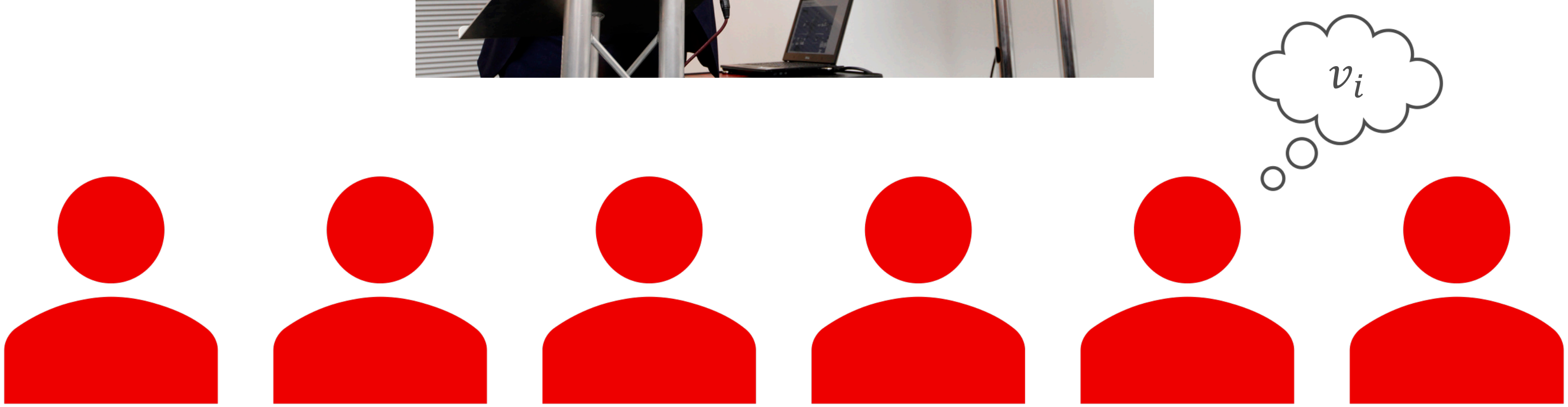
AUCTIONS

We will mostly talk about auctions

AUCTIONS: EXAMPLES

SINGLE ITEM AUCTIONS

- Single item for sale.
- n potential buyers: the bidders.
- Each bidder has a private value v_i for the item.



SEALED-BID AUCTIONS

1. Each bidder i privately communicates her bid b_i, possibly different than v_i, to the auctioneer (in a sealed envelope)
2. The auctioneer decides who to allocate the item to.
3. The auctioneer decides who pays what.

SEALED-BID AUCTIONS

- Obvious answer to (2): give the item to the highest bidder
- Reasonable ways to implement (3):
  - Highest bidder pays her bid, aka a **first price auction.**
  - Highest bidder pays the minimum bid required to win, i.e. the second highest bid. This is the **second price auction**.

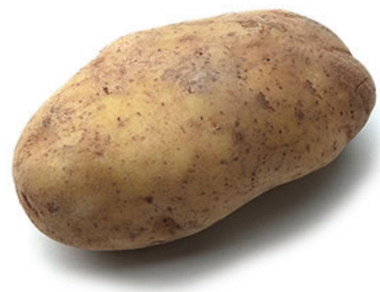
STRAWMAN

- Wait… Why charge in the first place?
- Proposal: give the item to the highest bidder and charge them nothing.
- Aka, “who can name the highest number?”
- Remember fair division?
  - In retrospect, truthful algorithms that eschew payments look even more amazing!

FIRST PRICE AUCTIONS

- How do I bid??
- If I bid my true value v_i I always get utility zero!
  - If I lose, I get nothing and pay nothing.
  - If I win, I pay v_i and get value v_i.
- So, I ``should'' bid something smaller than v_i
- How much smaller?

EXAMPLE



Poll 1

Assume your value = month + day of your birthday. E.g. 10/08/1997, value = 18.
How much would you bid?

FIRST PRICE AUCTIONS

- In order to argue about bidding behavior, we need to make more assumptions about the **information** agents have about other agents' bids.
- Common assumption: values come from known distribution D_i.
- Common question: what is an equilibrium bidding strategy? That is, if everyone follows this strategy, no one deviates.
- See homework.

SECOND PRICE AUCTIONS

- Who gets the item: highest bidder.
- What do they pay: the second highest bid.
- Claim: For a bidder to set $b_i = v_i$ (weakly) maximizes her utility *no matter what* everyone else is doing!
- Definition: When a player has a strategy that is (weakly) better than all other options, regardless of what the other player does, we will refer to it as a **dominant strategy.**

SECOND PRICE AUCTIONS

- Claim: Truth-telling is a dominant strategy.

Proof:

- Let $b_{-i} = (\mathrm{b}_1, \dots, \mathrm{b}_{\mathrm{i}-1}, \mathrm{b}_{i+1}, \dots, b_n)$ be the bids of all players except i. Let $B = \max_{j \neq i} b_j$
- There are two possible outcomes:
  1. $b_i < B, i$ loses and gets utility $u_i = 0$
  2. $b_i \geq B, i$ wins, pays B and gets utility $u_i = \mathrm{v_i} - \mathrm{B}$
- Effectively, i's utility is picking between 0 and $v_i - B$
  - If $v_i < B, \max\{0, v_i - B\} = 0$, which you can get by bidding $b_i = v_i$
  - If $v_i \geq \mathrm{B}, \max\{0, v_i - B\} = v_i - B$, which you can get by bidding $b_i = v_i$

SECOND PRICE AUCTIONS

- Theorem: The second price auction, aka the **Vickrey auction**, is awesome!
  - Dominant strategy incentive compatible (DSIC)!
  - Maximizes Social surplus! That is, the item always goes to the agent with the highest value!
  - Can be computed in polynomial (linear) time!

TOWARDS A MORE GENERAL RESULT

- If we have a single item and want to give it to the agent with the highest value, we can do so truthfully.
- What if we don’t want to give the item to the agent with the highest value?

SINGLE PARAMETER ENVIRONMENTS

- n buyers
- Buyer i has private valuation v_i and submits a bid b_i
- An auction is a pair of two functions (x, p)
- $x(b_1, \ldots, b_n) = (x_1, \ldots, x_n)$ is the **allocation** function.
    - $x_i =$ Probability that item goes to player i.
    - For single item auctions: $\sum_i x_i \leq 1$
    - Our next result will not use this fact!
- $p(b_1, \ldots, b_n) = (p_1, \ldots, p_n)$ is the **payment** function.
    - $p_i =$ Price player i pays.

MYERSON'S LEMMA

- Definition: An allocation rule x is implementable if there is a payment rule p such that the auction (x, p) is DSIC.
- We've seen that the allocation rule ``give the item to the highest bidder'' is implementable!
- What about the allocation rule ``give the item to the 3-rd highest bidder''?

MYERSON'S LEMMA

- Definition: An allocation rule x is monotone if for every bidder i and bids b_{-i} of the other agents, the allocation $x_i(b_i, b_{-i})$ is monotone non-decreasing in b_i.
- Lemma(Myerson):
  - An allocation is implementable iff it is monotone
  - If x is monotone, there exists a unique (up to a constant) payment rule p that makes (x, p) DSIC, given by

$$p_i(v, b_{-i}) = v x_i(v, b_{-i}) - \int_0^v x_i(z, b_{-i}) dz$$

POLL

Poll 2

Is the allocation rule “give the item to the third highest bidder” implementable?

1. Yes
2. No

MYERSON'S LEMMA: PROOF

- IC constraint between v and v':
  - $v\, x_i(v, b_{-i}) - p_i(v, b_{-i}) \geq v x_i(v', b_{-i}) - p_i(v', b_{-i})$
  - $v' x_i(v', b_{-i}) - p_i(v', b_{-i}) \geq v' x_i(v, b_{-i}) - p_i(v, b_{-i})$
- $v(x_i(v, b_{-i}) - x_i(v', b_{-i})) \geq$

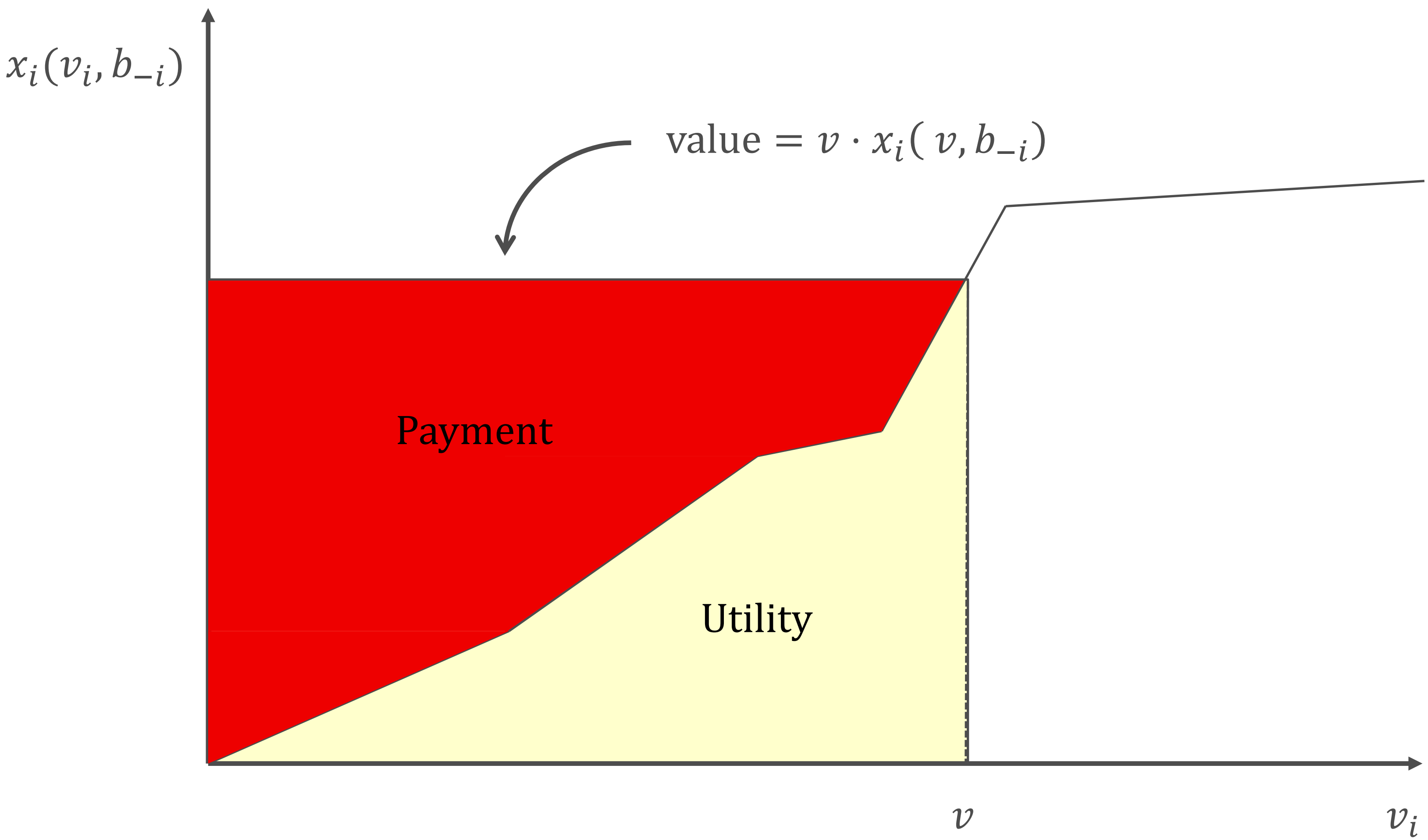
$$p_i(v, b_{-i}) - p_i(v', b_{-i})$$

$$\geq v'(x_i(v, b_{-i}) - x_i(v', b_{-i}))$$

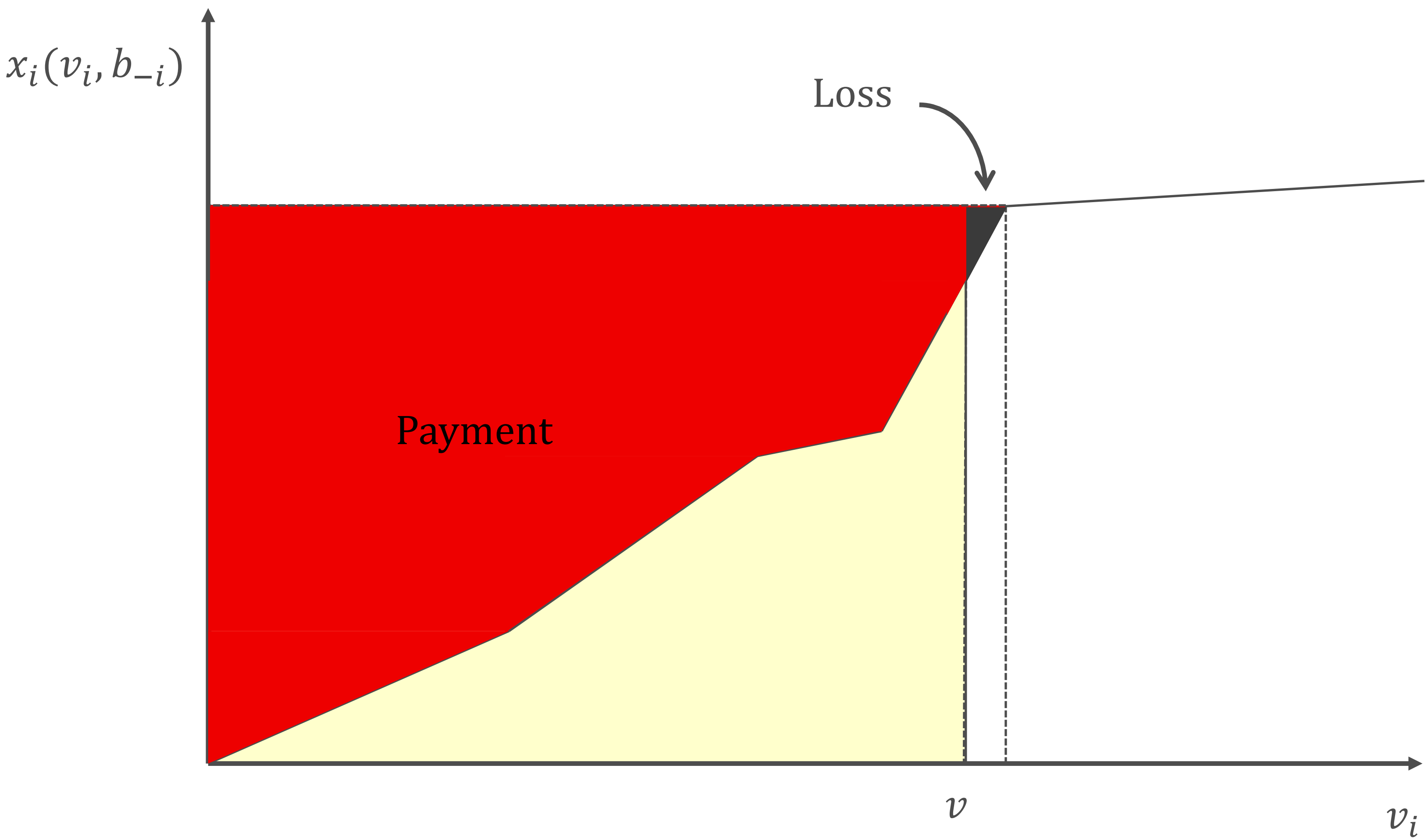
MYERSON'S LEMMA: PROOF

- $v(x_i(v, b_{-i}) - x_i(v', b_{-i})) \geq$
  $$p_i(v, b_{-i}) - p_i(v', b_{-i})$$
  $$\geq v'(x_i(v, b_{-i}) - x_i(v', b_{-i}))$$
- $v \geq v'$ implies monotonicity of the allocation!
- Take $v' = v - \epsilon$, and take the limit as ϵ goes to zero.
  - $p'_i(v, b_{-i}) = v x_i'(v, b_{-i})$
  - $p_i(v, b_{-i}) = v x_i(v, b_{-i}) - \int_0^v x_i(z, b_{-i}) dz + p_i(0, b_{-i}) + c(b_{-i})$
- Assuming that $p_i(0, b_{-i}) = 0$ (**Individual rationality**) we get the desired result.

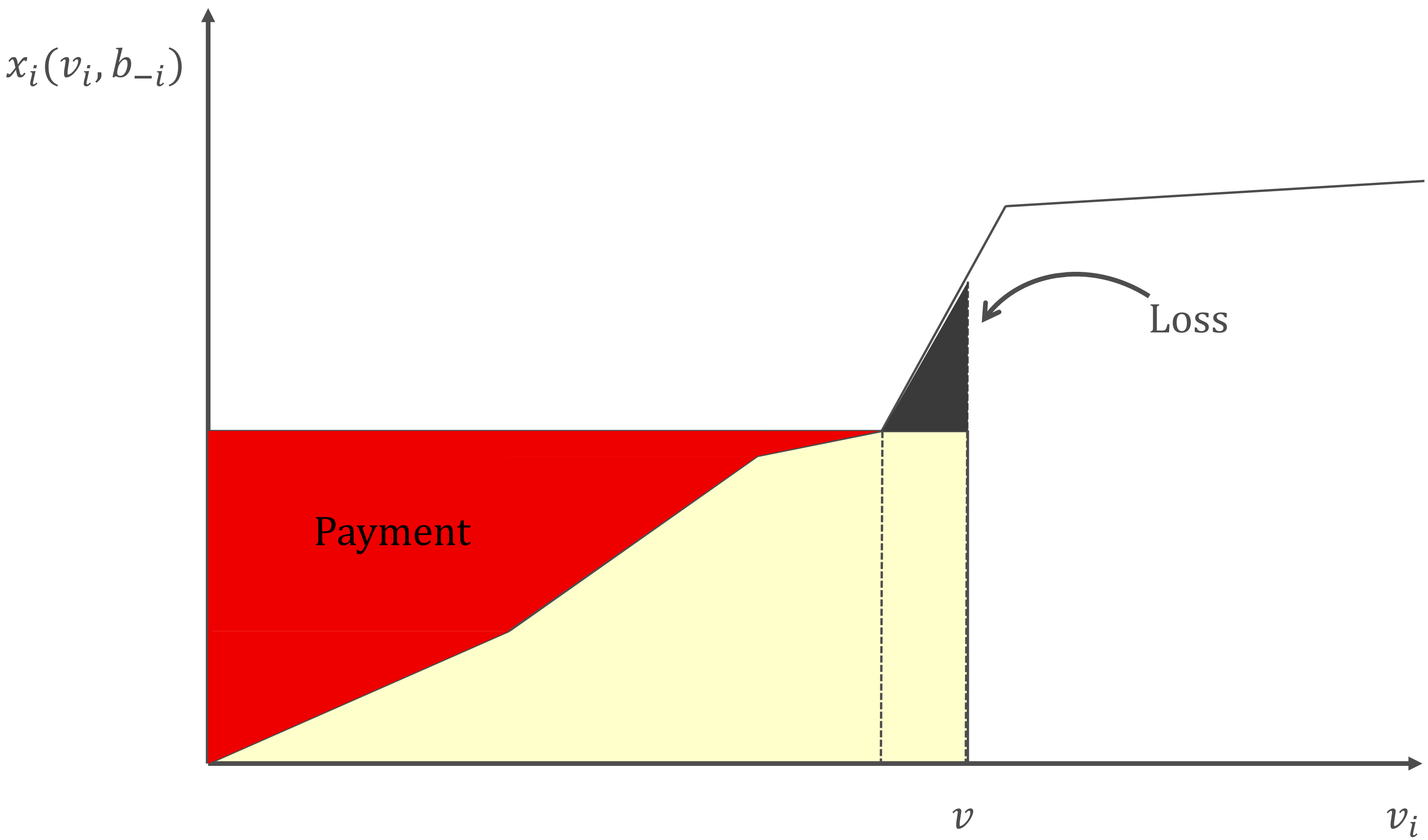
MYERSON'S LEMMA PICTORIALLY



MYERSON'S LEMMA PICTORIALLY



MYERSON'S LEMMA PICTORIALLY



SUMMARY

- Basic definitions of single parameter environments
- Second price auctions
- Myerson’s lemma