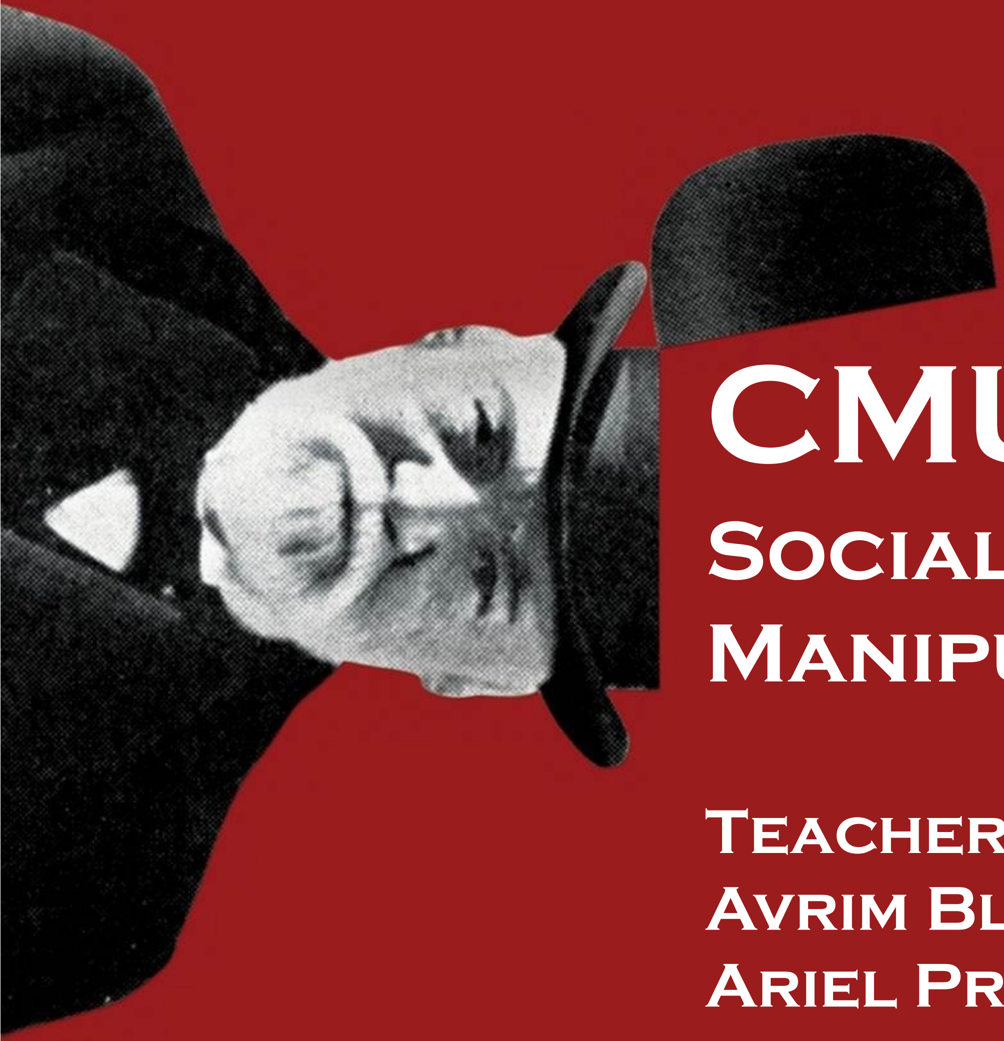


CMU 15-896

Social choice: Manipulation

Teachers:
Avrim Blum
Ariel Procaccia (this time)

Reminder: Voting

- Set of voters $N = \{1, \ldots, n\}$
- Set of alternatives $A, |A| = m$
- Each voter has a ranking over the alternatives
- $x \succ_i y$ means that voter i prefers x to y
- Preference profile = collection of all voters' rankings
- Voting rule = function from preference profiles to alternatives
- Important: so far voters are honest!

Manipulation

- Using Borda count
- Top profile: b wins
- Bottom profile: a wins
- By changing his vote, voter 3 achieves a better outcome!
- Borda responded: “My scheme is intended only for honest men!”

1	2	3
b	b	a
a	a	b
c	c	c
d	d	d

1	2	3
b	b	a
a	a	c
c	c	d
d	d	b

Strategyproofness

- A voting rule is strategyproof (SP) if a voter can never benefit from lying about his preferences:

  $$\forall \prec, \forall i \in N, \forall \prec_i', f(\prec) \succcurlyeq_i f(\prec_i', \prec_{-i})$$

- Vote: value of m for which plurality is SP
- Vote: are constant functions and dictatorships SP?

Gibbard-Satterthwaite

- A voting rule is dictatorial if there is a voter who always gets his most preferred alternative
- A voting rule is onto if any alternative can win
- **Theorem (Gibbard-Satterthwaite)**: If $m \geq 3$ then any voting rule that is SP and onto is dictatorial
- In other words, any voting rule that is onto and nondictatorial is manipulable

Proof of G-S

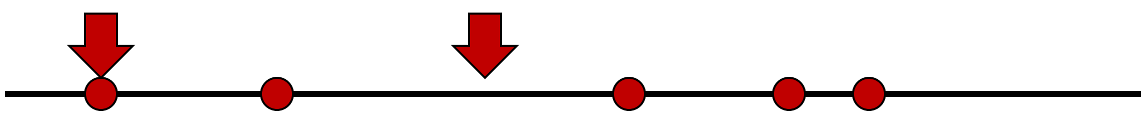
- Lemmas (prove in HW2):
  - Strong monotonicity: f is SP rule, $\prec$ profile, $f(\prec) = a$. Then $f(\prec') = a$ for all profiles $\prec'$ s.t. $\forall x \in A, i \in N$: $[a \succ_i x \Rightarrow a \succ_i' x]$
  - Pareto optimality: f is SP+onto rule, $\prec$ profile. If $a \succ_i b$ for all $i \in N$ then $f(\prec) \neq b$
- We prove the G-S Theorem for $n = 2$ on the board

Circumventing G-S

- Restricted preferences (this lecture)
- Money $\Rightarrow$ mechanism design (Avrim)
- Computational complexity (this lecture)

Single peaked preferences

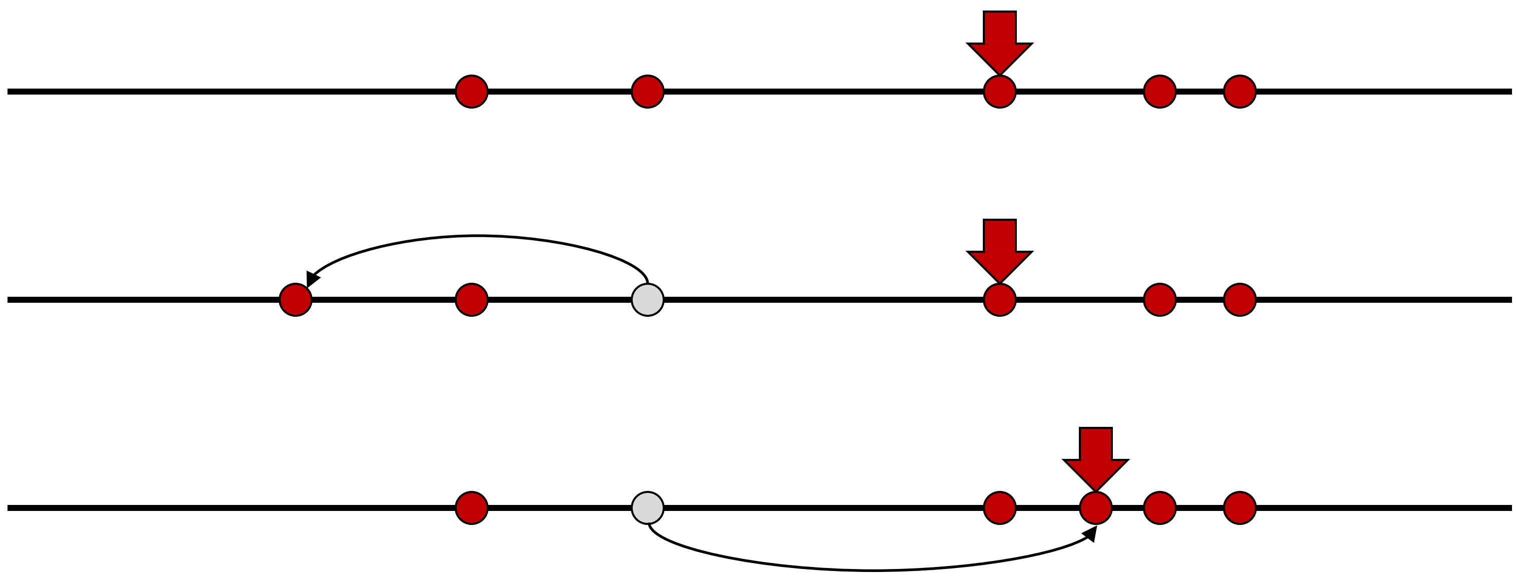
- We want to choose a location for a public good (e.g., library) on a street
- Alternatives = possible locations
- Each voter has an ideal location (peak)
- The closer the library is to a voter's peak, the happier he is
- Vote: leftmost and midpoint are SP?



The median

- Select the median peak
- The median is a Condorcet winner!
- The median is onto
- The median is nondictatorial

The median is SP



Complexity of manipulation

- Manipulation is always possible in theory
- But can we design voting rules where it is difficult in practice?
- Are there "reasonable" voting rules where manipulation is a hard computational problem? [Bartholdi et al., SC&W 1989]

The computational problem

- R-Manipulation problem:
  - Given votes of nonmanipulators and a preferred candidate p
  - Can manipulator cast vote that makes p (uniquely) win under R?
- Example: Borda, $p = a$

1	2	3
b	b	
a	a	
c	c	
d	d	

1	2	3
b	b	a
a	a	c
c	c	d
d	d	b

A greedy algorithm

- Rank p in first place
- While there are unranked alternatives:
  - If there is an alternative that can be placed in next spot without preventing p from winning, place this alternative
  - Otherwise return false

Example: Borda

1	2	3
b	b	a
a	a	
c	c	
d	d	

1	2	3
b	b	a
a	a	b
c	c	
d	d	

1	2	3
b	b	a
a	a	c
c	c	
d	d	

1	2	3
b	b	a
a	a	c
c	c	b
d	d	

1	2	3
b	b	a
a	a	c
c	c	d
d	d	

1	2	3
b	b	a
a	a	c
c	c	d
d	d	b

Example: Copeland

1	2	3	4	5
a	b	e	e	a
b	a	c	c	
c	d	b	b	
d	e	a	a	
e	c	d	d	

Preference profile

	a	b	c	d	e
a	-	2	3	5	3
b	3	-	2	4	2
c	2	2	-	3	1
d	0	0	1	-	2
e	2	2	3	2	-

Pairwise elections

Example: Copeland

1	2	3	4	5
a	b	e	e	a
b	a	c	c	c
c	d	b	b	
d	e	a	a	
e	c	d	d	

Preference profile

	a	b	c	d	e
a	-	2	3	5	3
b	3	-	2	4	2
c	2	3	-	4	2
d	0	0	1	-	2
e	2	2	3	2	-

Pairwise elections

Example: Copeland

1	2	3	4	5
a	b	e	e	a
b	a	c	c	c
c	d	b	b	d
d	e	a	a	
e	c	d	d	

Preference profile

	a	b	c	d	e
a	-	2	3	5	3
b	3	-	2	4	2
c	2	3	-	4	2
d	0	1	1	-	3
e	2	2	3	2	-

Pairwise elections

Example: Copeland

1	2	3	4	5
a	b	e	e	a
b	a	c	c	c
c	d	b	b	d
d	e	a	a	e
e	c	d	d	

Preference profile

	a	b	c	d	e
a	-	2	3	5	3
b	3	-	2	4	2
c	2	3	-	4	2
d	0	1	1	-	3
e	2	3	3	2	-

Pairwise elections

Example: Copeland

1	2	3	4	5
a	b	e	e	a
b	a	c	c	c
c	d	b	b	d
d	e	a	a	e
e	c	d	d	b

Preference profile

	a	b	c	d	e
a	-	2	3	5	3
b	3	-	2	4	2
c	2	3	-	4	2
d	0	1	1	-	3
e	2	3	3	2	-

Pairwise elections

When does the alg work?

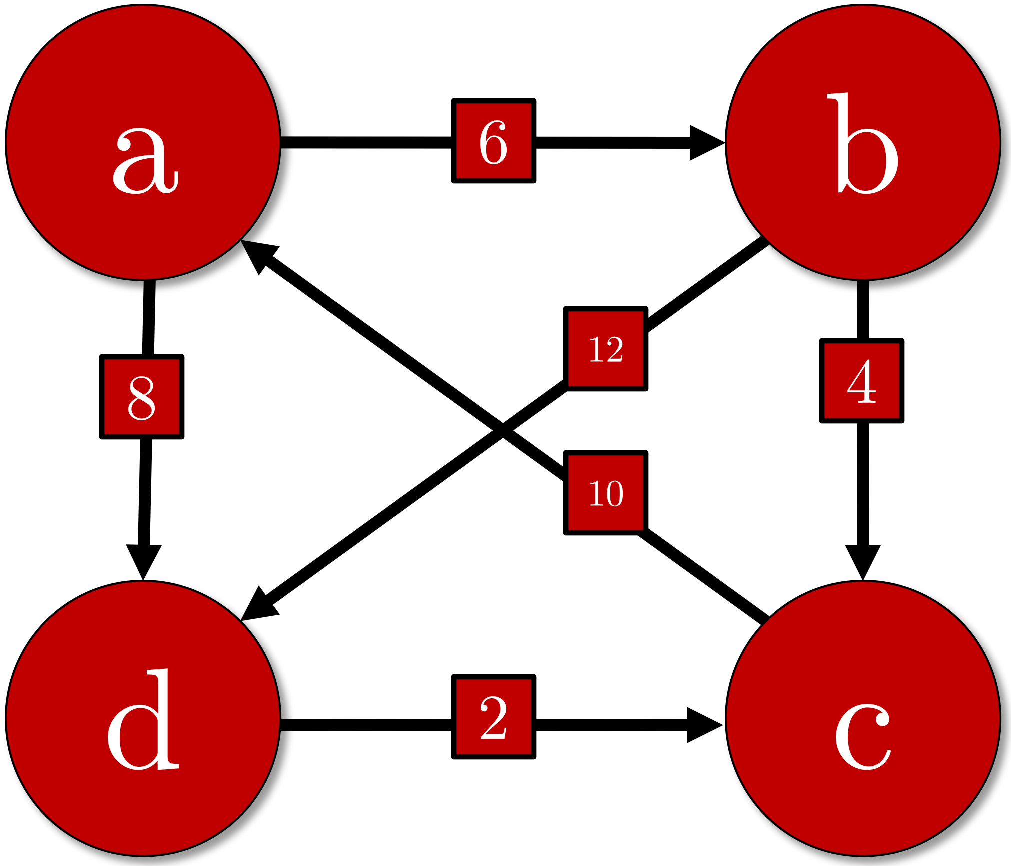
- **Theorem [Bartholdi et al., SCW 89]:** Fix $i \in N$ and the votes of other voters. Let R be a rule s.t. $\exists$function $s(\prec_i, x)$ such that:
  - For every $\prec_i$ chooses a candidate that uniquely maximizes $s(\prec_i, x)$
  - $\{y \colon y \prec_i x\} \subseteq \{y \colon y \prec_i' x\} \Rightarrow s(\prec_i, x) \leq s(\prec_i', x)$

  Then the algorithm always decides R-Manipulation correctly
- Vote: which rule does the theorem *not* capture?
- We will prove the theorem on Thursday

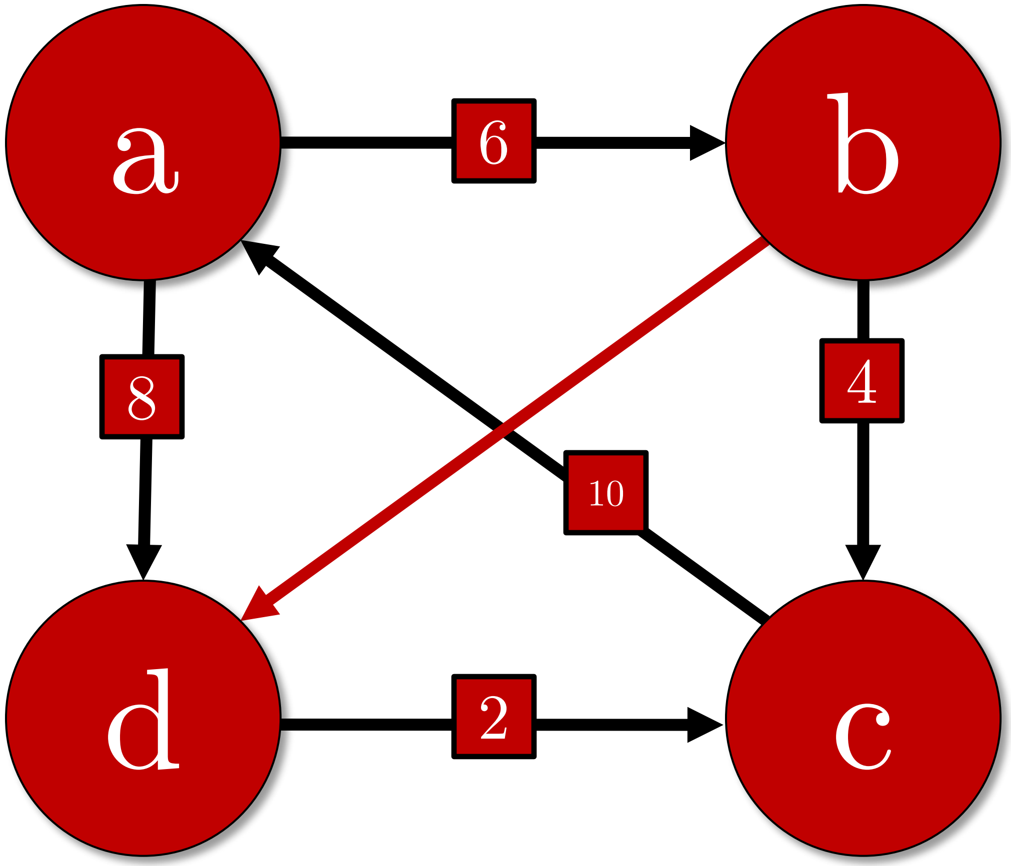
Voting rules that are hard to manipulate

- Natural rules
  - Copeland with second order tie breaking [Bartholdi et al., SCW 89]
  - STV [Bartholdi&Orlin, SCW 91]
  - Ranked Pairs [Xia et al., IJCAI 09]
    - Order pairwise elections by decreasing strength of victory
    - Successively lock in results of pairwise elections unless it leads to cycle
    - Winner is the top ranked candidate in final order
- Can also "tweak" easy to manipulate voting rules [Conitzer&Sandholm, IJCAI 03]

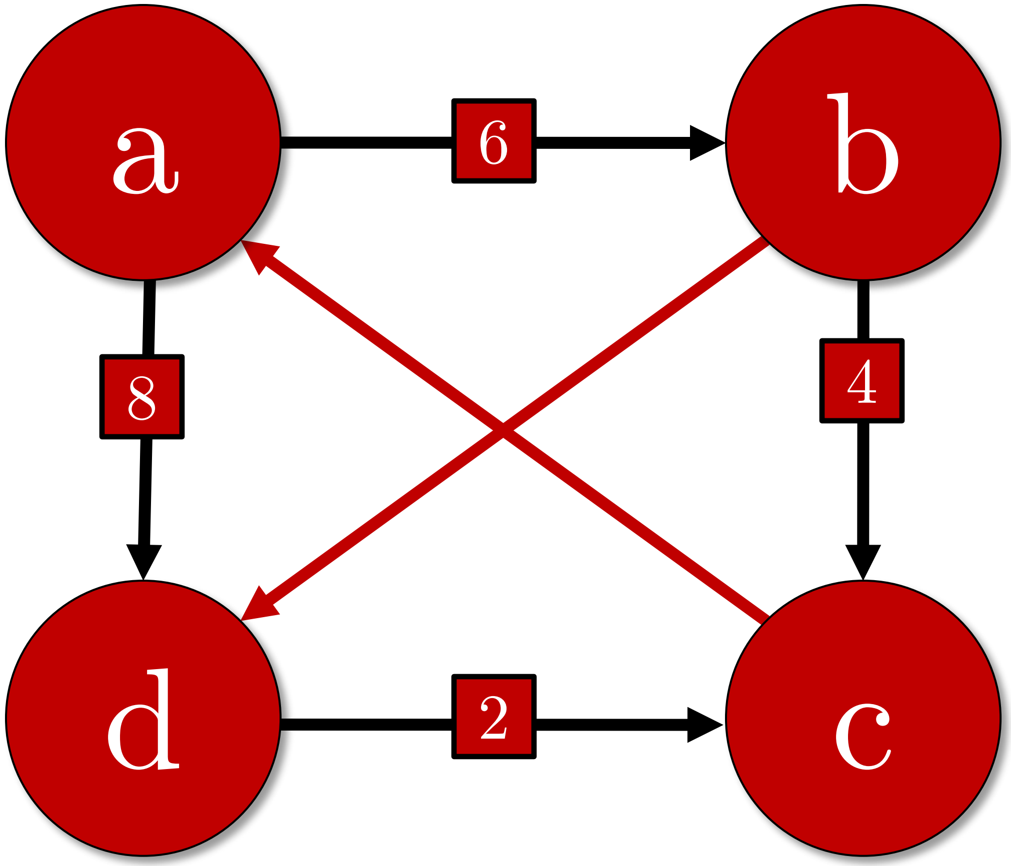
Example: ranked pairs



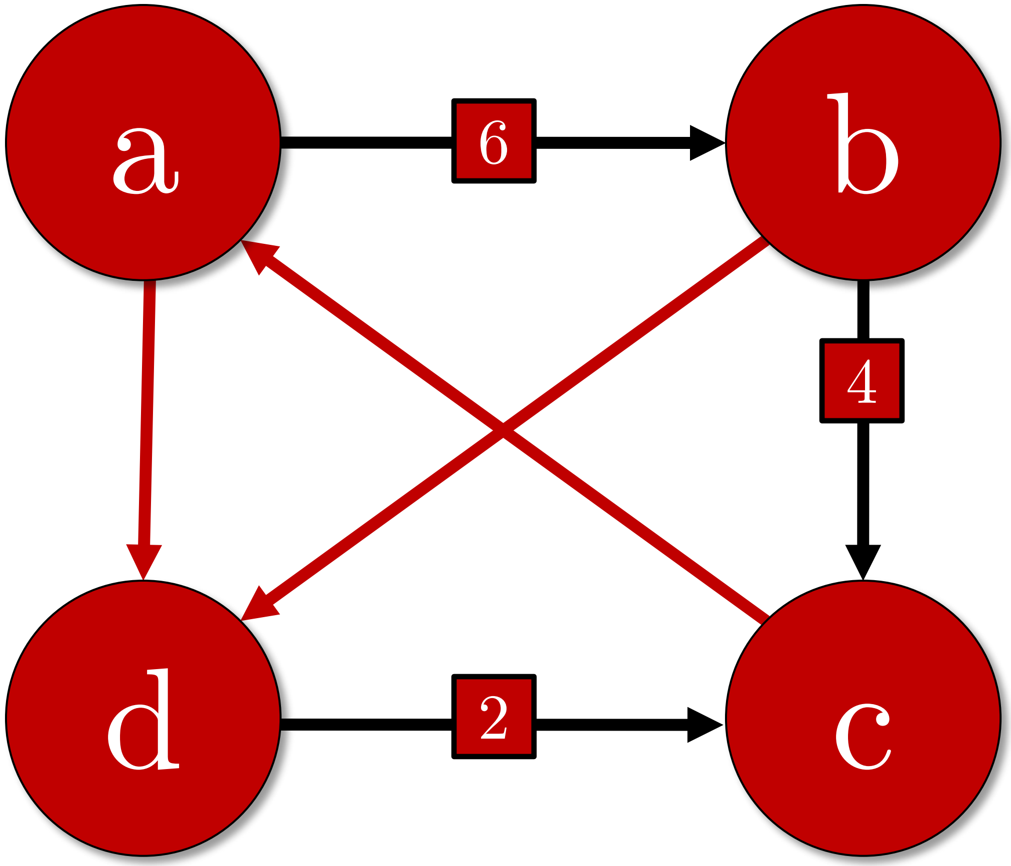
Example: ranked pairs



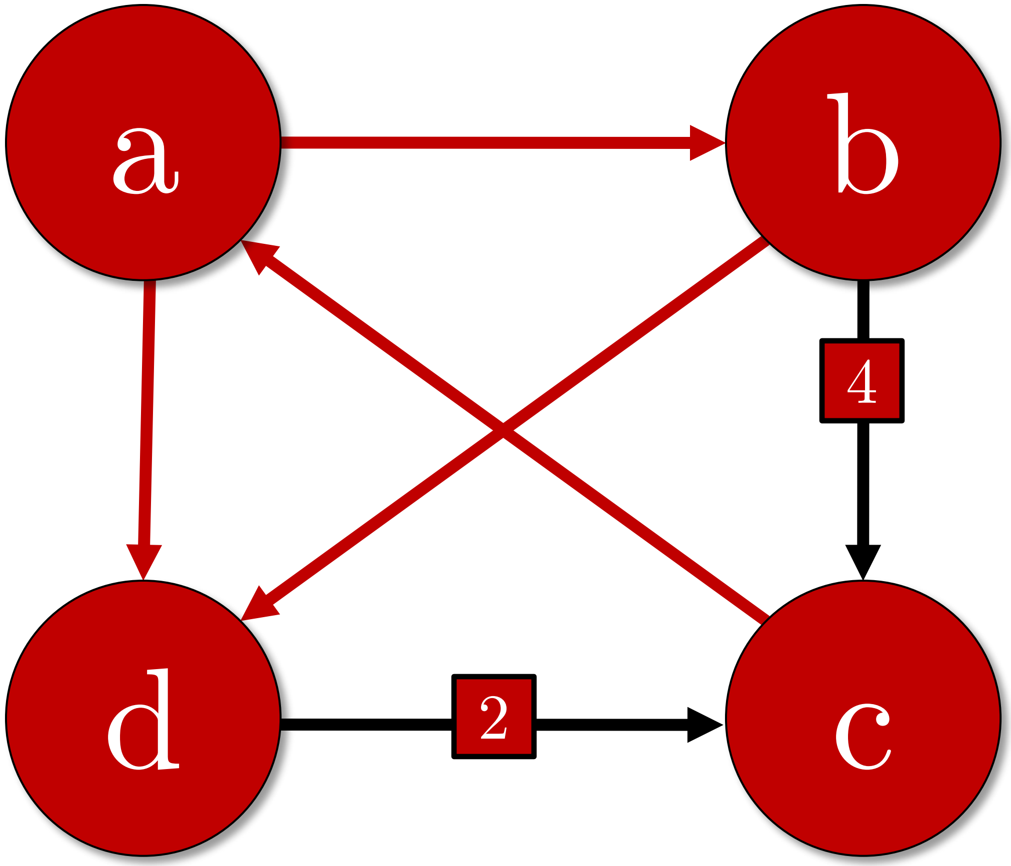
Example: ranked pairs



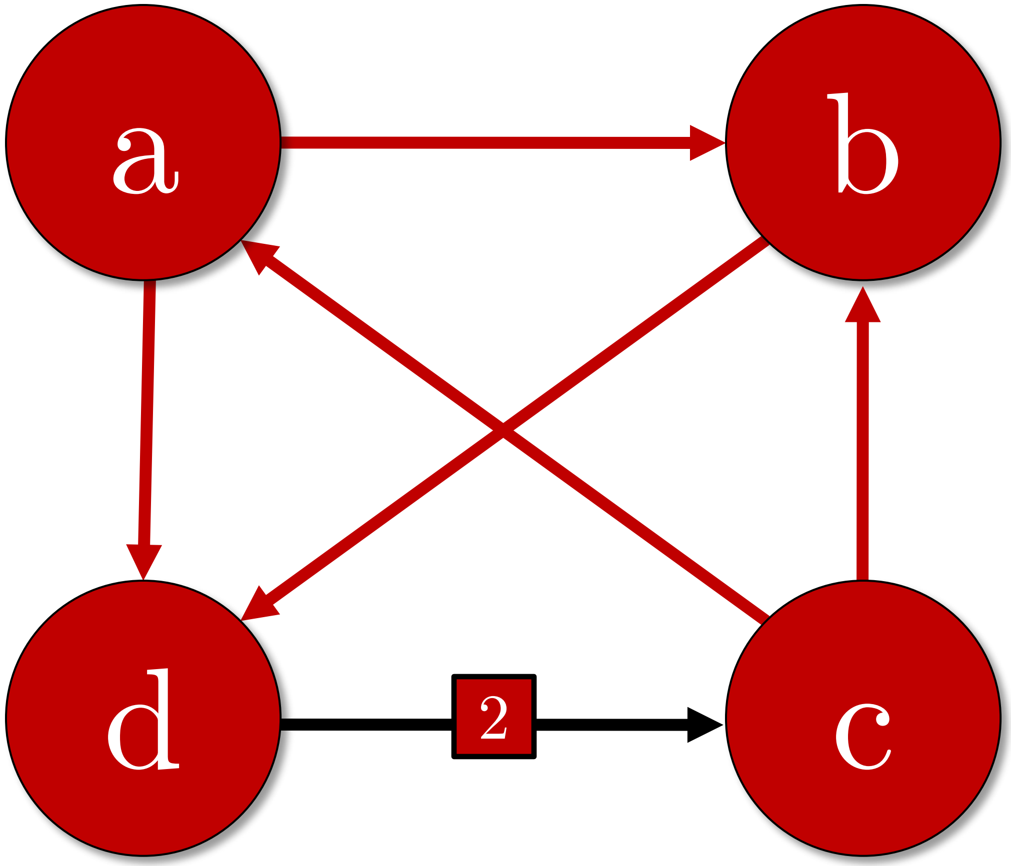
Example: ranked pairs



Example: ranked pairs



Example: ranked pairs



Example: ranked pairs