CMU 15-896

Social choice:
Voting rules + axioms

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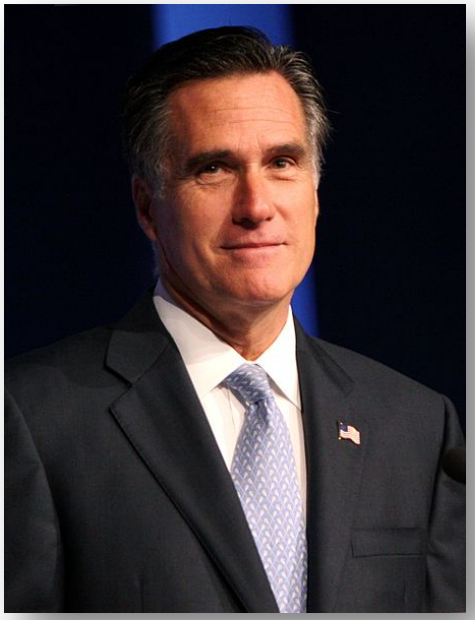
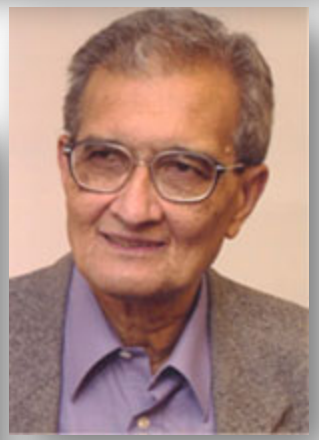
Social choice: example

vs.



Social choice theory

- A mathematical theory that deals with aggregation of individual preferences
- Origins in ancient Greece
- Formal foundations: 18th Century (Condorcet and Borda)
- 19th Century: Charles Dodgson
- 20th Century: Nobel prizes to Arrow and Sen



The voting model

- Set of voters $N = \{1, \ldots, n\}$
- Set of alternatives $A, |A| = m$
- Each voter has a ranking over the alternatives
- $x \succ_i y$ means that voter i prefers x to y
- Preference profile = collection of all voters' rankings

1	2	3
a	c	b
b	a	c
c	b	a

Voting rules

- Voting rule = function from preference profiles to alternatives that specifies the winner of the election
- Plurality
  - Each voter awards one point to top alternative
  - Alternative with most points wins
  - Used in almost all political elections

More voting rules

- Borda count
  - Each voter awards $m - k$ points to alternative ranked k'th
  - Alternative with most points wins
  - Proposed in the 18th Century by the chevalier de Borda
  - Used for elections to the national assembly of Slovenia
  - Similar to rule used in the Eurovision song contest

Lordi, Eurovision 2006 winners

More voting rules

- Veto
  - Each voter vetoes his least preferred alternative
  - Alternative with least vetoes wins
- Positional scoring rules
  - Defined by a vector $(s_1, \ldots, s_m)$
  - Each voter gives s_k points to k'th position
  - Plurality: $(1,0, \ldots, 0)$;
    Borda: $(m-1, m-2, \ldots, 0)$;
    Veto: $(1, \ldots, 1,0)$

More voting rules

- x beats y in a pairwise election if the majority of voters prefer x to y
- Plurality with runoff
  - First round: two alternatives with highest plurality scores survive
  - Second round: pairwise election between these two alternatives

More voting rules

- Single Transferable vote (STV)
  - $m-1$ rounds
  - In each round, alternative with least plurality votes is eliminated
  - Alternative left standing is the winner
  - Used in Ireland, Malta, Australia, and New Zealand (and Cambridge, MA)

STV: EXAMPLE

2 voters	2 voters	1 voter
a	b	c
b	a	d
c	d	b
d	c	a

2 voters	2 voters	1 voter
a	b	c
b	a	b
c	c	a

2 voters	2 voters	1 voter
a	b	b
b	a	a

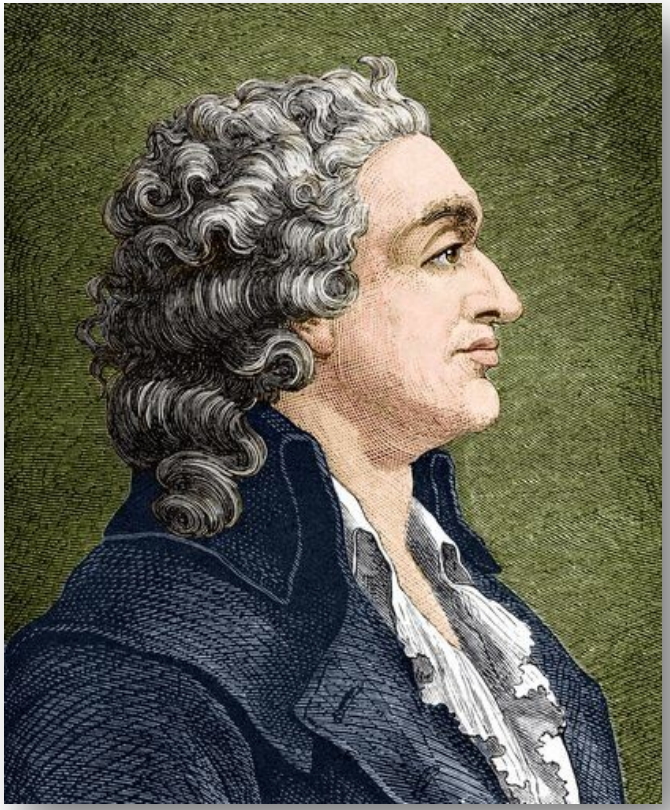
2 voters	2 voters	1 voter
b	b	b

Social choice axioms

- How do we choose among the different voting rules? Via desirable properties!
- Majority consistency = if a majority of voters rank alternative x first, then x should be the winner
- Vote: which voting rules are majority consistent?

Marquis de Condorcet

- 18th Century French Mathematician, philosopher, political scientist
- One of the leaders of the French revolution
- After the revolution became a fugitive
- His cover was blown and he died mysteriously in prison



Condorcet winner

- Recall: x beats y in a pairwise election if a majority of voters rank x above y
- Condorcet winner beats every other alternative in pairwise election
- Condorcet paradox = cycle in majority preferences

1	2	3
a	c	b
b	a	c
c	b	a

Condorcet consistency

- Condorcet consistency = select a Condorcet winner if one exists
- Vote: relation between majority consistency and Condorcet consistency
- Vote: Condorcet consistent rules

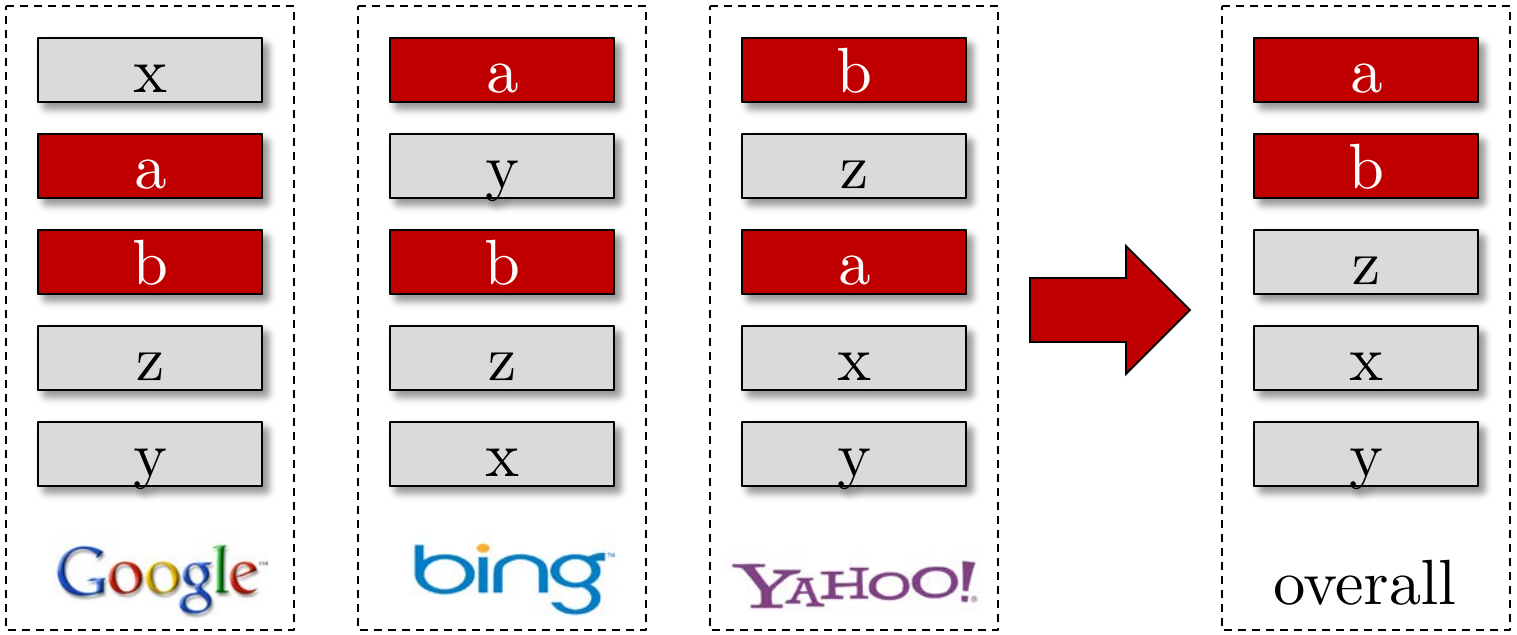
More voting rules

- Copeland
  - Alternative's score is #alternatives it beats in pairwise elections
  - Why does Copeland satisfy the Condorcet criterion?
- Maximin
  - Score of x is $\min_y |\{i \in N: x \succ_i y\}|$
  - Why does Maximin satisfy the Condorcet criterion?

Application: web search

- Generalized Condorcet: if there is a partition X, Y of A such that a majority prefers every $x \in X$ to every $y \in Y$, then X is ranked above Y
- Assumption: spam website identified by a majority of search engines
- When aggregating results from different search engines, spam websites will be ranked last [Dwork et al., WWW 2001]

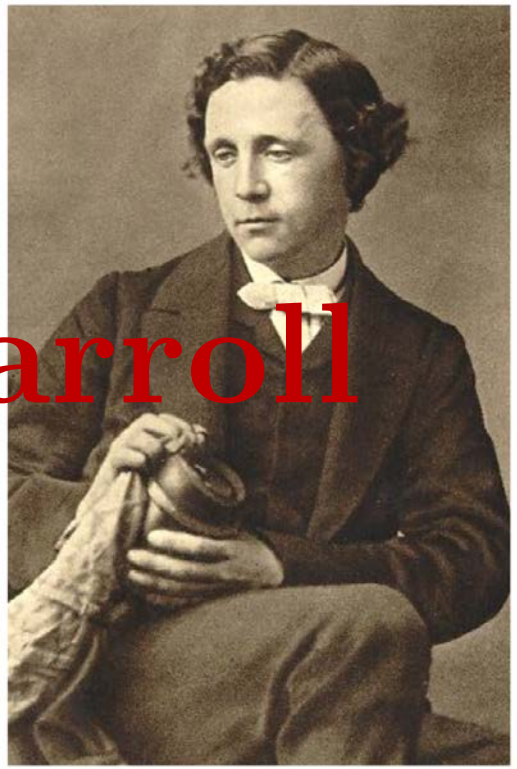
Application: Web Search



Metamorphosis

rles
dge
gson

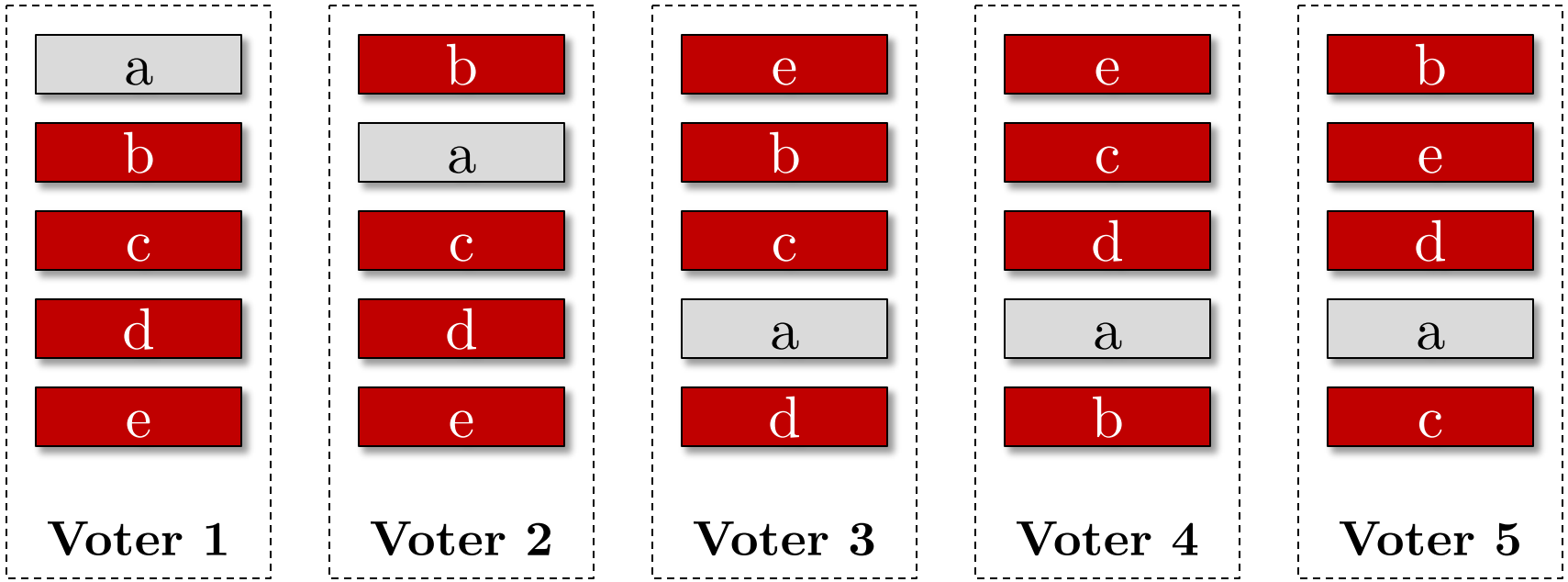
Lewis Carroll



Dodgson's Rule

- Distance function between profiles: #swaps between adjacent candidates
- Dodgson score of x = the min distance from a profile where x is a Condorcet winner
- Dodgson's rule: select candidate that minimizes Dodgson score
- The problem of computing the Dodgson score is NP-complete!

Dodgson Unleashed



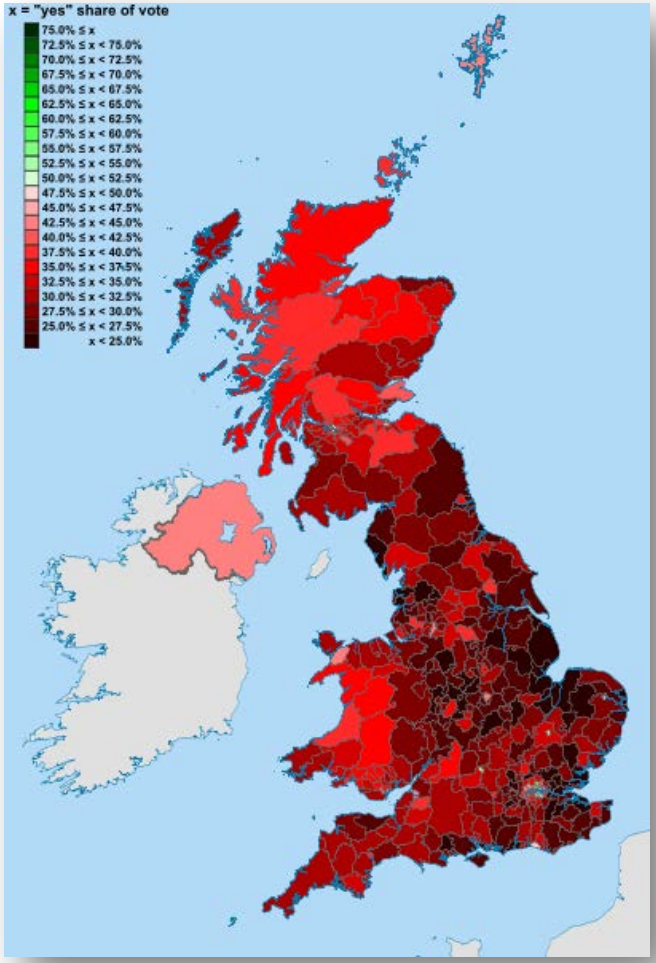
Awesome example

- Plurality: a
- Borda: b
- Condorcet winner: c
- STV: d
- Plurality with runoff: e

33 voters	16 voters	3 voters	8 voters	18 voters	22 voters
a	b	c	c	d	e
b	d	d	e	e	c
c	c	b	b	c	b
d	e	a	d	b	d
e	a	e	a	a	a

Is social choice practical?

- UK referendum: Choose between plurality and STV as a method for electing MPs
- Academics agreed STV is better...
- ... but STV seen as beneficial to the hated Nick Clegg
- Hard to change political elections!



Computational social choice

- However:
  - in human computation systems...
  - in multiagent systems...

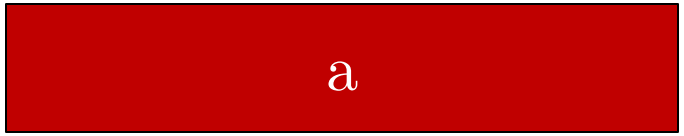
  the designer is free to employ any voting rule!
- Computational social choice focuses on positive results through computational thinking

Example: Robobees

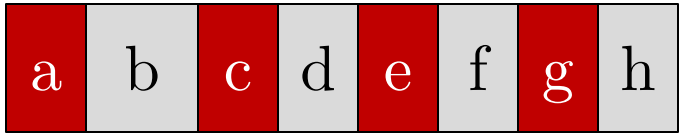
- Robobees need to decide on a joint plan (alternative)
- Many possible plans
- Each robobee (agent) has a numerical evluation (utility) for each alternative
- Want to maximize sum of utilities = *social welfare*
- Communication is restricted

Application: Robobees

- Approach 1: communicate utilities
  - May be infeasible
- Approach 2: each agent votes for favorite alternative (plurality)
  - log$m$ bits per agent
  - May select a bad alternative



n/2 – 1 agents



n/2 + 1 agents

Application: Robobees

- Approach 3: each agent votes for an alternative with probability proportional to its utility
- **Theorem (informal):**
  if $n = \omega(m \log m)$ then this approach gives almost optimal social welfare in expectation [Caragiannis & P, AIJ 2011]