CMU 15-896 Algorithms, Games, & Networks Spring 2013

01/22/13

Bandit algorithms, internal & swap regret, and correlated equilibria

Your guide:

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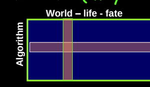
[Readings: Ch. 4.4-4.6 of AGT book]

Recap

"No-regret" algorithms for repeated decisions:

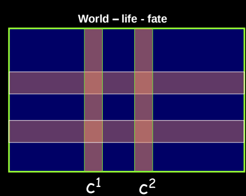
- Algorithm has N options. World chooses cost vector. Can view as matrix like this (maybe infinite # cols)



- At each time step, algorithm picks row, life picks column.
  - Alg pays cost (or gets benefit) for action chosen.
  - Alg gets column as feedback (or just its own cost/benefit in the "bandit" model).
  - Goal: do nearly as well as best fixed row in hindsight.

RWM

$(1-\varepsilon c_1^2)(1-\varepsilon c_1^1)1$
$(1-\varepsilon c_2^2)(1-\varepsilon c_2^1)1$
$(1-\varepsilon c_3^2)(1-\varepsilon c_3^1)1$
. . 1
. . 1
$(1-\varepsilon c_n^2)(1-\varepsilon c_n^1)1$



scaling so costs in [0,1]

Guarantee: $E[\text{cost}] \leq OPT + 2(OPT \cdot \log n)^{1/2}$

Since $OPT \leq T$, this is at most $OPT + 2(T \log n)^{1/2}$.

So, regret/time step $\leq 2(T \log n)^{1/2}/T \to 0$.

[ACFS02]: applying RWM to bandit setting

- What if only get your own cost/benefit as feedback?

- Use of RWM as subroutine to get algorithm with cumulative regret $O((TN \log N)^{1/2})$.
  [average regret $O(((N \log N)/T)^{1/2})$.]
- Will do a somewhat weaker version of their analysis (same algorithm but not as tight a bound).
- For fun, talk about it in the context of online pricing...

Online pricing

- Say you are selling lemonade (or a cool new software tool, or bottles of water at the world cup).
- For t=1,2,...T
  - Seller sets price p^t
  - Buyer arrives with valuation v^t
  - If $v^t \geq p^t$, buyer purchases and pays p^t, else doesn't.
  - Repeat.

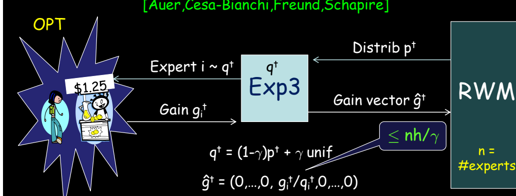
View each possible price as a different row/expert

- Assume all valuations $\leq h$.
- Goal: do nearly as well as best fixed price in hindsight.
- If v^t revealed, run RWM. $E[\text{gain}] \geq OPT(1-\epsilon) - O(\epsilon^{-1} h \log n)$.

Multi-armed bandit problem

Exponential Weights for Exploration and Exploitation (exp^3)

[Auer,Cesa-Bianchi,Freund,Schapire]

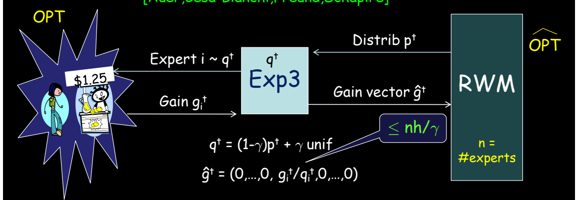


1. RWM believes gain is: $p^t \cdot \hat{g}^t = p_i^t(g_i^t/q_i^t) \equiv g^t_{RWM}$
2. $\sum_t g^t_{RWM} \geq \widehat{OPT}(1-\epsilon) - O(\epsilon^{-1}\, nh/\gamma \log n)$
3. Actual gain is: $g_i^t = g^t_{RWM}(q_i^t/p_i^t) \geq g^t_{RWM}(1-\gamma)$
4. $E[\widehat{OPT}] \geq OPT$. Because $E[\hat{g}_j^t] = (1-q_j^t)0 + q_j^t(g_j^t/q_j^t) = g_j^t$, so $E[\max_j[\sum_t \hat{g}_j^t]] \geq \max_j[E[\sum_t \hat{g}_j^t]] = OPT$.

Multi-armed bandit problem

Exponential Weights for Exploration and Exploitation (exp³)

[Auer,Cesa-Bianchi,Freund,Schapire]



Conclusion ($\gamma = \epsilon$):

$E[\text{Exp3}] \geq \text{OPT}(1-\epsilon)^2 - O(\epsilon^{-2}\, nh \log(n))$

Balancing would give $O((\text{OPT}\; nh \log n)^{2/3})$ in bound because of ϵ^{-2}. But can reduce to ϵ^{-1} and $O((\text{OPT}\; nh \log n)^{1/2})$ more care in analysis.

Summary

Algorithms for online decision-making with strong guarantees on performance compared to best fixed choice.

- Application: play repeated game against adversary. Perform nearly as well as fixed strategy in hindsight.

Can apply even with very limited feedback.

- Application: which way to drive to work, with only feedback about your own paths; online pricing, even if only have buy/no buy feedback.

Internal/Swap Regret and Correlated Equilibria

What if all players minimize regret?

- In zero-sum games, empirical frequencies quickly approaches minimax optimal.
- In general-sum games, does behavior quickly (or at all) approach a Nash equilibrium?
  - After all, a Nash Eq is exactly a set of distributions that are no-regret wrt each other. So *if they converge at all*, they must converge to a Nash equil.
- Well, unfortunately, no.

A bad example for general-sum games

- Augmented Shapley game from [Zinkevich04]:
  - First 3 rows/cols are Shapley game (rock / paper / scissors but if both do same action then both lose).
  - 4th action "play foosball" has slight negative if other player is still doing r/p/s but positive if other player does 4th action too.

  RWM will cycle among first 3 and have no regret, but do worse than only Nash Equilibrium of both playing foosball.

- We didn't really expect this to work given how hard NE can be to find…

A bad example for general-sum games

- [Balcan-Constantin-Mehta12]:
  - Failure to converge even in Rank-1 games (games where R+C has rank 1).
  - Interesting because one can find equilibria efficiently in such games.

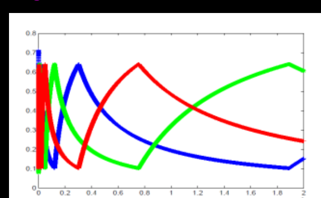


Figure 4. c_is of symmetric Shapley game with $a = 10, b = 1$

What *can* we say?

If algorithms minimize "internal" or "swap" regret, then empirical distribution of play approaches *correlated* equilibrium.

- Foster & Vohra, Hart & Mas-Colell,...
- Though doesn't imply play is stabilizing.

What are internal/swap regret and correlated equilibria?

More general forms of regret

1. "best expert" or "external" regret:
   - Given n strategies. Compete with best of them in hindsight.
2. "sleeping expert" or "regret with time-intervals":
   - Given n strategies, k properties. Let S_i be set of days satisfying property i (might overlap). Want to simultaneously achieve low regret over each S_i.
3. "internal" or "swap" regret: like (2), except that S_i = set of days in which we chose strategy i.

Internal/swap-regret

- E.g., each day we pick one stock to buy shares in.
  - Don't want to have regret of the form "every time I bought IBM, I should have bought Microsoft instead".
- Formally, swap regret is wrt optimal function $f:\{1,\ldots,n\}\to\{1,\ldots,n\}$ such that every time you played action j, it plays f(j).

Weird... why care?

"Correlated equilibrium"

- Distribution over entries in matrix, such that if a trusted party chooses one at random and tells you your part, you have no incentive to deviate.
- E.g., Shapley game.

	R	P	S
R	-1,-1	-1,1	1,-1
P	1,-1	-1,-1	-1,1
S	-1,1	1,-1	-1,-1

In general-sum games, if all players have low swap-regret, then empirical distribution of play is apx correlated equilibrium.

Connection

- If all parties run a low swap regret algorithm, then empirical distribution of play is an apx correlated equilibrium.
  - Correlator chooses random time $t \in \{1,2,\ldots,T\}$. Tells each player to play the action j they played in time t (but does not reveal value of t).
  - Expected incentive to deviate: $\sum_j \Pr(j)(\text{Regret}|j)$ = swap-regret of algorithm
  - So, this suggests correlated equilibria may be natural things to see in multi-agent systems where individuals are optimizing for themselves

Correlated vs Coarse-correlated Eq

In both cases: a distribution over entries in the matrix. Think of a third party choosing from this distr and telling you your part as "advice".

"Correlated equilibrium"

- You have no incentive to deviate, even after seeing what the advice is.

"Coarse-Correlated equilibrium"

- If only choice is to see and follow, or not to see at all, would prefer the former.

Low external-regret $\Rightarrow$ apx coarse correlated equilib.

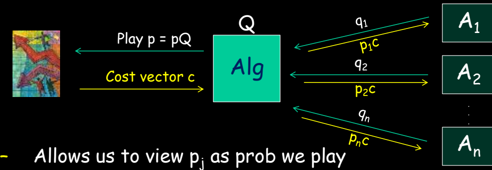
Internal/swap-regret, contd

Algorithms for achieving low regret of this form:

- Foster & Vohra, Hart & Mas-Colell, Fudenberg & Levine.
- Will present method of [BM05] showing how to convert any "best expert" algorithm into one achieving low swap regret.
- Unfortunately, #steps to achieve low swap regret is O(n log n) rather than O(log n).

Can convert any "best expert" algorithm A into one achieving low swap regret. Idea:

- Instantiate one copy A_j responsible for expected regret over times we play j.



- Allows us to view p_j as prob we play action j, or as prob we play alg A_j.
- Give A_j feedback of p_jc.
- A_j guarantees $\sum_t (p_j^tc^t)\cdot q_j^t \leq \min_i \sum_t p_j^tc_i^t$ + [regret term]
- Write as: $\sum_t p_j^t(q_j^t\cdot c^t) \leq \min_i \sum_t p_j^tc_i^t$ + [regret term]

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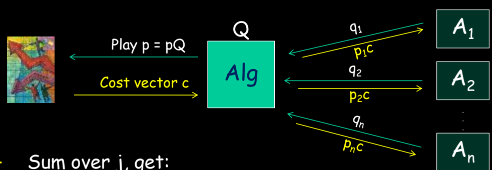

- Sum over j, get:

$$\sum_t p^tQ^tc^t \leq \sum_j \min_i \sum_t p_j^tc_i^t + n\text{[regret term]}$$

Our total cost | For each j, can move our prob to its own i=f(j)

- Write as: $\sum_t p_j^t(q_j^t\cdot c^t) \leq \min_i \sum_t p_j^tc_i^t$ + [regret term]

More on Correlated Equilib

Can solve for them using linear programming.

p_{11}	p_{12}	p_{13}
p_{21}	p_{22}	p_{23}
p_{31}	p_{32}	p_{33}

- Variables are p_{ij}.
- Constraints for each row i.
  - For all i', $\sum_j (p_{ij}/p_i) R_{ij} \geq \sum_j (p_{ij}/p_i) R_{i'j}$
  - Make linear by multiplying LHS,RHS by p_i.
- Constraints for each column j.
  - Similarly for column player.
- This is for 2-player games. In m-player games it's trickier but can use Ellipsoid alg.
- Or, just run a swap-regret-minimizing alg for each player to get an ϵ-CE.