



**CMU 15-896**

**SOCIAL NETWORKS:  
INFLUENCE MAXIMIZATION**

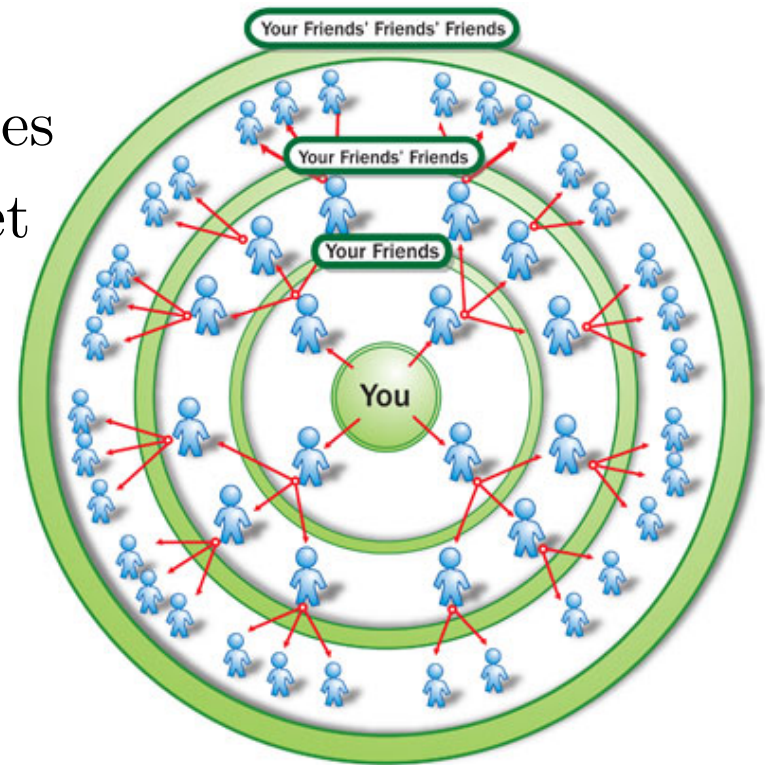
**TEACHERS:**

**AVRIM BLUM**

**ARIEL PROCACCIA (THIS TIME)**

# SIX DEGREES OF SEPARATION

- Stanley Milgram's famous experiments (1960's)
- Find short chains of acquaintances
- Source person in Nebraska, target in MA
- Deliver letter by passing to a friend
- Anyone receiving the letter gets same instructions
- Avg number of hops between 5 and 6



# QUESTIONS

- Why should there exist short chains linking arbitrary pairs of strangers?
  - This question has been well understood for a long time
- Why should arbitrary pairs of strangers be able to **find** short chains that link them?
  - Short chains may exist, but no “decentralized” alg may find them
  - This is the topic of [Kleinberg, 2000]



# MODEL

- Nodes reside on an  $n \times n$  lattice

$$V = \{(i, j) | i, j \in \{1, \dots, n\}\}$$

- Each node has a directed edge to each of its 4 neighbors

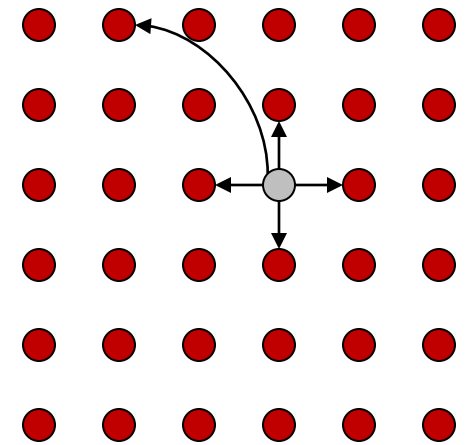
- The **Manhattan distance** is

$$d((i, j), (k, l)) = |i - k| + |j - l|$$

- Each node  $u$  has an additional long-distance edge  $v$  with probability

$$\frac{d(u, v)^{-r}}{\sum_{v'} d(u, v')^{-r}}, \text{ where } r \geq 0 \text{ is a parameter}$$

- The source  $s$  and target  $t$  are chosen u.a.r.



# DECENTRALIZED ALGS

- A local decentralized alg knows only:
  1. The underlying grid structure
  2. The lattice coordinates of  $t$  and nodes that held the message
  3. The coordinates of long-range contacts of nodes that held the message
- 3 may seem restrictive, but it just strengthens negative result, positive result doesn't use it



# A NEGATIVE RESULT

- Consider the case of  $r = 0$ : the long-distance neighbor is chosen uniformly at random
- **Fact:**  $\mathbb{E}[d(s, t)] = O(\log n)$
- **Theorem [Kleinberg, 2000]:** When  $r = 0$ , the expected number of hops under any local decentralized alg is  $\Omega(n^{2/3})$
- We prove the theorem on the board



# A POSITIVE RESULT

- Greedy alg: forward the message to the neighbor that minimizes distance from target
- The alg doesn't use information about long-distance neighbors of previous nodes
- **Theorem [Kleinberg, 2000]:** When  $r = 2$  the greedy alg requires  $O(\log^2 n)$  hops in expectation
- We prove the theorem on the board



# MORE GENERALLY

- $r = 2$  is the only value for which there is a local decentralized alg that requires polylogarithmic #hops
- **Theorem [Kleinberg, 2000]:**
  1. For  $0 \leq r < 2$ , expected #hops is  $\Omega(n^{(2-r)/3})$
  2. For  $r > 2$ , expected #hops is  $\Omega(n^{(r-2)/(r-1)})$

