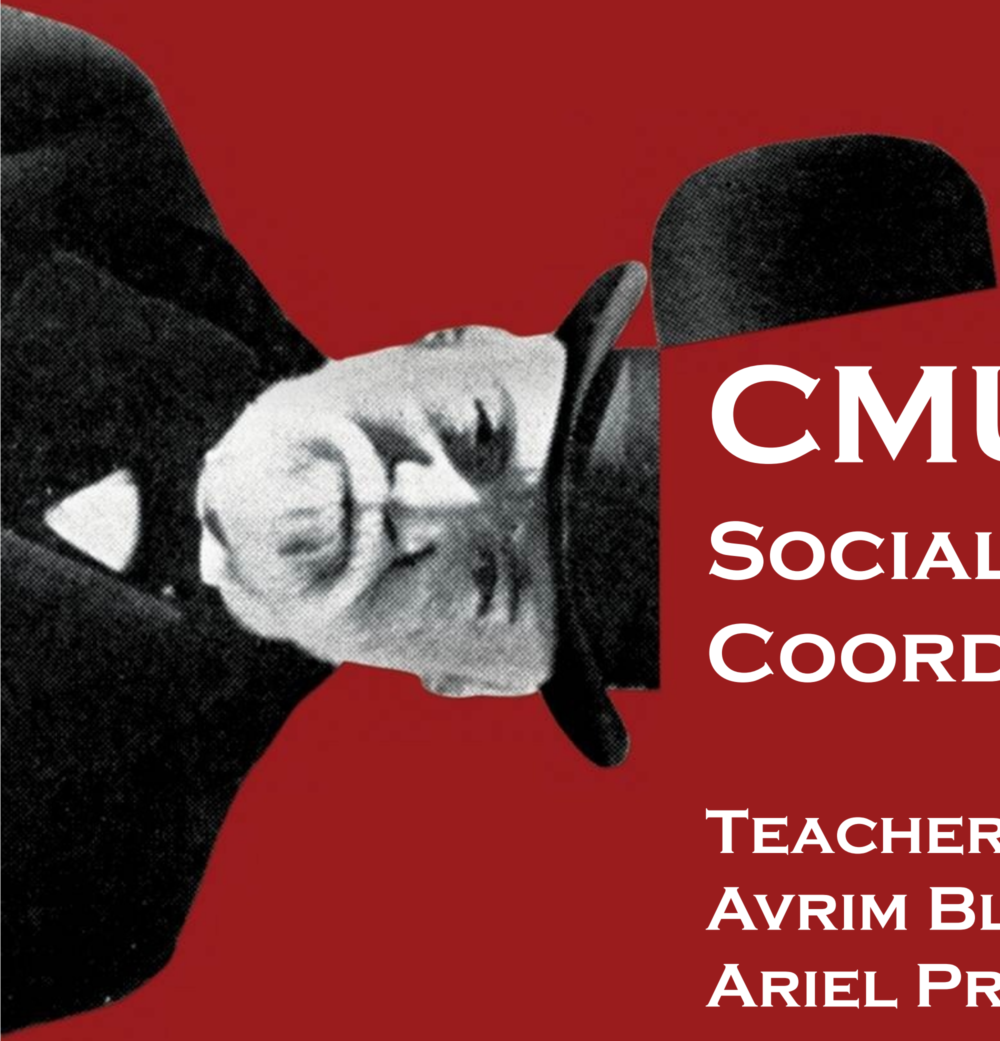

CMU 15-896

Social networks: Coordination games

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Background

- We follow chapter 24 in the AGT book
- Spread of ideas and new behaviors through a population
- Examples:
  - Religious beliefs and political movements
  - Adoption of technological innovations
  - Success of new product
- Process starts with early adopters and spreads through the social network

Networked coordination games

- Simple model for the diffusion of ideas and innovations
- Social network is undirected graph $G = (V, E)$
- Choice between old behavior A and new behavior B
- Parametrized by $q \in (0,1)$

Networked coordination games

- Rewards for u and v when $(u, v) \in E$:
  - If both choose A, they receive q
  - If both choose B, they receive $1 - q$
  - Otherwise both receive 0
- Overall payoff to v = sum of payoffs
- Denote d_v = degree of v, d_v^X = #neighbors playing X
- Payoff to v from choosing A is $q d_v^A$; reward from choosing B is $(1 - q) d_v^B$
- v adopts B if $d_v^B \geq q d_v \Rightarrow q$ is a threshold

Cascading behavior

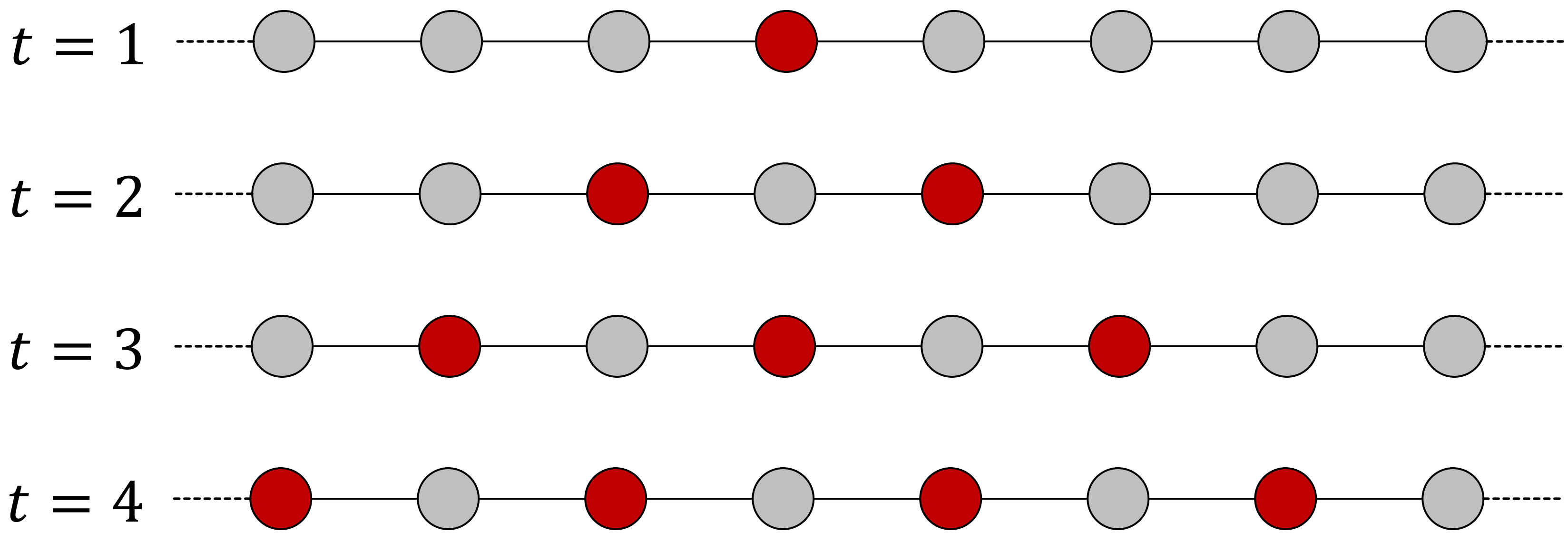
- Each node simultaneously updates its behavior in discrete time steps $t = 1, 2, \ldots$
- Nodes in S initially adopt B
- $h_q(S)$ = set of nodes adopting B after one round
- $h_q^k(S)$ = after k rounds of updates
- Question: When does a small set of nodes convert the entire population?

Contagion threshold

- V is countably infinite and each d_v is finite
- v is converted by S if $\exists k$ s.t. $v \in h_q^k(S)$
- S is contagious if every node is converted
- It is easier to be contagious when q is small
- Contagion threshold of $G = \max q$ s.t. $\exists$finite contagious set

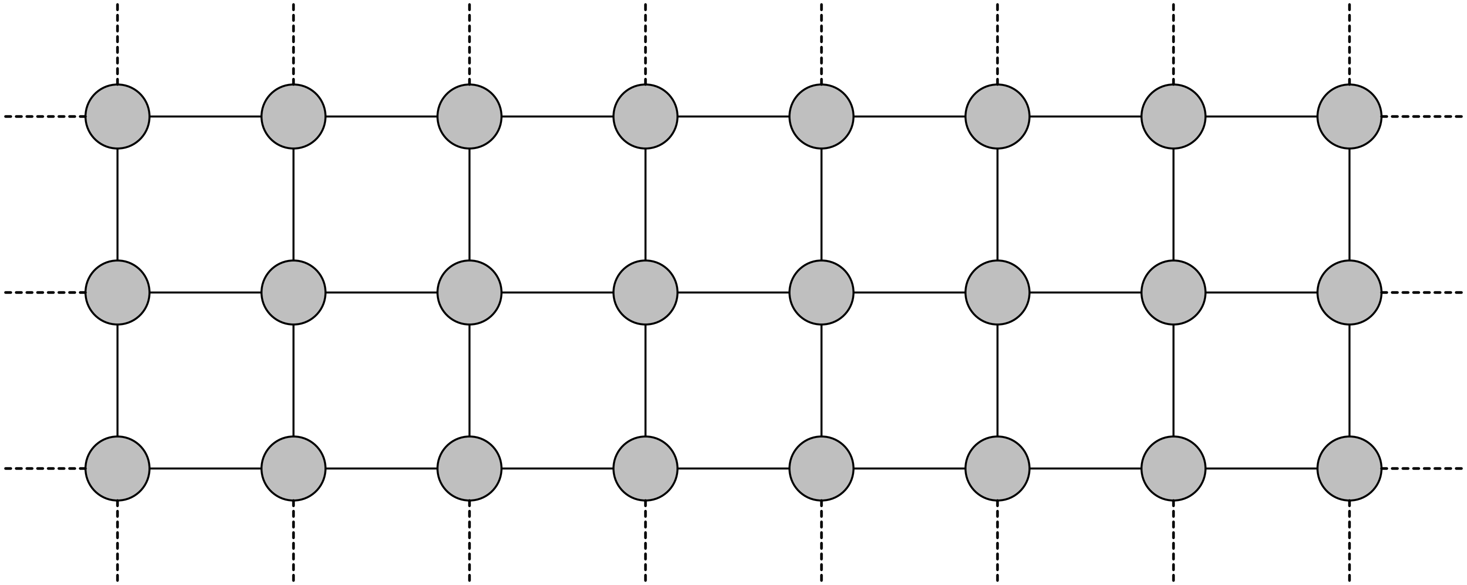
Example

$$q = \frac{1}{2}$$



What is the contagion threshold of G?

Example



Vote: What is the contagion threshold of G?

Progressive processes

- Nonprogressive process: Nodes can switch from A to B or B to A
- Progressive process: Nodes can only switch from A to B
- As before, a node switches to B if a q fraction of its neighbors follow B
- $\bar{h}_q(S) =$ set of nodes adopting B in progressive process; define $\bar{h}_q^k(S)$ as before

Progressive processes

- With progressive processes intuitively the contagion threshold should be lower
- **Theorem [Morris, 2000]:** For any graph G, $\exists$finite contagious set wrt $h_q \Leftrightarrow \exists$finite contagious set wrt $\bar{h}_q$
- I.e., the contagion threshold is identical under both models
- We prove the theorem on the board

Contagion threshold $\leq 1/2$

- Saw a graph with contagion threshold $1/2$
- Does there exist a graph with contagion threshold $> 1/2$?
- The previous theorem allows us to focus on the progressive case
- **Theorem [Morris, 2000]:** For any graph G, the contagion threshold $\leq 1/2$
- We prove the theorem on the board

More general models

- Directed graphs to model asymmetric influence
- $N(v) = \{u \in V: (u, v) \in E\}$
- Assume progressive contagion
- Node is active if it adopts B; activated if switches from A to B

Linear threshold model

- Nonnegative weight w_{uv} for each edge $(u, v) \in E$; $w_{uv} = 0$ otherwise
- Assume $\forall v \in V$, $\sum_u w_{uv} \le 1$
- Each $v \in V$ has threshold θ_v
- v becomes active if

$$\sum_{\text{active } u} w_{uv} \ge \theta_v$$

General threshold model

- Linear model assumes additive influences
  - Switch if two co-workers and three family members switch?
- v has a monotonic function $g_v(\cdot)$ defined on subsets $X \subseteq N(v)$
- v becomes activated if the activated subset $X \subseteq N(v)$ satisfies $g_v(X) \geq \theta_v$

The cascade model

- When $\exists(u,v) \in E$ s.t. u is active and v is not, u has one chance to activate v
- v has an incremental function $p_v(u,X) =$ probability that u activates v when X have tried and failed
- Special cases:
  - Diminishing returns: $p_v(u,X) \geq p_v(u,Y)$ when $X \subseteq Y$
  - Independent cascade: $p_v(u,X) = p_{uv}$