# 15780: GRADUATE AI (SPRING 2018)

# Practice Midterm 2

March 1, 2018

| Topic | Total Score | Score |
|---|---|---|
| Heuristic Search | 25 | |
| VC Dimension | 25 | |
| Integer Programming | 25 | |
| Convex Optimization | 25 | |
| Total | 100 | |

# 1 Heuristic Search [25 points]

Consider the problem of informed search with a heuristic. For each state $x$, let $h^*(x)$ be the length of the cheapest path from $x$ to a goal.

Prove or disprove the following statements:

1.1 [15 points] If $h(x) = 2h^*(x)$ for all states $x$, then $A^*$ tree search with the heuristic $h$ is optimal.

1.2 [10 points] If $h$ is a consistent heuristic, $A^*$ graph search with the heuristic $h'(x) = h(x)/2$ is optimal.

# 2 Learning Theory [25 points]

Determine the VC dimension of the following function classes.

2.1 [15 points] Define $F$ to be the set of strings of length 3 composed of the symbols 0, 1, and $*$. Each $f \in F$ acts as a pattern matcher; i.e., when applied to a binary string $s$, it either accepts or rejects $s$. For example, when we apply the schema $f = 1 * *$ to the string $s = 101$, it accepts, and when we apply $f$ to $s' = 010$, it rejects. What is the VC dimension of $F$?

2.2 [10 points] The union of $n$ intervals on the real line.

# 3 Integer Programming [25 points]

Consider an undirected graph $G = (V, E)$. A ***minimum dominating set*** is a smallest subset $S$ of $V$ such that every node not in $S$ is adjacent to at least one node in $S$. A ***minimum independent dominating set*** is a smallest subset $S$ of $V$ such that (1) every node not in $S$ is adjacent to at least one node in $S$ and (2) no pair of nodes in $S$ are adjacent. In your answer, you can use $N(i)$ to denote the set of neighbors of node $i$ (i.e., $N(i)$ is a set of nodes adjacent to $i$) for each node $i \in V$. Note that $i \notin N(i)$. You also can use $(i, j) \in E$ to denote the edge between node $i \in V$ and node $j \in V$.

3.1 [15 points] Formulate an integer linear program to find a minimum dominating set.

3.2 [10 points] Formulate an integer linear program to find a minimum independent dominating set.

# 4 Convex Optimization [25 points]

Consider a linear program of the standard form: minimize $\mathbf{c}^T\mathbf{x}$ such that $\mathbf{A}\mathbf{x} \leq \mathbf{b}$. Here $\mathbf{x} \in \mathbb{R}^n$ is the vector of variables, and $\mathbf{c} \in \mathbb{R}^n$, $\mathbf{A} \in \mathbb{R}^{m \times n}$, and $\mathbf{b} \in \mathbb{R}^m$ are constants.

Prove from the definitions that this is a convex program.