# Reinforcement Learning

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*(see Tom Mitchell's "Machine Learning" book)*

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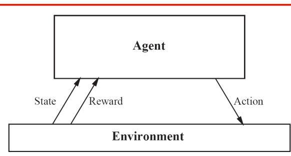
## Learning Conditions

- Assume world can be modeled as a Markov Decision Process, with rewards as a function of state and action.
- *Markov assumption:*
  New states and rewards are a function only of the current state and action, i.e.,
  - $s_{t+1} = \delta(s_t, a_t)$
  - $r_t = r(s_t, a_t)$
- *Unknown and uncertain environment:*
  Functions $\delta$ and $r$ may be nondeterministic and are not necessarily known to learner.

## Reinforcement Learning Problem




$$s_0 \xrightarrow[r_0]{a_0} s_1 \xrightarrow[r_1]{a_1} s_2 \xrightarrow[r_2]{a_2} \ldots$$

Goal: Learn to choose actions that maximize

$r_0 + \gamma r_1 + \gamma^2 r_2 + \ldots$ , where $0 \leq \gamma < 1$

## Control Learning Task

- Execute actions in world,
- Observe state of world,
- Learn action policy $\pi : S \rightarrow A$
- Maximize expected reward

$$E[r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \ldots]$$

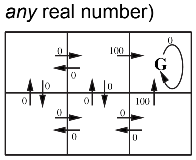
*from any starting state in* $S$.

– $0 \leq \gamma < 1$, discount factor for future rewards

## Statement of Learning Problem

- We have a target function to learn $\pi : S \rightarrow A$
- We have **no** training examples of the form $\langle s, a \rangle$
- We have training examples of the form $\langle\langle s, a\rangle, r\rangle$

(rewards can be *any* real number)




immediate reward values $r(s,a)$

## Policies

Assume deterministic world

- There are *many possible* policies, of course not necessarily *optimal*, i.e., with maximum expected reward
- There can be also *several OPTIMAL* policies.

## Value Function

- For each possible policy $\pi$, define an *evaluation function over states*

$$V^{\pi}(s) \equiv r_t + \gamma r_{t+1} + \gamma^2 r_{t+1} + \ldots$$
$$\equiv \sum_{i=0}^{\infty} \gamma^i r_{t+i}$$

where $r_t, r_{t+1}, \ldots$ are generated by following policy $\pi$ starting at state $s$

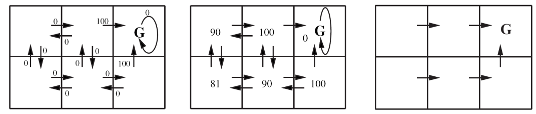
- Learning task: Learn OPTIMAL policy

$$\pi^* \equiv \text{argmax}_{\pi} V^{\pi}(s), (\forall s)$$

## Learn Value Function

- Learn the evaluation function $V^{\pi^*}$, i.e., $V^*$.
- Select the optimal action from any state $s$, i.e., have an optimal policy, by using $V^*$ with one step lookahead:

$$\pi^*(s) = \arg\max_a \left[ r(s,a) + \gamma V^*(\delta(s,a)) \right]$$




rewards $V^*(s)$ values **ONE** optimal policy

## Optimal Value to Optimal Policy

$$\pi^*(s) = \text{argmax}_a [r(s,a) + \gamma V^*(\delta(s,a))]$$

A problem:

- This works well if agent knows $\delta : S \times A \rightarrow S$, and $r : S \times A \rightarrow \Re$
- When it doesn't, it can't choose actions this way

## $Q$ Function

- Define new function very similar to $V^*$

$$Q(s,a) \equiv r(s,a) + \gamma V^*(\delta(s,a))$$

Learn $Q$ function – $Q$-learning

- If agent learns $Q$, it can choose optimal action even without knowing $\delta$ or $r$.

$$\pi^*(s) = \arg\max_a \left[ r(s,a) + \gamma V^*(\delta(s,a)) \right]$$

$$\pi^*(s) = \arg\max_a Q(s,a)$$

## $Q$-Learning

Note that $Q$ and $V^*$ are closely related:

$$V^*(s) = \max_{a'} Q(s,a')$$

Which allows us to write $Q$ recursively as

$$Q(s_t,a_t) = r(s_t,a_t) + \gamma V^*(\delta(s_t,a_t))$$
$$= r(s_t,a_t) + \gamma \max_{a'} Q(s_{t+1},a')$$

$Q$-learning actively generates examples.
It "processes" examples by updating its $Q$ values.
*While* learning, $Q$ values are approximations.

## Training Rule to Learn $Q$

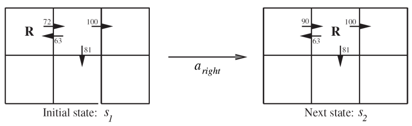
Let $\hat{Q}$ denote current approximation to $Q$.

Then Q-learning uses the following **training rule**:

$$\hat{Q}(s,a) \leftarrow r + \gamma \max_{a'} \hat{Q}(s',a')$$

where $s'$ is the state resulting from executing action $a$ in state $s$, and $r$ is the reward that is returned.

## Example - Updating $\hat{Q}$




$$\hat{Q}(s_1, a_{right}) \leftarrow r + \gamma \max_{a'} \hat{Q}(s_2, a')$$
$$\leftarrow 0 + 0.9 \max\{63, 81, 100\}$$
$$\leftarrow 90$$

## $Q$ Learning for Deterministic Worlds

For each $s$, $a$ initialize table entry $\hat{Q}(s,a) \leftarrow 0$

Observe current state $s$

Do forever:

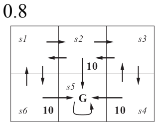
- Select an action $a$ and execute it
- Receive immediate reward $r$
- Observe the new state $s'$
- Update the table entry for $\hat{Q}(s,a)$ as follows:

$$\hat{Q}(s,a) \leftarrow r + \gamma \max_{a'} \hat{Q}(s',a')$$

- $s \leftarrow s'$

## *Example* - $Q$ Learning Iterations

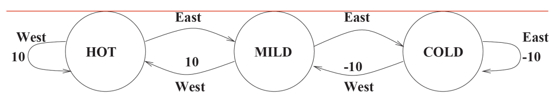
Starts at top left corner – moves clockwise around perimeter;
Initially $Q(s,a) = 0$; $\gamma = 0.8$




$$\hat{Q}(s,a) \leftarrow r + \gamma \max_{a'} \hat{Q}(s',a')$$

| Q(s1,E) | Q(s2,E) | Q(s3,S) | Q(s4,W) |
|---|---|---|---|
| 0 | 0 | 0 | r + $\gamma$ max{Q(s5,loop)}= 10 + 0.8 . 0 = 10 |
| 0 | 0 | r + $\gamma$ max{Q(s4,W),Q(s4,N)}= 0 + 0.8 max{10,0}= 8 | 10 |
| 0 | r + $\gamma$ max{Q(s3,W),Q(s3,S)}= 0 + 0.8 max{0,8}= 6.4 | 8 | 10 |

## Problem - Deterministic

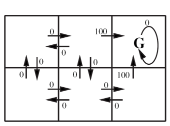



How many possible **policies** are there in this 3-state, 2-action deterministic world?

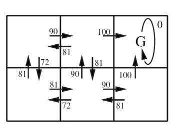
A robot starts in the state Mild. It moves for 4 steps choosing actions **West, East, East, West**. The initial values of its Q-table are 0 and the discount factor is $\gamma = 0.5$

| | Initial State: MILD | | Action: West New State: HOT | | Action: East New State: MILD | | Action: East New State: COLD | | Action: West New State: MILD | |
|---|---|---|---|---|---|---|---|---|---|---|
| | East | West | East | West | East | West | East | West | East | West |
| HOT | 0 | 0 | 0 | 0 | **5** | 0 | 5 | 0 | 5 | 0 |
| MILD | 0 | 0 | 0 | **10** | 0 | 10 | **0** | 10 | 0 | 10 |
| COLD | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | **-5** |

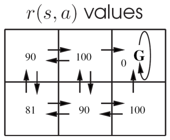
## Another Deterministic Example



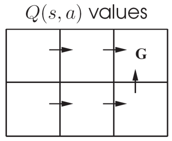

$r(s,a)$ values




$Q(s,a)$ values




$V^*(s)$ values




One optimal policy

## Nondeterministic Case

What if reward and next state are non-deterministic?

We redefine $V$, $Q$ by taking expected values

$$V^{\pi}(s) \equiv E[r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \ldots]$$
$$\equiv E\left[\sum_{i=0}^{\infty} \gamma^i r_{t+i}\right]$$

$$Q(s,a) \equiv E[r(s,a) + \gamma V^*(\delta(s,a))]$$

## Nondeterministic Case

$Q$ learning generalizes to nondeterministic worlds

Alter training rule to

$$\hat{Q}_n(s,a) \leftarrow (1-\alpha_n)\hat{Q}_{n-1}(s,a) + \alpha_n\left[r + \gamma \max_{a'} \hat{Q}_{n-1}(s',a')\right],$$

where $\alpha_n = \frac{1}{1+visits_n(s,a)}$, and $s' = \delta(s,a)$.

$\hat{Q}$ still converges to $Q^*$ (Watkins and Dayan, 1992)

## Discussion

- How should the learning agent use the *intermediate* $Q$ values?
  - Exploration
  - Exploitation
- Scaling up in the size of the state space
  - Function approximator (neural net instead of table)
  - Generalization
  - Reuse, use of macros
  - Abstraction, learning substructure

## Summary

- Markov Models with Reward
- Value iteration
- Markov Decision Process
- Value Iteration
- Policy Iteration
- Reinforcement Learning
- POMDPs – Chapter 17, Russell and Norvig