GRADUATE AI

LECTURE 23: GAME THEORY II

APRIL 16, 2012

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A CURIOUS GAME

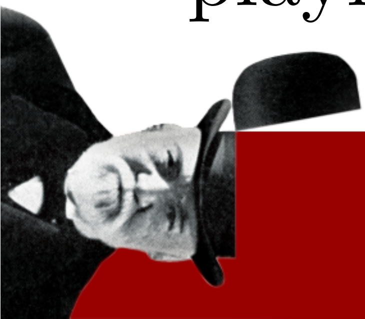
1,1	3,0
0,0	2,1

Iterated elimination ⇒ Unique NE at (up,left)

COMMITMENT IS GOOD

- Suppose the game is played as follows:
  - Row player commits to playing a row
  - Column player observes the commitment and chooses column
- Row player can commit to playing down!

1,1	3,0
0,0	2,1



COMMITMENT TO MIXED STRATEGY

- By committing to a mixed strategy, row player can guarantee a reward of 2.5
- Called a *Stackelberg* (mixed) strategy

	0	1
.49	1,1	3,0
.51	0,0	2,1

Computing Stackelberg

- **Theorem** [Conitzer and Sandholm, EC 2006]**:** In 2-player normal form games, an optimal Stackelberg strategy can be found in poly time
- **Theorem** [ditto]: the problem is NP-hard when the number of players is ≥ 3

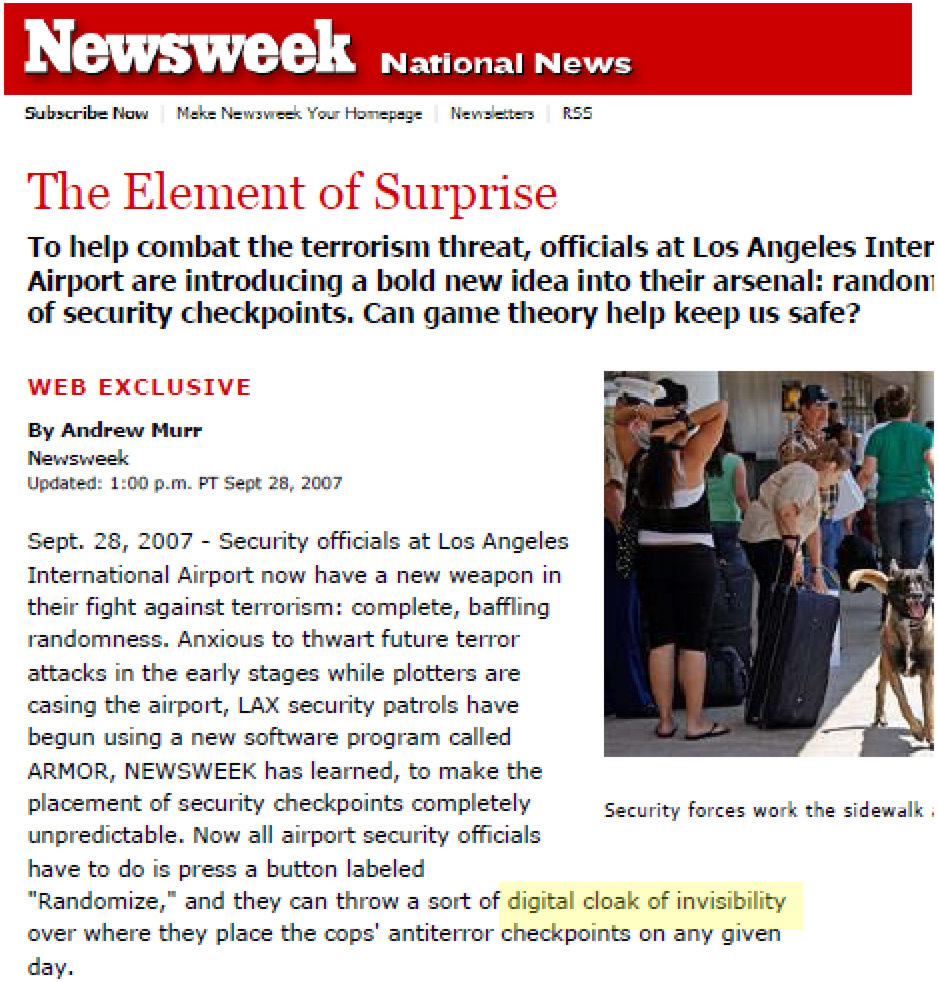
TRACTABILITY FOR 2 PLAYERS

- For each pure follower strategy t, we compute via the LP below a strategy for the leader such that
  - Playing t is a best response for the follower
  - Under this constraint, the leader strategy is optimal
- Choose t* that maximizes leader value

$$\begin{aligned}
&\textbf{maximize } \sum_{s \in S} p_s u_l(s,t) \\
&\textbf{subject to} \\
&\text{for all } t' \in T,\ \sum_{s \in S} p_s u_f(s,t) \geq \sum_{s \in S} p_s u_f(s,t') \\
&\sum_{s \in S} p_s = 1
\end{aligned}$$

APPLICATION: SECURITY

- Airport security: deployed at LAX
- Federal Air Marshals
- Coast Guard
- Idea:
  - Defender commits to mixed strategy
  - Attacker observes and best responds



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The Element of Surprise

To help combat the terrorism threat, officials at Los Angeles Inter Airport are introducing a bold new idea into their arsenal: random of security checkpoints. Can game theory help keep us safe?

WEB EXCLUSIVE

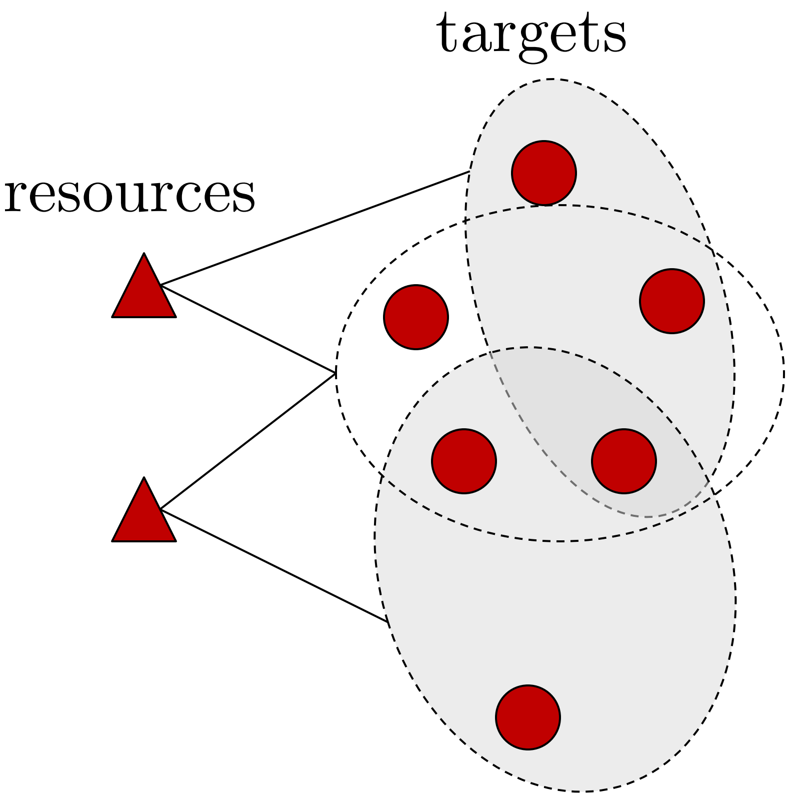
By Andrew Murr
Newsweek
Updated: 1:00 p.m. PT Sept 28, 2007

Sept. 28, 2007 - Security officials at Los Angeles International Airport now have a new weapon in their fight against terrorism: complete, baffling randomness. Anxious to thwart future terror attacks in the early stages while plotters are casing the airport, LAX security patrols have begun using a new software program called ARMOR, NEWSWEEK has learned, to make the placement of security checkpoints completely unpredictable. Now all airport security officials have to do is press a button labeled "Randomize," and they can throw a sort of digital cloak of invisibility over where they place the cops' antiterror checkpoints on any given day.

Security forces work the sidewalk

SECURITY GAMES

- Model due to [Kiekintveld et al., AAMAS 2009]
- Set of targets T
- Set of security resources Ω available to the defender (leader)
- Set of schedules $S \subseteq 2^T$
- Resource ω can be assigned to one of the schedules in $A(\omega) \subseteq S$
- Attacker chooses one target to attack
- Utilities depend on target and whether it is defended



SOLVING SECURITY GAMES

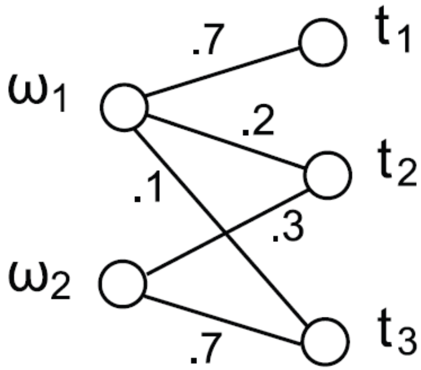
- Consider the case of S=T, i.e., resources are assigned to individual targets, i.e., schedules have size 1
- Nevertheless, number of leader strategies is exponential
- **Theorem** [Korzhyk et al., AAAI 2010]: Optimal leader strategy can be computed in poly time

A compact LP

- LP formulation similar to previous one
- Advantage: logarithmic in #leader strategies
- Disadvantage: do probabilities correspond to strategy?

$$
\begin{aligned}
&\text{maximize } U_d(t^*, \mathbf{c}) \\
&\text{subject to} \\
&\forall \omega \in \Omega, \forall t \in A(\omega) : 0 \leq c_{\omega,t} \leq 1 \\
&\forall t \in T : c_t = \sum_{\omega \in \Omega : t \in A(\omega)} c_{\omega,t} \leq 1 \\
&\forall \omega \in \Omega : \sum_{t \in A(\omega)} c_{\omega,t} \leq 1 \\
&\forall t \in T : U_a(t, \mathbf{c}) \leq U_a(t^*, \mathbf{c})
\end{aligned}
$$

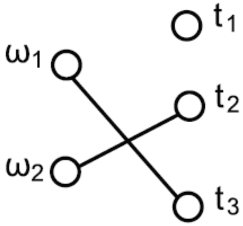
Fixing the probabilities



	t_1	t_2	t_3
ω_1	.7	.2	.1
ω_2	0	.3	.7

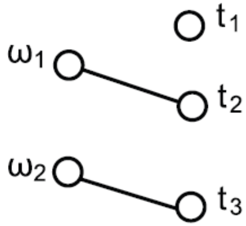
.1

	t_1	t_2	t_3
ω_1	0	0	1
ω_2	0	1	0



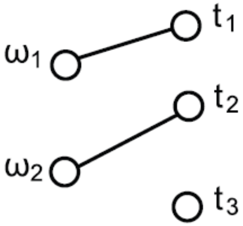
.2

	t_1	t_2	t_3
ω_1	0	1	0
ω_2	0	0	1



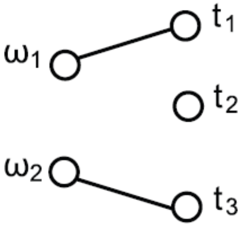
.2

	t_1	t_2	t_3
ω_1	1	0	0
ω_2	0	1	0



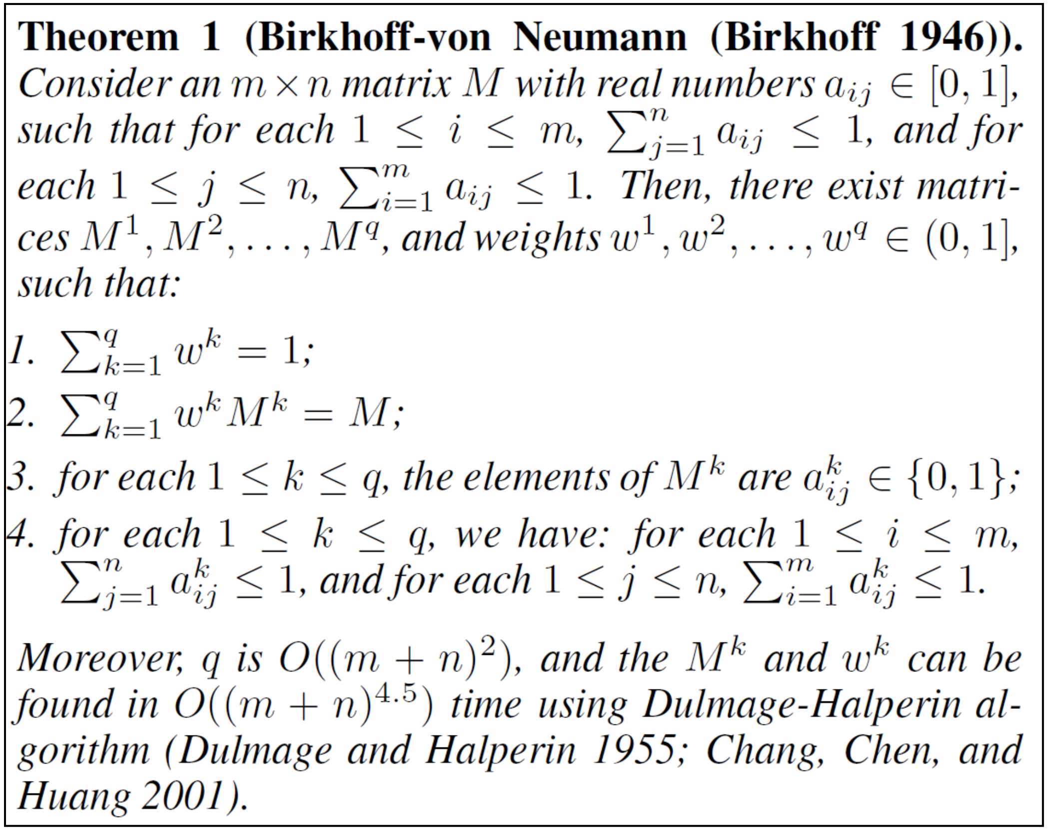
.5

	t_1	t_2	t_3
ω_1	1	0	0
ω_2	0	0	1



FIXING THE PROBABILITIES

- The probabilities $c_{\omega,t}$ satisfy theorem's conditions
- By 3, each matrix consists of {0,1} entries
- Interpretation by 4: ω assigned to t iff corresponding entry is 1
- By 1, we get a mixed strategy
- By 2, gives right probs



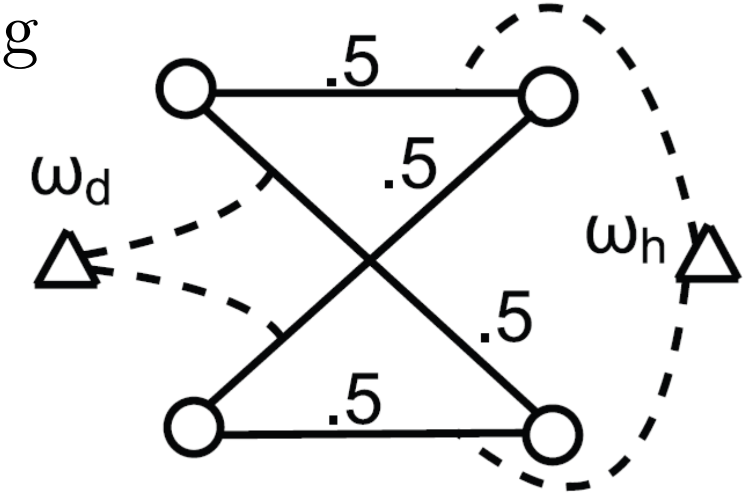
Theorem 1 (Birkhoff-von Neumann (Birkhoff 1946)). *Consider an* $m \times n$ *matrix* M *with real numbers* $a_{ij} \in [0,1]$, *such that for each* $1 \le i \le m$, $\sum_{j=1}^{n} a_{ij} \le 1$, *and for each* $1 \le j \le n$, $\sum_{i=1}^{m} a_{ij} \le 1$. *Then, there exist matrices* $M^1, M^2, \ldots, M^q$, *and weights* $w^1, w^2, \ldots, w^q \in (0,1]$, *such that:*

1. $\sum_{k=1}^{q} w^k = 1$;
2. $\sum_{k=1}^{q} w^k M^k = M$;
3. *for each* $1 \le k \le q$, *the elements of* M^k *are* $a_{ij}^k \in \{0,1\}$;
4. *for each* $1 \le k \le q$, *we have: for each* $1 \le i \le m$, $\sum_{j=1}^{n} a_{ij}^k \le 1$, *and for each* $1 \le j \le n$, $\sum_{i=1}^{m} a_{ij}^k \le 1$.

Moreover, q *is* $O((m+n)^2)$, *and the* M^k *and* w^k *can be found in* $O((m+n)^{4.5})$ *time using Dulmage-Halperin algorithm (Dulmage and Halperin 1955; Chang, Chen, and Huang 2001).*

GENERALIZING?

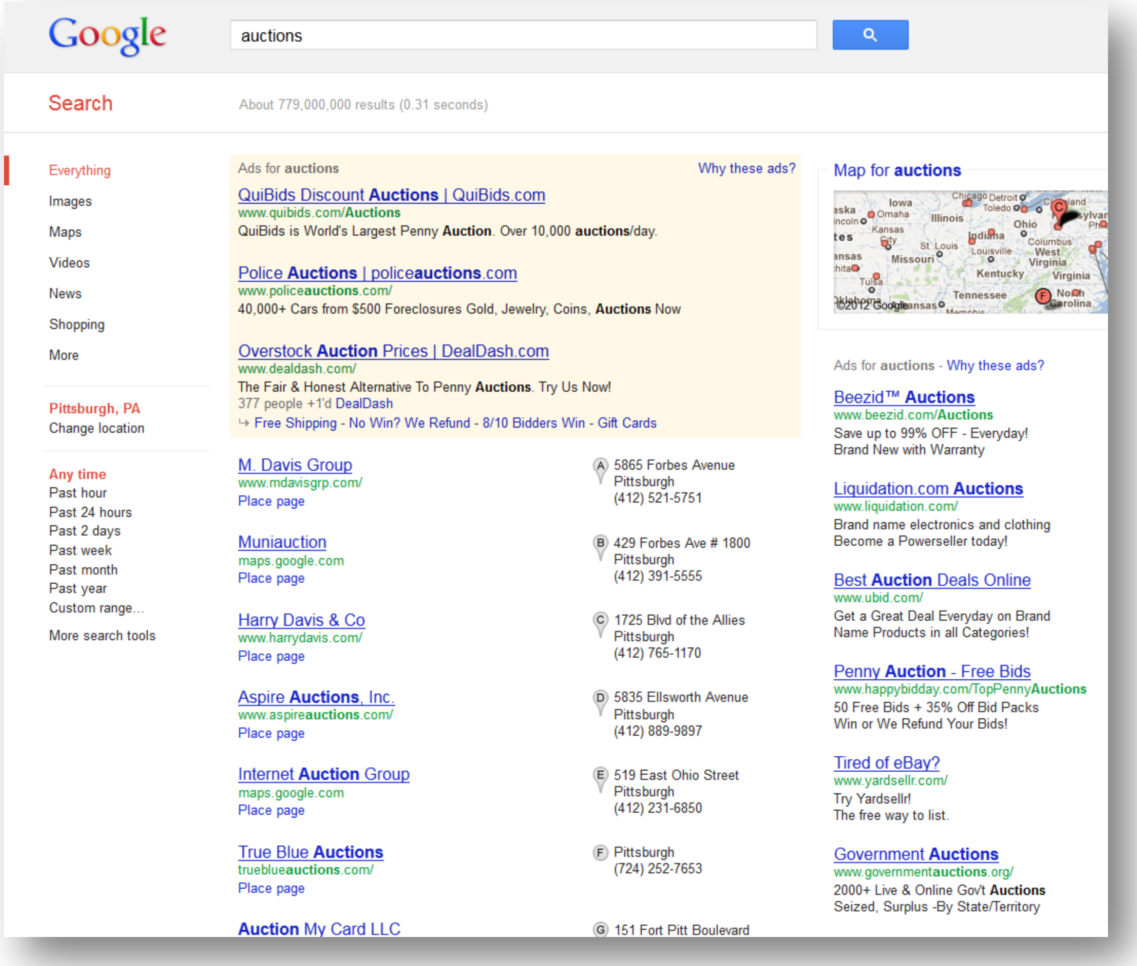
- Schedules of size 2
- Air Marshals domain has such schedules: outgoing+incoming flight (bipartite graph)
- Previous apporoach fails
- **Theorem** [Korzhyk et al., AAAI 2010]: (even bipartite) problem is NP-hard



MECHANISM DESIGN!

- A subfield of game theory that focuses on designing the rules of the game to achieve desirable properties
- We will only cover a tiny fraction of the very basics of auction theory

AD AUCTIONS



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ENGLISH AUCTIONS

- Most well-known type of auctions
    - Ascending
    - Open cry
    - First price
- Dominant strategy: successively bid slightly more than current highest bid until price reaches valuation
- Susceptible to:
    - Winner’s curse: why doesn’t anyone else want the good at the final price?
    - Shills: work for auctioneer and drive prices up

OTHER BORING AUCTIONS

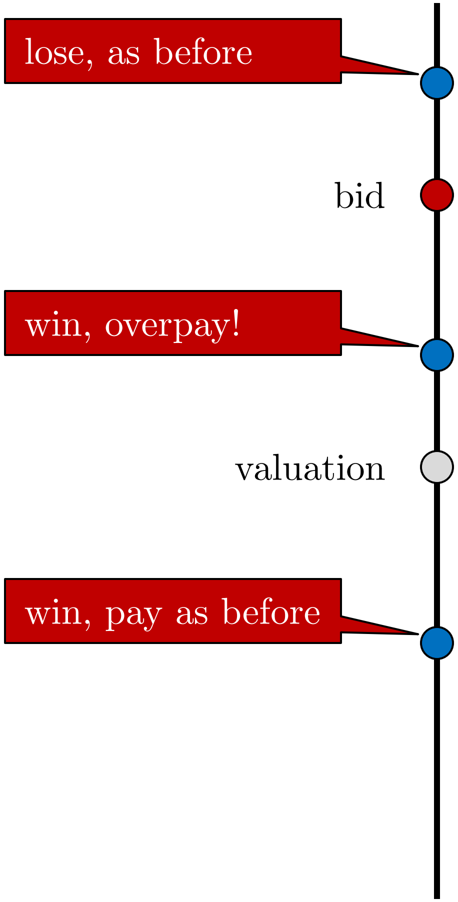
- Dutch
  - Auctioneer starts at high price
  - Auctioneer lowers price until a bidder makes a bid at current price
- First-price sealed-bid auction
  - Bidders submit sealed bids
  - Good is allocated to highest bidder
  - Winner pays price of highest bid
- Bids generally do not match valuation!

VICKREY AUCTION

- Bidders submit sealed bids
- Good is allocated to highest bidder
- Winner pays price of *second highest* bid!!
- Amazing observation: bidding true valuation is a dominant strategy!!

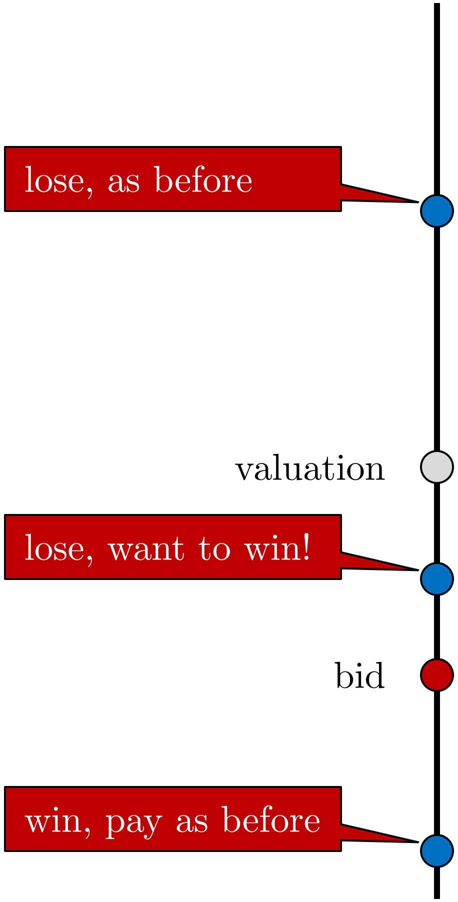
TRUTHFULNESS: BIDDING HIGH

- Three cases based on highest other bid (blue dot)
- Higher than bid: lose before and after
- Lower than valuation: win before and after, pay same
- Between bid and valuation: lose before, win after but overpay



TRUTHFULNESS: BIDDING LOW

- Three cases based on highest other bid (blue dot)
- Higher than valuation: lose before and after
- Lower than bid: win before and after, pay the same
- Between valuation and bid: win before with profit, lose after



SEQUENTIAL AUCTIONS ARE BAD

- A computer and screen are sold in two Vickrey auctions
- Each is worthless alone but together their value to you is $500
- What should bid in the first auction?
    - Say you bid $200 and lose to a $300 bid; the screen may sell for $50
    - Say you bid $200 and win; the screen may sell for $500

COMBINATORIAL AUCTIONS

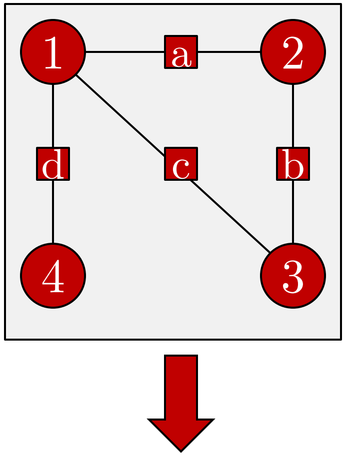
- Bidders submit bids for *subsets* of goods
- Example:
  - ({A, C, D}, 7)
  - ({B, E}, 7)
  - ({C}, 3)
  - ({A, B, C, E}, 9)
  - ({D}, 4)
  - ({A, B, C}, 5)
  - ({B, D}, 5)
- What is the optimal solution?

WINNER DETERMINATION

- Allocate to maximize social welfare
- Consider the special case of *single minded bidders*: each bidder i values a subset S_i of items at v_i and any subset that does not contain S_i at 0
- **Theorem (folk):** optimal winner determination is NP-complete, even with single minded bidders

NP-HARDNESS+PIC

- INDEPENDENT SET (IS): given a graph, is there a set of vertices of size k such that no two are connected?
- Given an instance of IS:
  - The set of items is E
  - Player for each vertex
  - Desired bundle is adjacent edges, value is 1
- A set of winners W satisfies $S_i \cap S_j$ for every $i \neq j \in W$ iff the vertices in W are an independent set



1: {a,c,d}
2: {a,b}
3: {b,c}
4: {d}

FINAL REMARKS

- Vickrey auction can be generalized to yield a truthful mechanism (VCG) for combinatorial auctions
- Requires optimally solving the winner determination problem
- Resorting to approximation is no longer truthful
- *Tons* of research on practical algorithms for solving CAs, and on approximation algorithms that are truthful