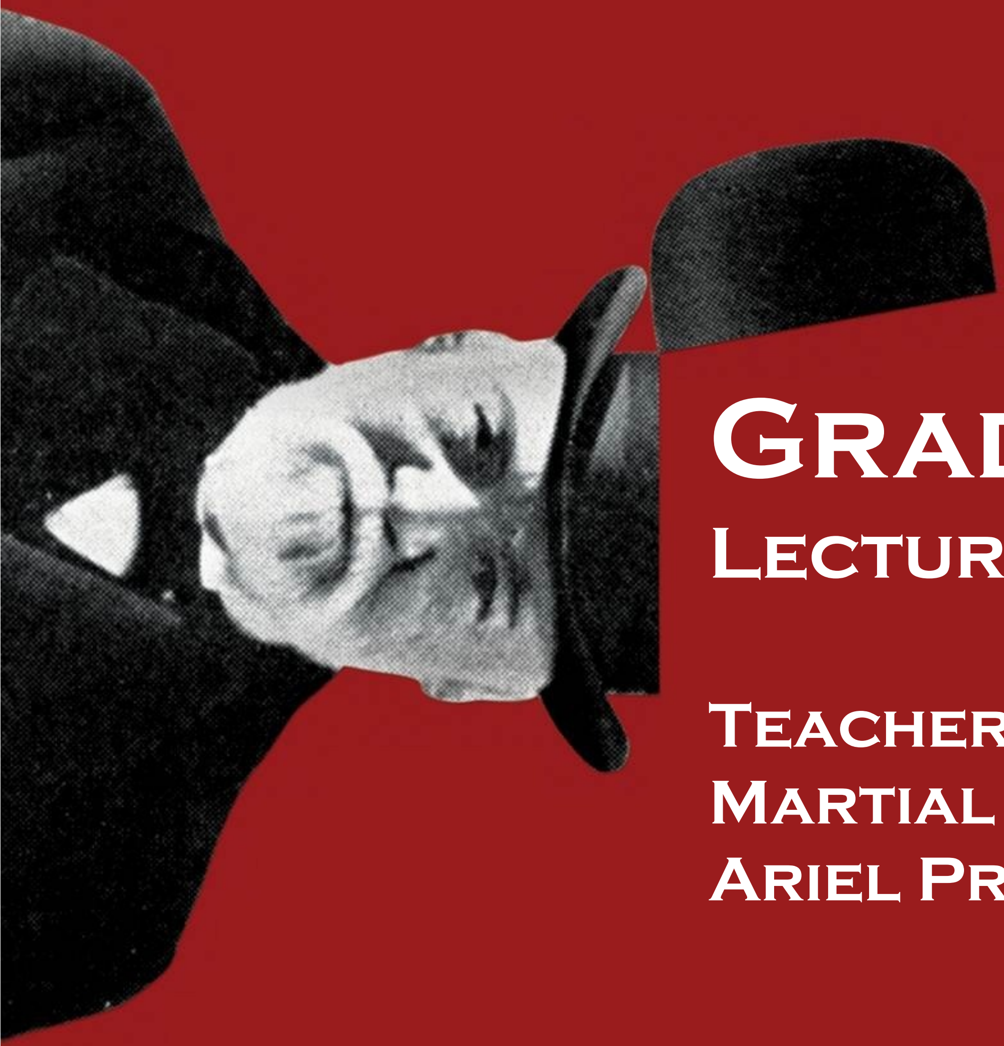


Graduate AI

Lecture 22: Game theory I

Teachers:

Martial Hebert

Ariel Procaccia (this time)

The prisoner's dilemma

- Two men are charged with a crime
- They are told that:
  - If one rats out and the other does not, the rat will be freed, other jailed for nine years
  - If both rat out, both will be jailed for six years
- They also know that if neither rats out, both will be jailed for one year

The prisoner's dilemma

	Cooperate	Defect
Cooperate	-1,-1	-9,0
Defect	0,-9	-6,-6

What would you do?

Understanding the dilemma

- Defection is a *dominant* strategy
- (Defect,Defect) is a *dominant strategy equilibrium*
- Defection is the only rational outcome
- But the players can do much better by cooperating
- Related to the *tragedy of the commons*

In real life

- Republican primaries
    - Cooperate = positive ads
    - Defect = negative ads
- Nuclear arms race
    - Cooperate = destroy arsenal
    - Defect = build arsenal
- Climate change
    - Cooperate = curb CO_2 emissions
    - Defect = do not curb

The professor's dilemma

		Class	
		Listen	Sleep
Professor	Make effort	$10^6,10^6$	-10,0
	Slack off	0,-10	0,0

Are there dominant strategies?

Nash equilibrium

- Each player's strategy is a *best response* to strategies of others
- Formally, a *Nash equiblibrium* is a vector of strategies $s=(s_1,\ldots,s_n)$ such that
- $\forall i \in N$, $s'_i \in S_i$, $u_i(s) \geq u_i(s'_i, s_{-i})$
- What are the Nash equilibria of the professor's dilemma?
  - (effort,listen) and (slack off,sleep)

Rock-paper-scissors

	R	P	S
R	0,0	-1,1	1,-1
P	1,-1	0,0	-1,1
S	-1,1	1,-1	0,0

Is there a Nash equilibrium?

Mixed strategies

- A mixed strategy is a randomization over pure strategies
- For two players, if player 1 (2) chooses strategy s_j with probability x_j (y_j) then the utility is $u_i(x,y) = \Sigma_{j,k} x_j y_k u_i(s_j,s_k)$
- Is ((1/2,1/2,0),(1/2,1/2,0)) a NE for Rock-Paper-Scissors?
  - Each player can improve by playing (0,1,0)
- Is ((1/3,1/3,1/3),(1/3,1/3,1/3)) a NE?
  - Yes!

Nash's Theorem

- **Theorem [Nash, 1950]:** if everything is finite then there exists at least one (possibly mixed) Nash equilibrium
- However, how does one *compute* a Nash equilibrium?
- Standard complexity classes are irrelevant because this is not a decision problem

NE IS PPAD COMPLETE

- **Theorem [Chen and Deng, STOC 2007]:** Finding a NE is PPAD-complete
- But what is PPAD?
- Formally defined by its complete problem
  - G is a directed graph with every vertex having at most one predecessor and at most one successor
  - G is specified by giving a function f(v) that returns the predecessor and successor of v
  - Given a vertex s in G with a successor but no predecessor, find a vertex t≠s with no predecessor or no successor
- Such a vertex exists at the end of the path starting with the source s

Application: Interdomain routing

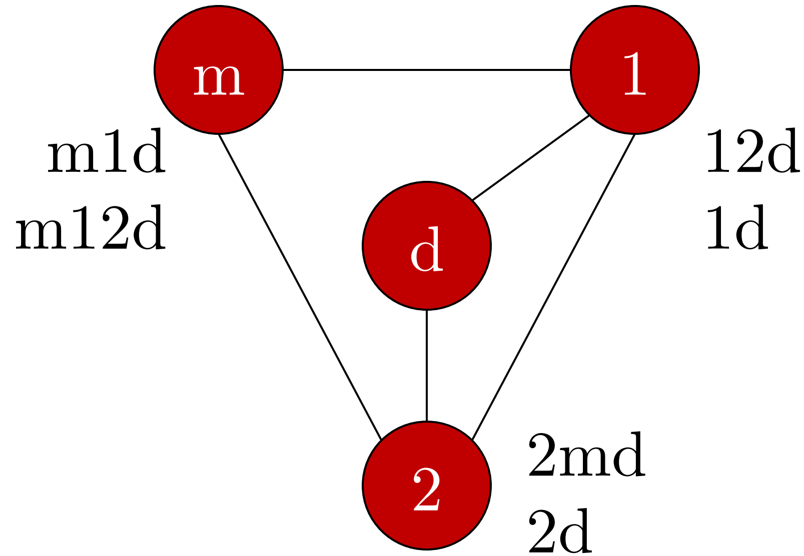
- Internet composed of smaller networks called *autonomous systems* (AS)
- Owned by competing entities (Microsoft, AT&T, etc.)
- Interdomain routing = establishing routes between ASes
- Standard protocol: BGP

Application: Interdomain routing

- Graph with n source nodes (players) and a destination node
- Each player has preferences over routes to the destination
- Under BGP ASes continuously:
  - Receive updates about routes of neighbors
  - Choose a neighbor to send traffic to
  - Announce new route to neighboring nodes

Application: Interdomain routing

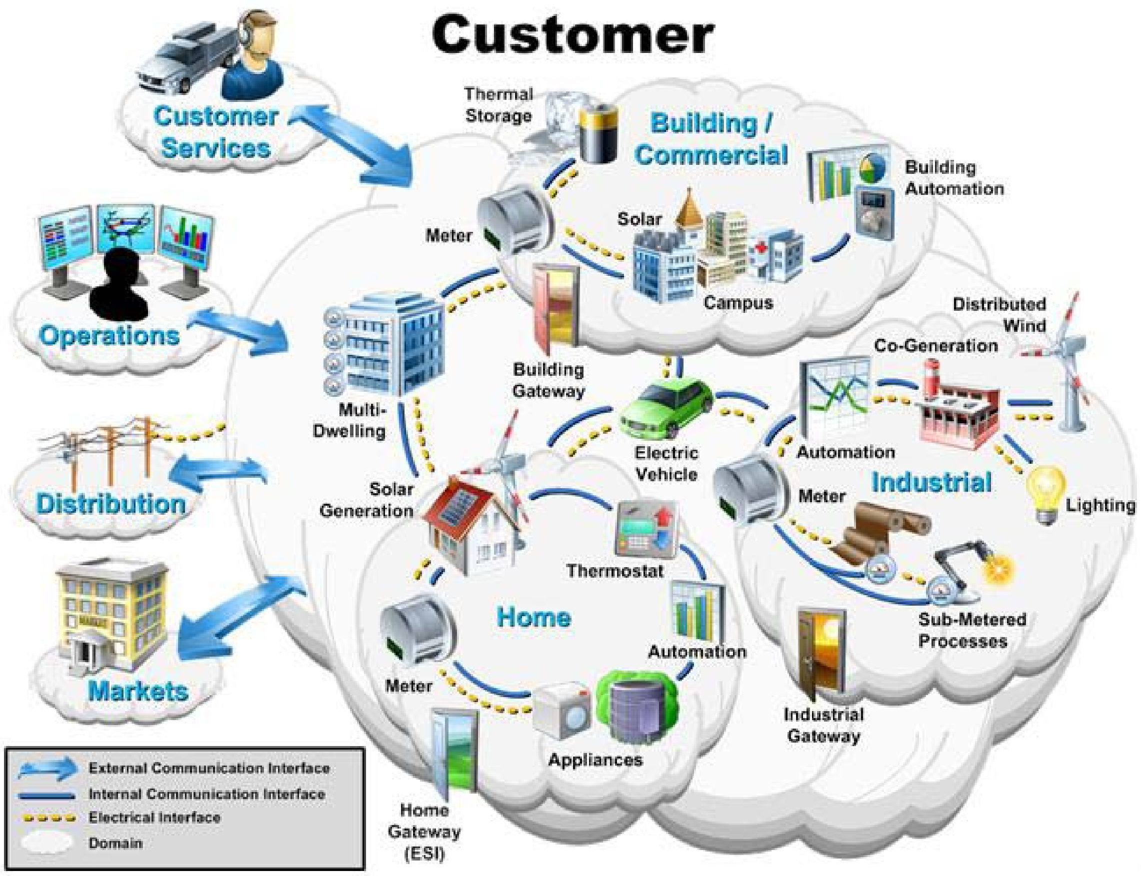
- **Theorem [Levin et al, STOC 2008]:** Following BGP is not an (ex-post) NE
- BGP converges to the NE (12d,2d,m12d)
- But… if m repeatedly announces to 2 the route md
- 2 would go with 2md
- 1 would go with 1d
- m gets m1d!



Application: Interdomain routing

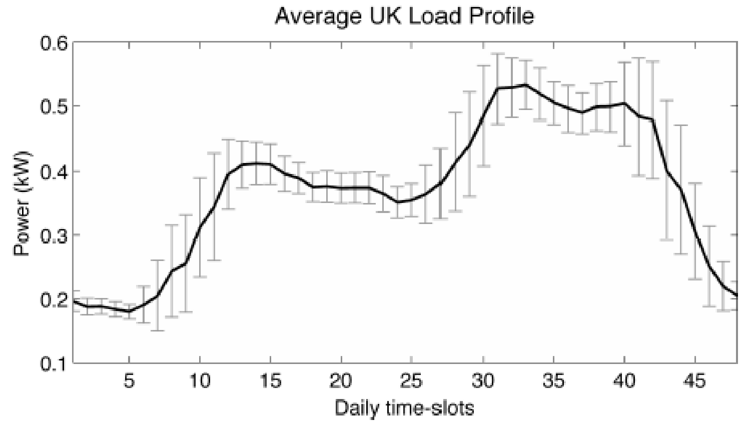
- Route verification = players can verify that neighbors' declared paths actually exist
- **Theorem [Levin et al., STOC 2008]:** Assuming route verification (+mild technical condition), following BGP is an (ex-post) Nash equilibrium!
- Provides partial explanation for why interdomain routing functions so well!

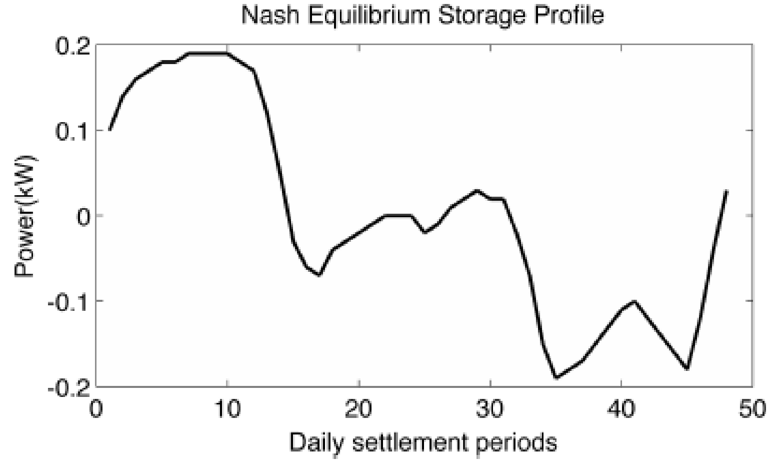
Application: Smart grid



Application: Smart grid

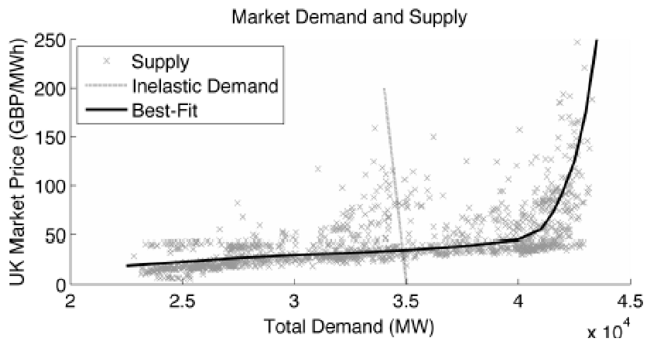
- Energy storage devices advocated for saving energy in future smart grid
- Bad if all are charged at the same time
- Solution: agent-based management system that allows storage devices to converge to equilibrium [Vytelingum et al., AAMAS 2010]





Application: Smart grid

- Strategy of an agent: how much to charge in each half hour of the day
- The behavior of electricity suppliers is specified by a supply curve
- Equilibrium can be analytically computed
- Simulations show that in eq., savings of 13% on electricity bill in UK



Back to jail

- Let us revisit the prisoner's dilemma
- Only mixed NE plays defect with prob 1
- Idea: allow a mediator [Monderer+Tennenholtz, AAAI 2006]
- Players can choose to let the mediator play for them

Dilemma with mediator

	M	C	D
M	-1,-1	0,-9	-6,-6
C	-9,0	-1,-1	-9,0
D	-6,-6	0,-9	-6,-6

(M,M) is a *strong* Nash equilibrium

Correlated equilibrium

- Imagine a mediator choosing a pair of strategies (s_i,s_j) according to a distribution p over pairs
- Reveal s_i to player 1 and s_j to player 2
- When player 1 gets s_i, he knows that the distribution over strategies of player 2 is
  $\Pr[s_j \mid s_i] = p_{ij}/\Sigma_k p_{ik}$
- Player 1 is best responding if for all s'_i
  $\Sigma_j\ p_{ij}u_1(s_i,s_j) \geq \Sigma_j\ p_{ij}u_1(s'_i,s_j)$
- p is a correlated eq. (CE) if all players are best responding
- Every NE is a CE

Game of chicken

- Pure NE: (C,D) and (D,C), social welfare=5
- Mixed NE: both (1/2,1/2), social welfare=4
- Optimal social welfare is 6

	Dare	Chicken
Dare	0,0	4,1
Chicken	1,4	3,3

Game of chicken

- Correlated equilibrium:
  - (D,D): 0
  - (D,C): 1/3
  - (C,D): 1/3
  - (C,C): 1/3
- Social welfare of correlated eq. is 16/3

	Dare	Chicken
Dare	0,0	4,1
Chicken	1,4	3,3

Implementation of CE

- We need a mediator
- Mediator can be replaced with correlation device
- Correlation device for game of chicken:
    - Hat, two balls labeled “chicken”, one ball labeled “dare”
    - Each player draws ball without looking
- There is work in crypto on secure implementation of CEs

Computation of CE

- These inequalities are linear:
  $\Sigma_j\ p_{ij}u_1(s_i,s_j) \geq \Sigma_j\ p_{ij}u_1(s'_i,s_j)$
- Add the inequality $\Sigma_{ij}p_{ij}=1$
- We get… a linear program!
- Can be solved in polynomial time, even if we want to maximize a linear objective such as the social welfare
- Contrast with computation of NE
- Why isn't NE a linear program?