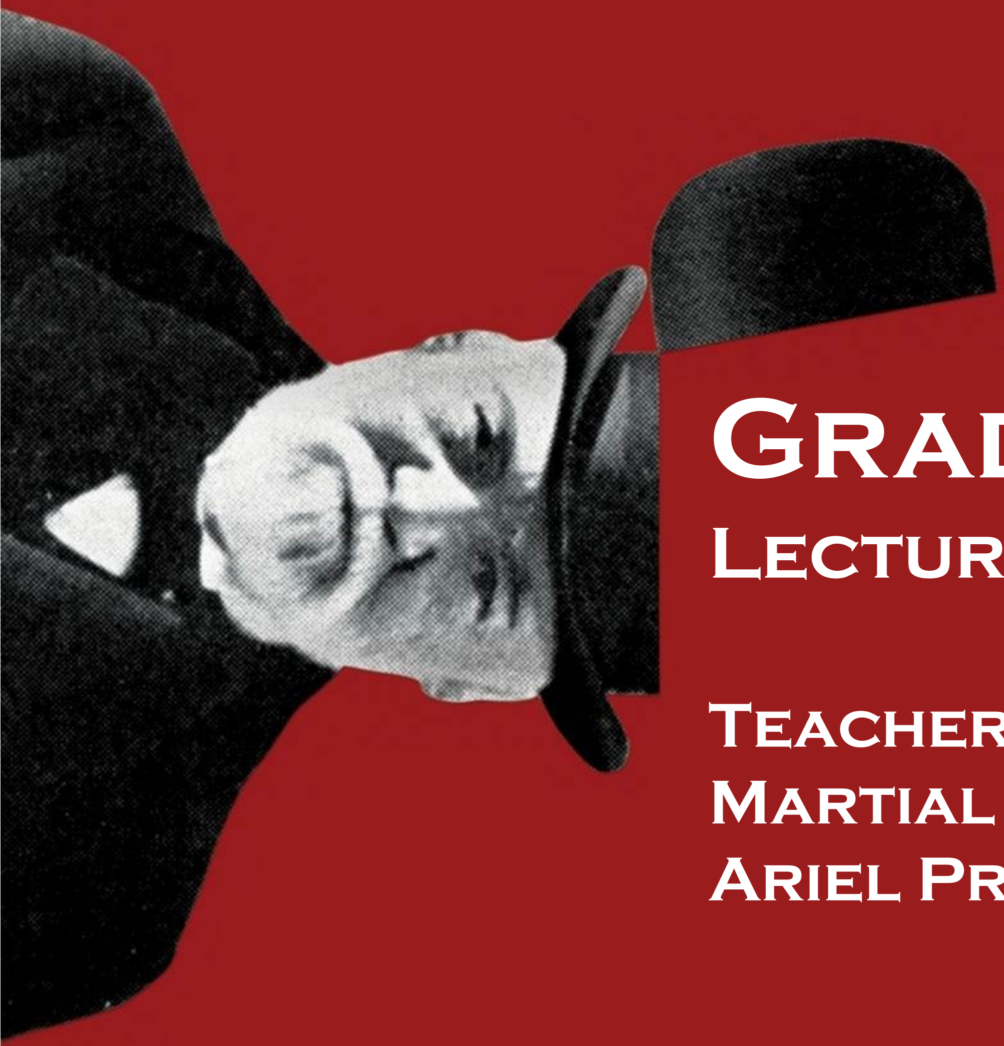

Graduate AI

Lecture 20: Social Choice I

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Social choice theory

- A mathematical theory that deals with aggregation of individual preferences
- Origins in ancient Greece
- Formal foundations: 18th Century (Condorcet and Borda)
- 19th Century: Charles Dodgson
- 20th Century: Nobel prizes to Kenneth Arrow and Amartya Sen

Computational social choice

- Two-way interaction with AI
- AI $\Rightarrow$ social choice
  - Algorithms and computational complexity
  - Machine learning in social choice
  - Knowledge representation
  - Markov decision processes

Computational social choice

- Social choice ⇒ AI
    - Multiagent systems: reducing communication
    - Human computation: aggregating peoples' opinions

The voting model

- Set of *voters* N={1,...,n}
- Set of alternatives A, |A|=m
- Each voter has a ranking over the candidates
- $x >_i y$ means that voter i prefers x to y
- *Preference profile* = collection of all voters' rankings

1	2	3
a	c	b
b	a	c
c	b	a

Voting rules

- *Voting rule* = function from preference profiles to alternatives that specifies the winner of the election
- Plurality
  - Each voter awards one point to top alternative
  - Alternative with most points wins
  - Used in almost all political elections

More voting rules

- Borda count
  - Each voter awards m-k points to alternative ranked k'th
  - Alternative with most points wins
  - Proposed in the 18th Century by the chevalier de Borda
  - Used in the national assembly of Slovenia
  - Similar to rule used in the Eurovision song contest

Lordi, Eurovision 2006 winners

More voting rules

- Veto
  - Each voter vetoes his least preferred alternative
  - Alternative with least vetoes wins
- Positional scoring rules
  - Defined by a vector $(s_1,\ldots,s_m)$
  - Each voter gives s_k points to k'th position
  - Plurality: (1,0,...,0); Borda: (m-1,m-2,...,0), Veto: (1,...,1,0)

More voting rules

- a beats b in a *pairwise election* if the majority of voters prefer a to b
- Plurality with runoff
  - First round: two alternatives with highest plurality scores survive
  - Second round: pairwise election between these two alternatives

More voting rules

- Single Transferable vote (STV)
  - m-1 rounds
  - In each round, alternative with least plurality votes is eliminated
  - Alternative left standing is the winner
  - Used in Ireland, Malta, Australia, and New Zealand (and Cambridge, MA)

STV: EXAMPLE

2 voters	2 voters	1 voter
a	b	c
b	a	d
c	d	b
d	c	a

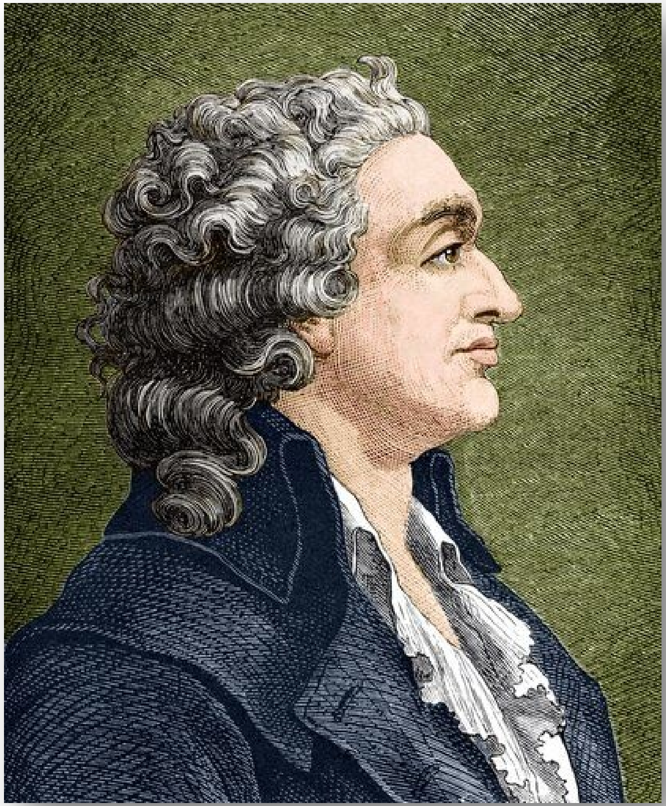
2 voters	2 voters	1 voter
a	b	c
b	a	b
c	c	a

2 voters	2 voters	1 voter
a	b	b
b	a	a

2 voters	2 voters	1 voter
b	b	b

Marquis de Condorcet

- 18th Century French Mathematician, philosopher, political scientist
- One of the leaders of the French revolution
- After the revolution became a fugitive
- His cover was blown and he died mysteriously in prison



Condorcet winner

- Condorcet winner = alternative that beats every other alternative in pairwise election
- Condorcet paradox = Condorcet winner may not exist
- Condorcet criterion = elect a Condorcet winner if one exists
- Does plurality satisfy criterion? Borda?

1	2	3
a	c	b
b	a	c
c	b	a

More voting rules

- Copeland
  - Alternative's score is #alternatives it beats in pairwise elections
  - Why does Copeland satisfy the Condorcet criterion?
- Maximin
  - Score of x is $\min_y |\{i \in N: x >_i y\}|$
  - Why does Maximin satisfy the Condorcet criterion?

Awesome example

- Plurality: a
- Borda: b
- Condorcet winner: c
- STV: d
- Plurality with runoff: e

33 voters	16 voters	3 voters	8 voters	18 voters	22 voters
a	b	c	c	d	e
b	d	d	e	e	c
c	c	b	b	c	b
d	e	a	d	b	d
e	a	e	a	a	a

Manipulation

- Using Borda count
- Top profile: b wins
- Bottom profile: a wins
- By changing his vote, voter 3 achieves a better outcome!

1	2	3
b	b	a
a	a	b
c	c	c
d	d	d

1	2	3
b	b	a
a	a	c
c	c	d
d	d	b

Strategyproofness

- A voting rule is *strategyproof (SP)* if a voter can never benefit from lying about his preferences:
  $\forall <, \forall i \in N, \forall <'_i, f(<) \geq_i f(<'_i, <_{-i})$
- If there are two candidates then plurality is SP

Gibbard-Satterthwaite

- A voting rule is *dictatorial* if there is a voter who always gets his most preferred alternative
- A voting rule is *onto* if any alternative can win
- **Theorem (Gibbard-Satterthwaite)**: If $m \geq 3$ then any voting rule that is SP and onto is dictatorial
- In other words, any voting rule that is onto and nondictatorial is manipulable

Proof of G-S Theorem

- We prove the following statement on the board
- If $m \geq 3$ and $n = 2$ then any voting rule that is SP and onto is dictatorial
- The proof also appears in:
  L.-G. Svensson. The proof of the Gibbard-Satterthwaite Theorem revisited, Theorem 1 (available from course website)

Lemmas

- A voting rule satisfies *monotonicity* if: $f(\prec) = a, \forall i \in N, x \in A, [x \preceq a \Rightarrow x \preceq' a]$ implies that $f(\prec') = a$
- **Lemma:** Any SP voting rule is monotonic
- A voting rule satisfies *Pareto optimality (PO)* if: $\forall i \in N, x \succ_i y \Rightarrow f(\prec) \neq y$
- **Lemma:** Any SP and onto voting rule is PO