CMU 15-781

Lecture 24: Game theory III

Teachers:
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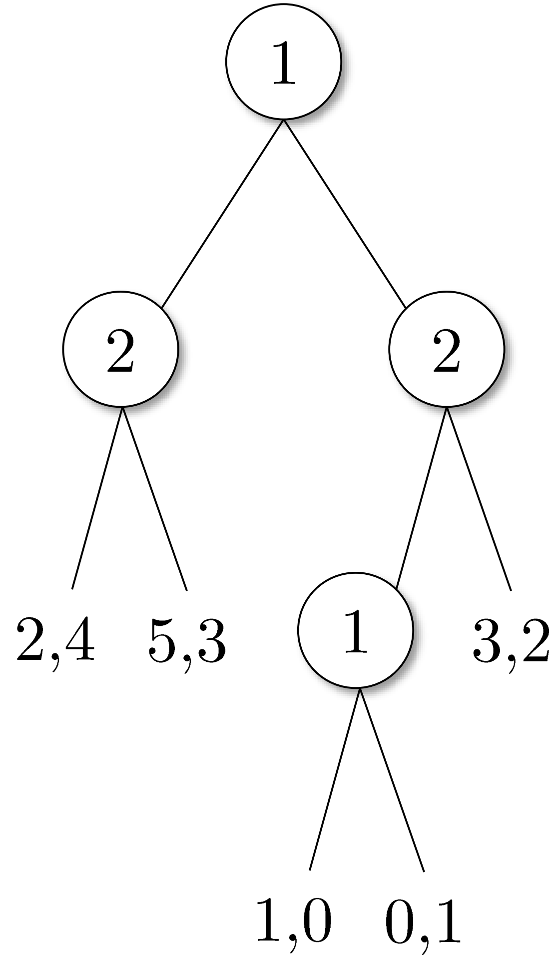
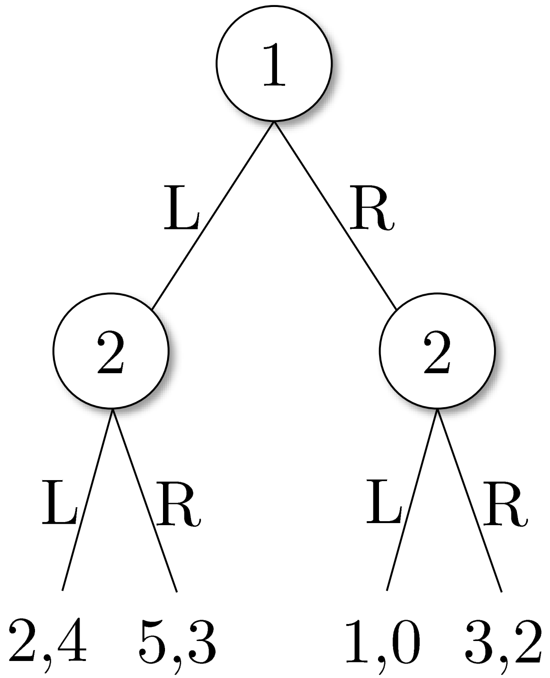
Extensive-form games

- Moves are done sequentially, not simultaneously
- Game forms a tree
- Nodes are labeled by players
- Leaves show payoffs



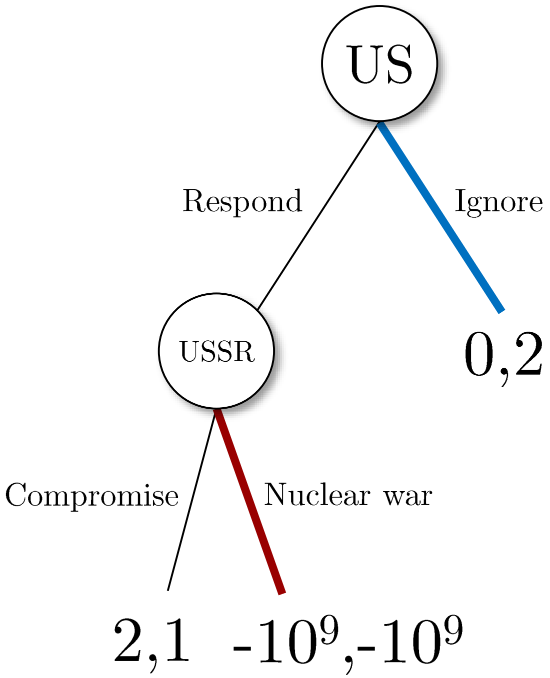
Extensive vs. normal form



	L/L	L/R	R/L	R/R
L	2,4	2,4	5,3	5,3
R	1,0	3,2	1,0	3,2

Problem: Normal-form representation is exponential in the size of the extensive-form representation

Extensive vs. normal form

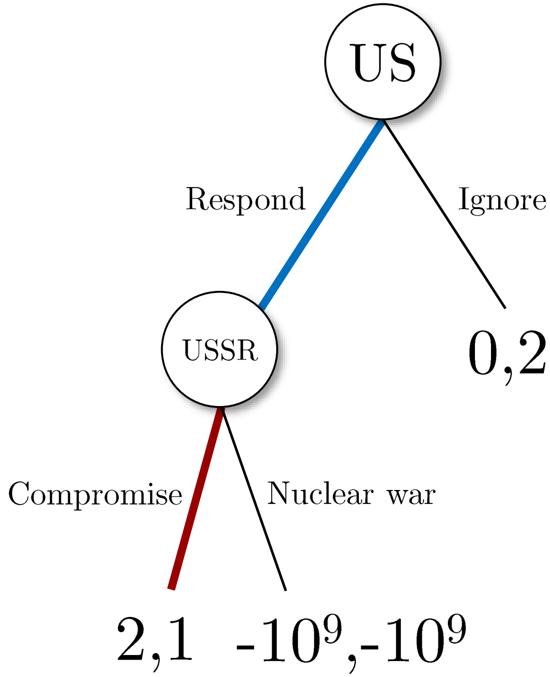


	Compromise	Nuclear war
Respond	2,1	$-10^9,-10^9$
Ignore	0,2	0,2

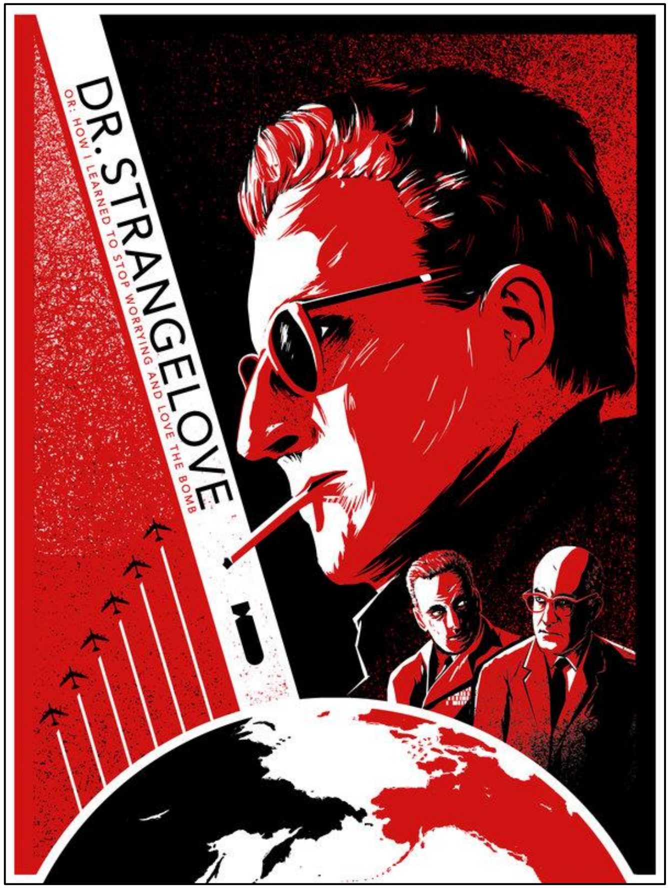
Problem: (ignore, nuclear war) is a Nash equilibrium, but threat isn't credible!

Subgame-perfect equilibrium

- Each subtree forms a subgame
- A set of strategies is a subgame-perfect equilibrium if it is a Nash equilibrium in each subgame
- A player may be able to improve his equilibrium payoff by eliminating strategies!

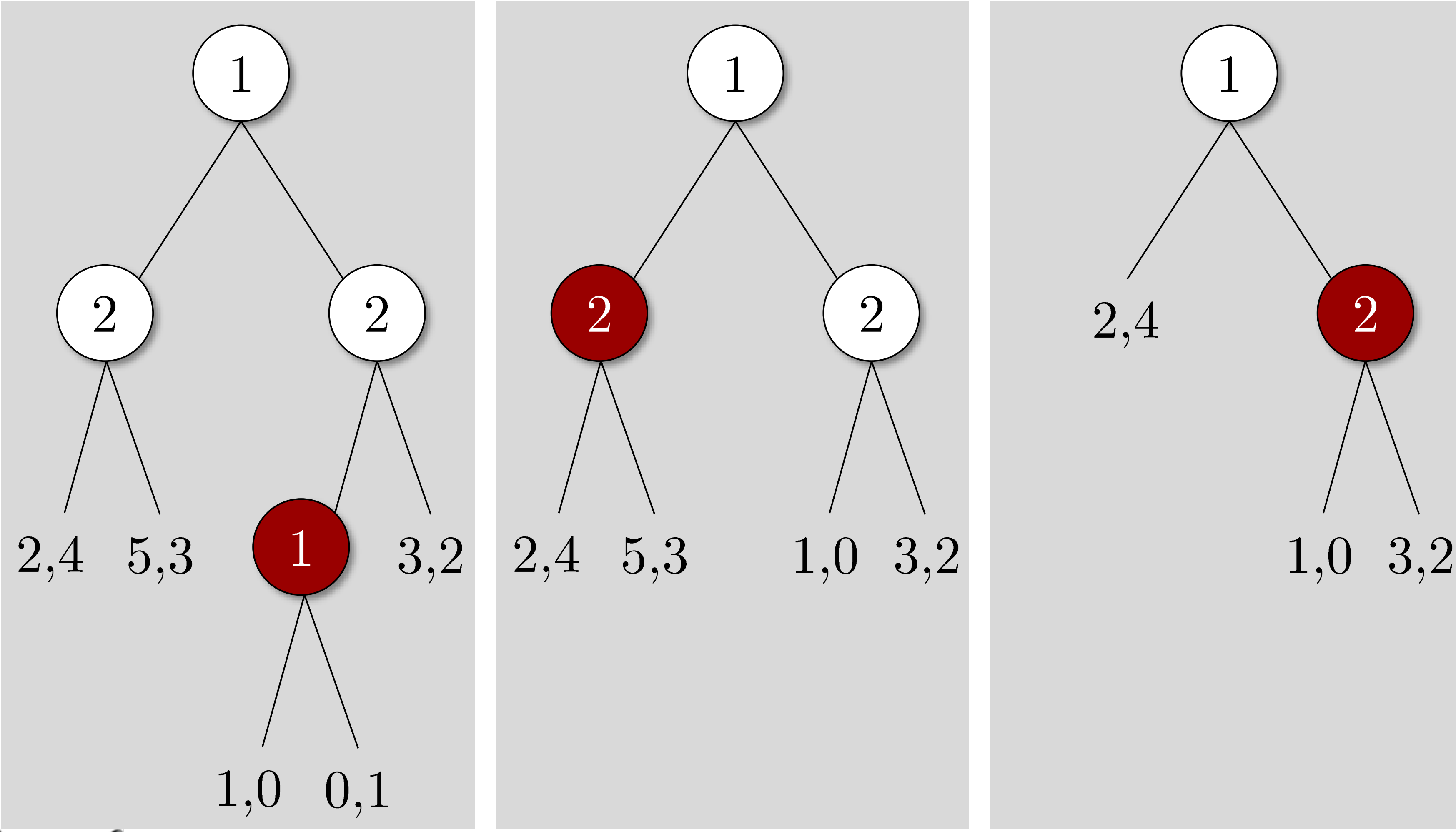


Doomsday machine

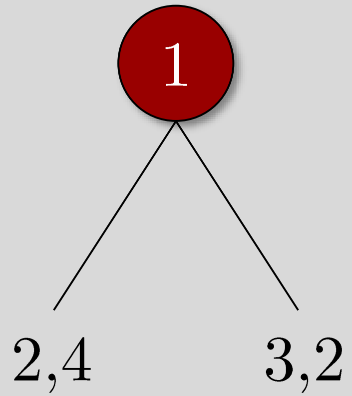


https://youtu.be/2yfXgu37iyI

Backward induction

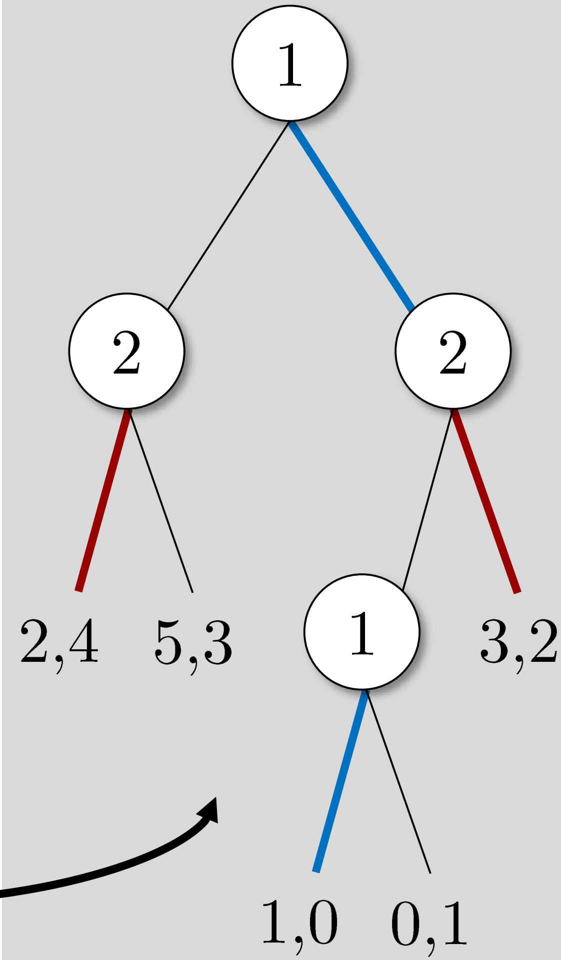


Backward induction

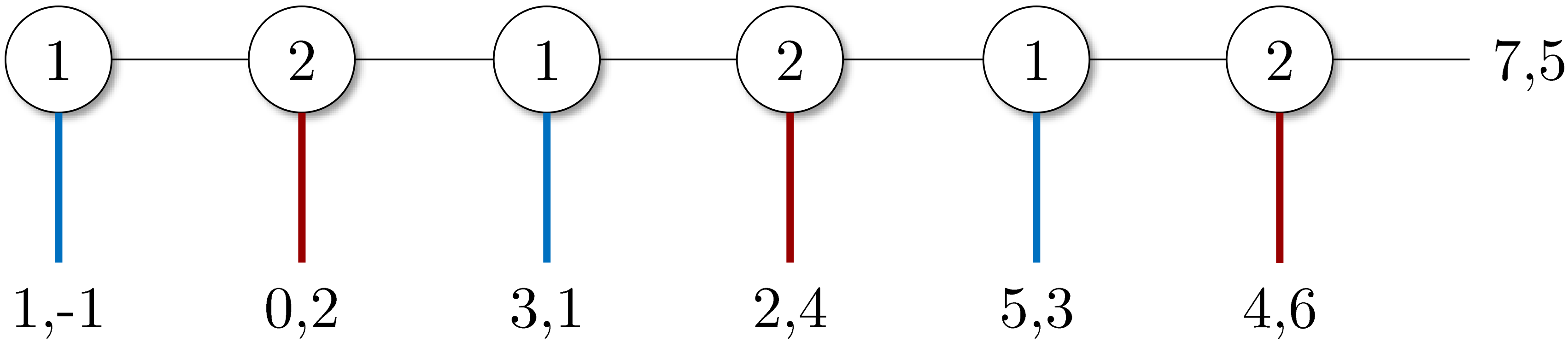


3,2

Subgame-perfect
equilibrium!



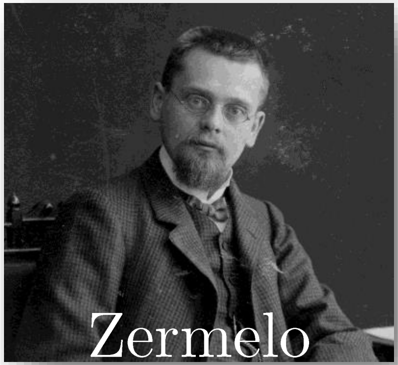
Example: Centipede game



Even subgame-perfect equilibrium can lead to strange outcomes!

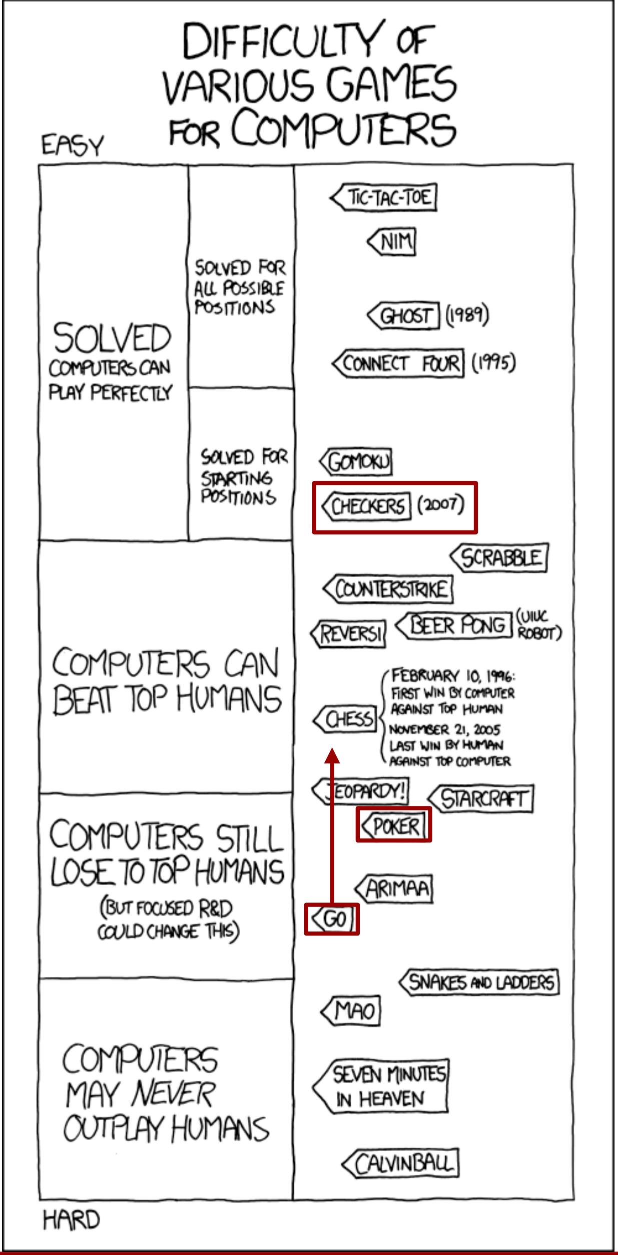
Checkers is solved

- Zermelo's Theorem [1913]: Either white can force a win, or black can force a win, or both sides can force a draw
- Proof: Backward induction ■
- Schaeffer solved the game in 2007, after 18 years of computation: It's a tie!
- Checkers game tree has 10^{20} nodes; chess has 10^{40}; go has 10^{170}



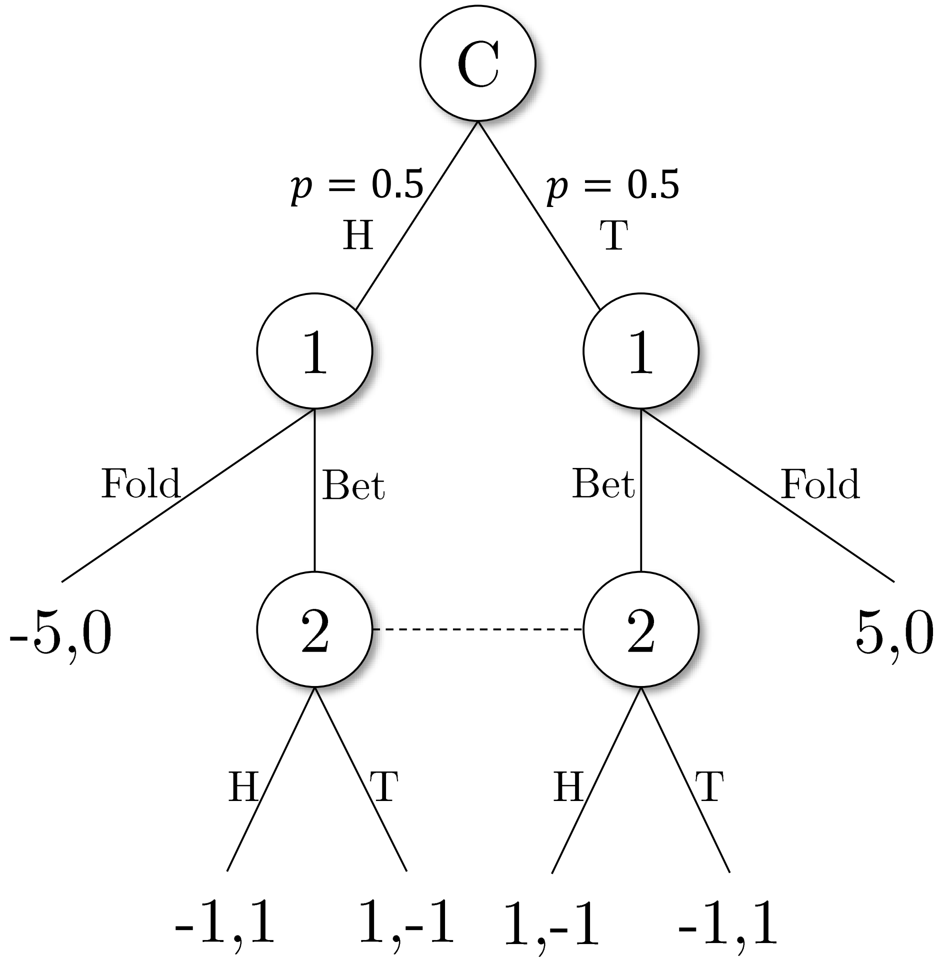
AlphaGo

- In 2016, AlphaGo beat Lee Sedol, one of the strongest players in the history of go, in a 5-game match
- A milestone that experts thought was a decade away
- Combination of tree search techniques and deep reinforcement learning


DIFFICULTY OF VARIOUS GAMES FOR COMPUTERS
EASY
SOLVED
COMPUTERS CAN PLAY PERFECTLY
SOLVED FOR ALL POSSIBLE POSITIONS
SOLVED FOR STARTING POSITIONS
TIC-TAC-TOE
NIM
GHOST (1989)
CONNECT FOUR (1995)
GOMOKU
CHECKERS (2007)
COMPUTERS CAN BEAT TOP HUMANS
SCRABBLE
COUNTERSTRIKE
REVERSI
BEER PONG (UIUC ROBOT)
CHESS
FEBRUARY 10, 1996: FIRST WIN BY COMPUTER AGAINST TOP HUMAN
NOVEMBER 21, 2005 LAST WIN BY HUMAN AGAINST TOP COMPUTER
JEOPARDY!
STARCRAFT
POKER
COMPUTERS STILL LOSE TO TOP HUMANS
(BUT FOCUSED R&D COULD CHANGE THIS)
ARIMAA
GO
COMPUTERS MAY NEVER OUTPLAY HUMANS
SNAKES AND LADDERS
MAO
SEVEN MINUTES IN HEAVEN
CALVINBALL
HARD

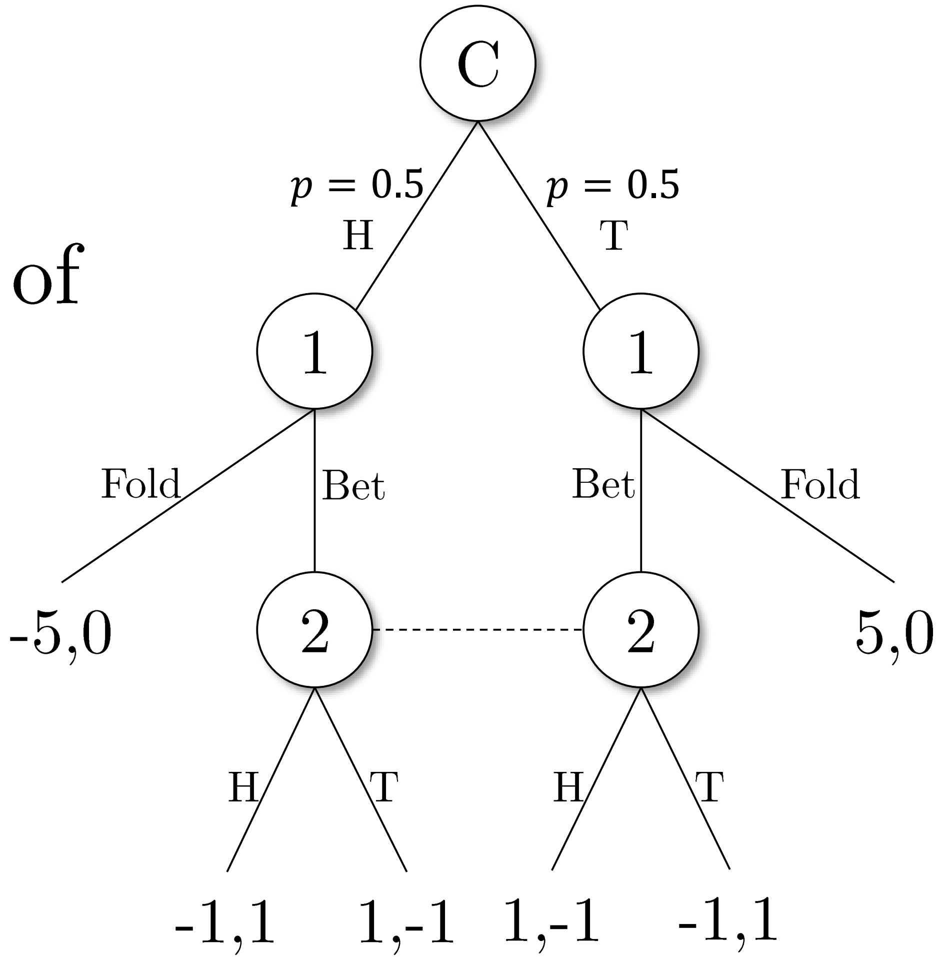
Imperfect-information games

- A chance node chooses between several actions according to a known probability distribution
- An information set is a set of nodes that a player may be in, given the available information
- A strategy must be identical for all nodes in an information set

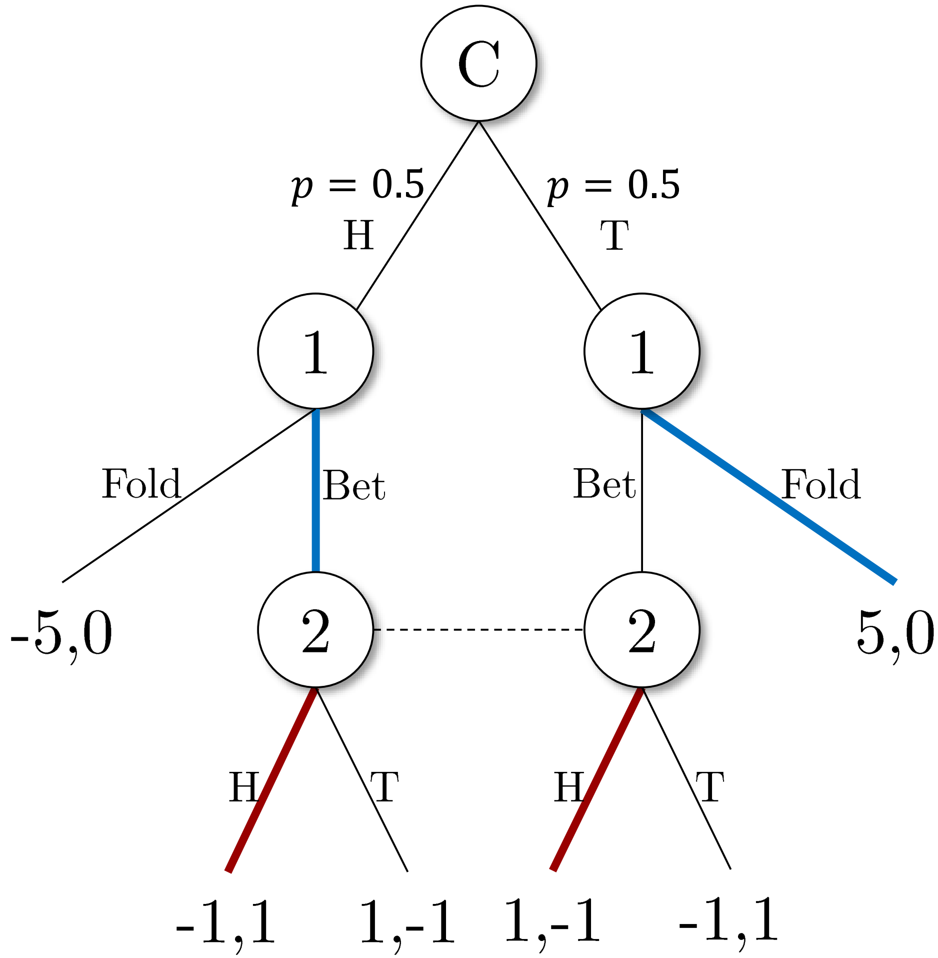


Example: Spaceship game

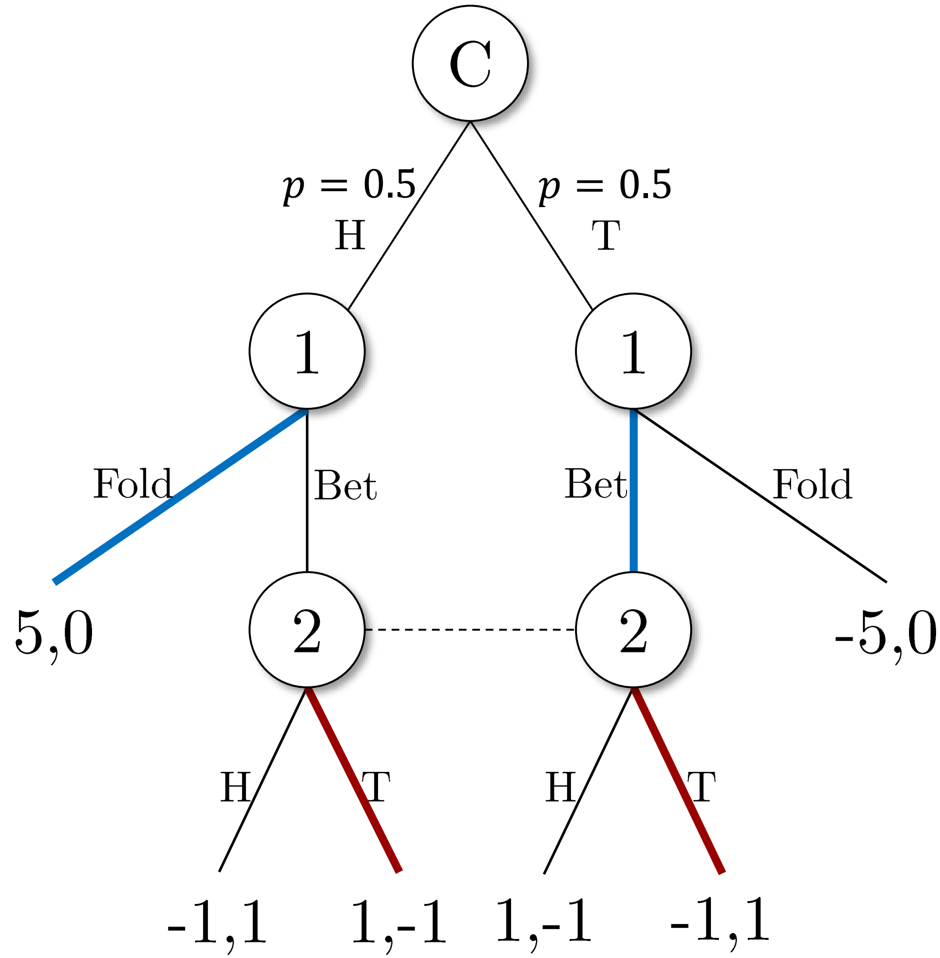
- Poll 1: In Nash equilibrium, what is the expected payoff of player 1?
  1. 0.5
  2. 1
  3. 1.5
  4. 2
  5. 2.5



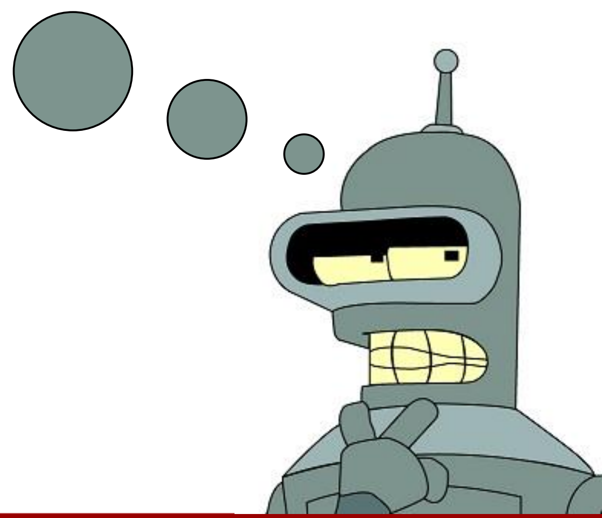
Example: Spaceship game



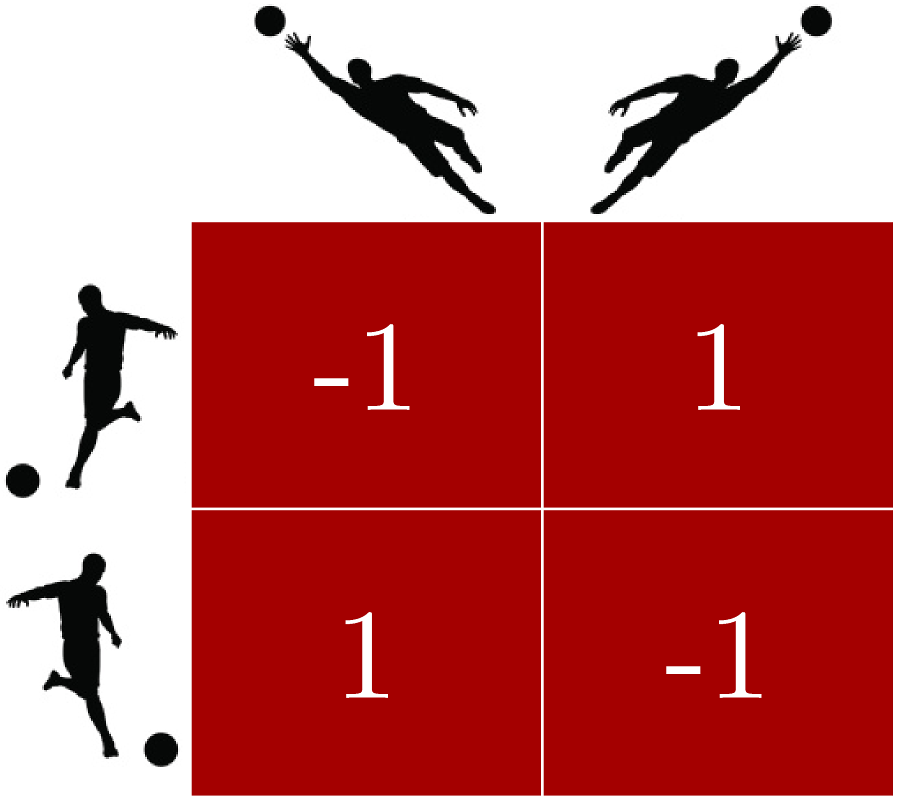
Example: Spaceship game



Impossible to compute the optimal strategy of a subgame in isolation, unlike prefect info games!



Interlude: zero-sum games



Interlude: zero-sum games

- Maximin (randomized) strategy of player 1 maximizes the worst-case expected payoff
- In the penalty shot game, optimal strategy for both players is playing $\left(\frac{1}{2}, \frac{1}{2}\right)$
- In the game below, if shooter uses $(p, 1-p)$:
  - Jump left: $-\frac{p}{2} + 1 - p = 1 - \frac{3}{2}p$
  - Jump right: $p - 1 + p = 2p - 1$
  - Maximize $\min\{1 - \frac{3}{2}p, 2p - 1\}$ over p

$-\frac{1}{2}$	1
1	-1

Interlude: zero-sum games

- Denote the reward of player 1 from strategies (s_1, s_2) by $R(s_1, s_2)$
- Maximin strategy is computed via LP:

$$
\begin{aligned}
&\max\ w \\
&\text{s.t.}\quad \forall s_2 \in S,\ \textstyle\sum_{s_1 \in S} p(s_1) R(s_1, s_2) \geq w \\
&\qquad\ \sum_{s_1 \in S} p(s_1) = 1 \\
&\qquad\ \forall s_1 \in S, p(s_1) \geq 0
\end{aligned}
$$

Interlude: The Minimax Theorem

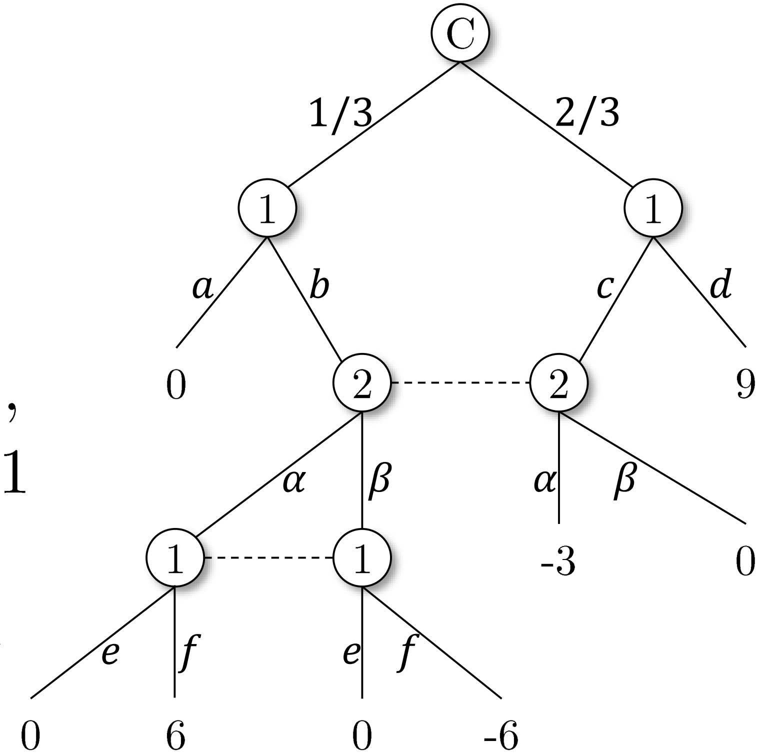
- Theorem [von Neumann 1928]: Every 2-player zero-sum game has a unique value v such that:
  - Player 1 can guarantee value at least v
  - Player 2 can guarantee loss at most v
- Poll 2: How many Nash equilibrium payoffs do zero-sum games have?
  1. At most one
  2. At least one
  3. Exactly one

Solving imperfect info games

- Focus on zero-sum games (such as poker)
- We just saw that linear programming solves normal-form, zero-sum games in polynomial time
- But size of the normal-form game is exponential in the extensive-form representation!
- Work directly on extensive-form game

Solving imperfect info games

- Player 1 constraints are linear:
  - $p_a + p_b = 1$
  - $p_c + p_d = 1$
  - $p_e + p_f = 1$
  - $\forall x, p_x \geq 0$
- Fix a strategy q_α, q_β for player 2, then the best response of player 1 is:
  $\max_{\boldsymbol{p}} \; 2p_b q_\alpha p_f - 2p_b q_\beta p_f - 2p_c q_\alpha + 6p_d$
  which leads to a nonconvex problem!

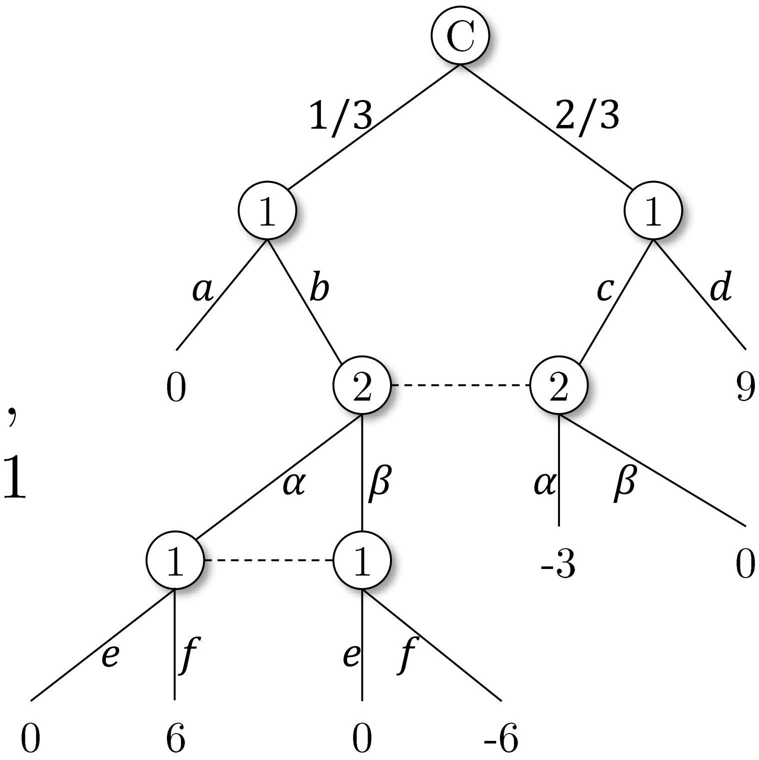


Sequence form

- Insight: last action taken by a player is the same for all nodes in an information set
  - Perfect recall: A player never forgets something he knew in the past
  - This is a restriction on the structure of the game
- Introduce scaled probability variables p'_x
- Information set constraint: $\sum_{x \in A_I} p'_x = p'_y$, where A_I is the set of actions in information set I, and y is the last action before reaching I
- To recover probabilities, set $p_x = p'_x / p'_y$

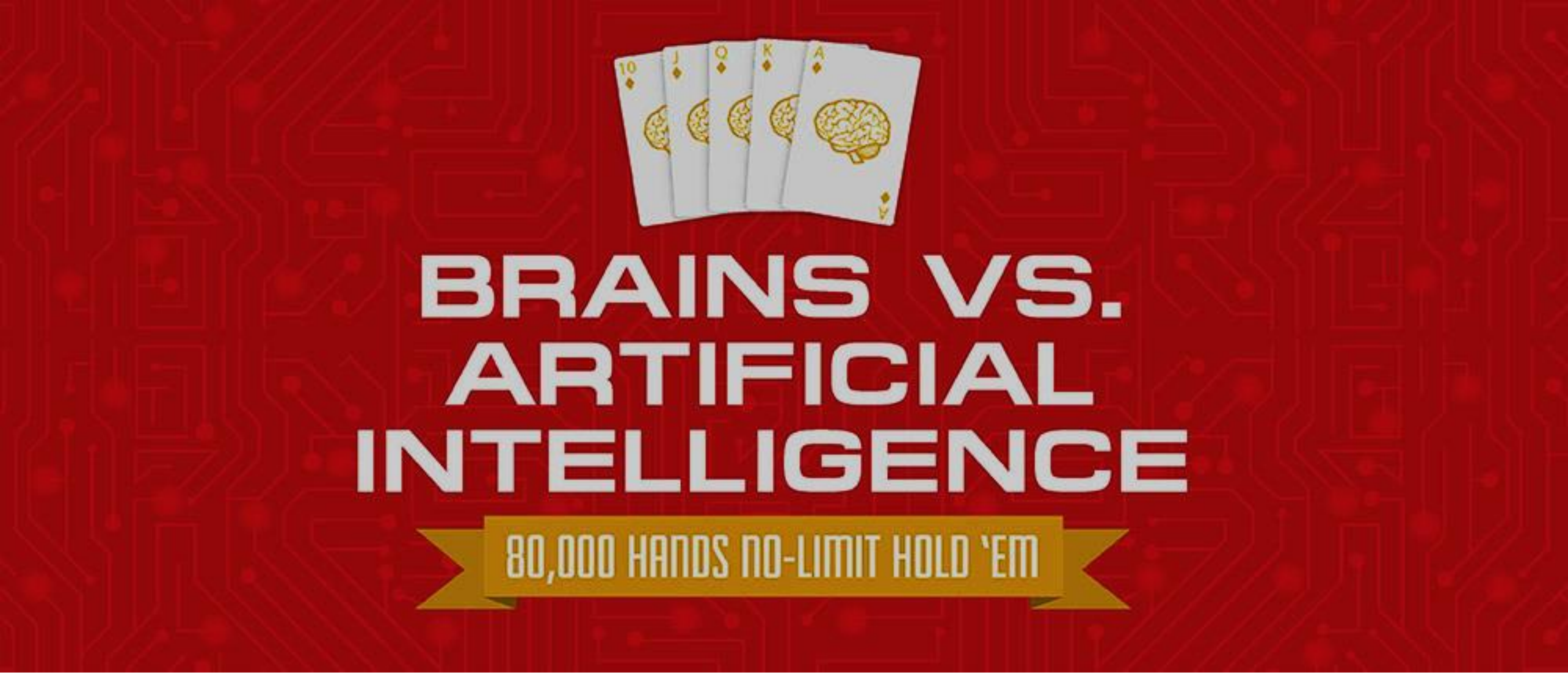
Sequence form

- Player 1 constraints are linear:
  - $p'_a + p'_b = 1$
  - $p'_c + p'_d = 1$
  - $p'_e + p'_f = p'_b$
  - $\forall x, p'_x \geq 0$
- Fix a strategy q_α, q_β for player 2, then the best response of player 1 is:
  $\max_{\boldsymbol{p}} 2q_\alpha p'_f - 2q_\beta p'_f - 2p'_c q_\alpha + 6p'_d$
  which is linear!



Sequence form

- We showed how to compute a best response for a fixed opponent strategy
- Fact: Using "LP duality", we can compute best responses for both players simultaneously
- Fact: This gives a method for computing optimal strategies
- Used to compute optimal strategies for Rhode Island Hold'em poker, which has roughly 10^8 nodes [Gilpin and Sandholm 2007]
- But No Limit Texas Hold'em has 10^{167} nodes



April 24–May 8, 2015, at Rivers Casino, Pittsburgh
The first time a computer program has played human pros in a heads-up, no-limit poker game

Summary

- Terminology:
  - Extensive-form game
  - Subgame perfect equilibrium
  - Imperfect information, information set
  - Perfect recall
- Algorithms:
  - Solving zero-sum games via LP
  - Sequence form-based approach to solving imperfect information games