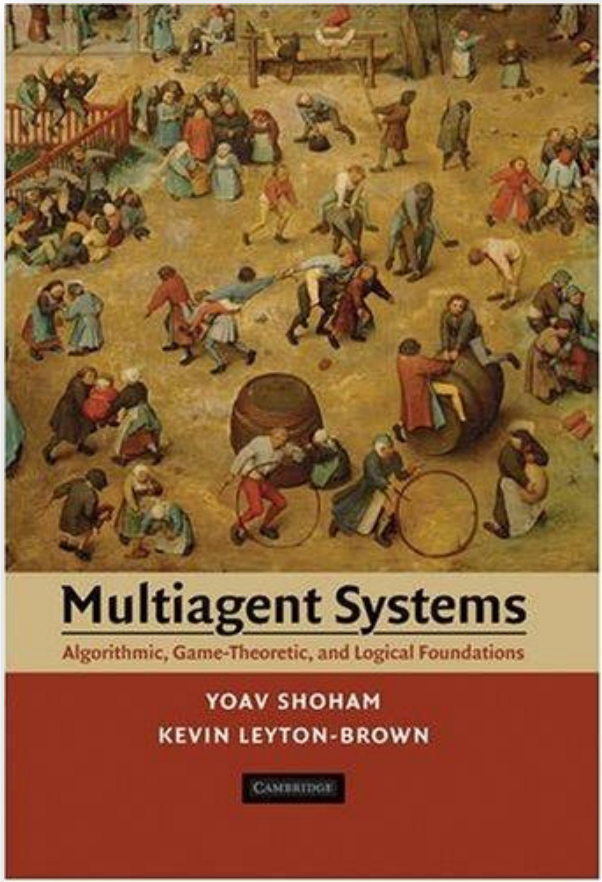
CMU 15-781

Lecture 22:
Game Theory I

Teachers:
Emma Brunskill
Ariel Procaccia (this time)

MULTIAGENT SYSTEMS



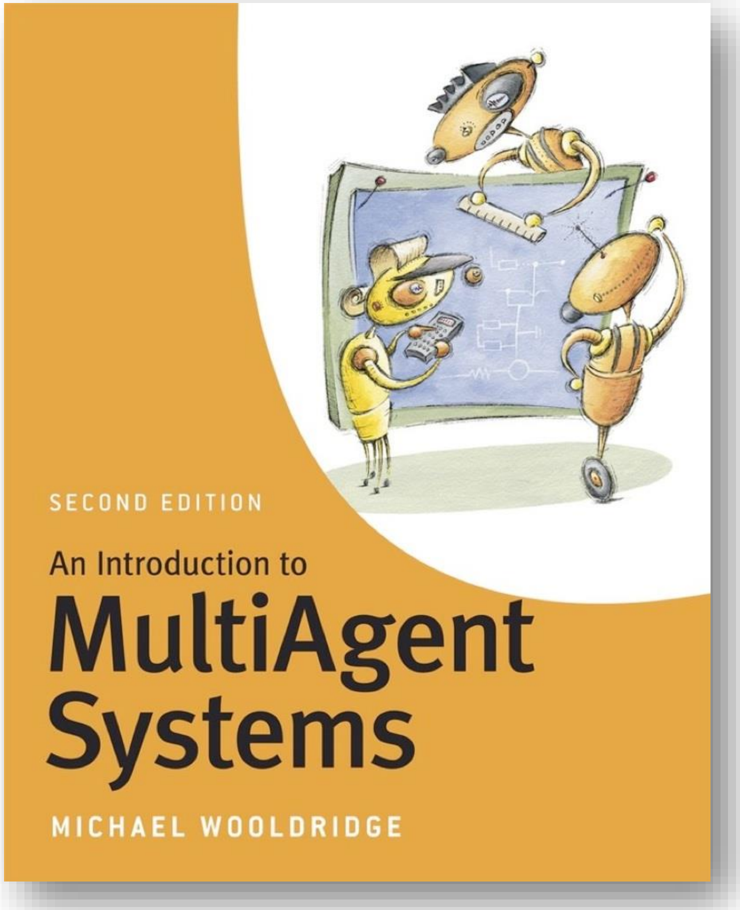
Multiagent Systems

Algorithmic, Game-Theoretic, and Logical Foundations

YOAV SHOHAM

KEVIN LEYTON-BROWN

CAMBRIDGE



SECOND EDITION

An Introduction to

MultiAgent Systems

MICHAEL WOOLDRIDGE

MULTIAGENT SYSTEMS

Chapters of the Shoham and Leyton-Brown book:

1. Distributed constraint satisfaction
2. Distributed optimization
3. Games in normal form
4. Computing solution concepts of normal-form games
5. Games with sequential actions
6. Beyond the normal and extensive forms
7. Learning and teaching
8. Communication
9. Social choice
10. Mechanism design
11. Auctions
12. Coalitional game theory
13. Logics of knowledge and belief
14. Probability, dynamics, and intention

Legend:

🟥 "Game theory"

■ Not "game theory"

MULTIAGENT SYSTEMS

Mike Wooldridge's 2014 publications:

2014

- [j111] Anthony Hunter, Simon Parsons, Michael Wooldridge: **Measuring Inconsistency in Multi-Agent Systems.** KI 28(3): 169-178 (2014)
- ➡ [j110] John Grant, Sarit Kraus, Michael Wooldridge, Inon Zuckerman: **Manipulating Games by Sharing Information.** Studia Logica 102(2): 267-295 (2014)
- [c191] Javier Morales, Maite López-Sánchez, Juan Antonio Rodriguez-Aguilar, Michael Wooldridge, Wamberto Vasconcelos: **Minimality and simplicity in the on-line automated synthesis of normative systems.** AAMAS 2014: 109-116
- ➡ [c190] Oskar Skibski, Tomasz P. Michalak, Talal Rahwan, Michael Wooldridge: **Algorithms for the shapley and myerson values in graph-restricted games.** AAMAS 2014: 197-204
- ➡ [c189] Liat Sless, Noam Hazon, Sarit Kraus, Michael Wooldridge: **Forming coalitions and facilitating relationships for completing tasks in social networks.** AAMAS 2014: 261-268
- ➡ [c188] Enrico Marchioni, Michael Wooldridge: **Lukasiewicz games.** AAMAS 2014: 837-844
- ➡ [c187] Paul Harrenstein, Paolo Turrini, Michael Wooldridge: **Hard and soft equilibria in boolean games.** AAMAS 2014: 845-852
- ➡ [c186] S. Shaheen Fatima, Michael Wooldridge: **Majority bargaining for resource division.** AAMAS 2014: 1393-1394
- ➡ [c185] Shaheen Fatima, Tomasz P. Michalak, Michael Wooldridge: **Power and welfare in noncooperative bargaining for coalition structure formation.** AAMAS 2014: 1439-1440
- [c184] Javier Morales, Iosu Mendizabal, David Sanchez-Pinsach, Maite López-Sánchez, Michael Wooldridge, Wamberto Vasconcelos: **NormLab: a framework to support research on norm synthesis.** AAMAS 2014: 1697-1698
- ➡ [c183] Julian Gutierrez, Michael Wooldridge: **Equilibria of concurrent games on event structures.** CSL-LICS 2014: 46
- ➡ [c182] S. Shaheen Fatima, Michael Wooldridge: **Multilateral Bargaining for Resource Division.** ECAI 2014: 309-314
- ➡ [c181] S. Shaheen Fatima, Tomasz P. Michalak, Michael Wooldridge: **Bargaining for Coalition Structure Formation.** ECAI 2014: 315-320
- ➡ [c180] Piotr L. Szczepanski, Tomasz P. Michalak, Michael Wooldridge: **A Centrality Measure for Networks With Community Structure Based on a Generalization of the Owen Value.** ECAI 2014: 867-872
- ➡ [c179] Julian Gutierrez, Paul Harrenstein, Michael Wooldridge: **Reasoning about Equilibria in Game-Like Concurrent Systems.** KR 2014

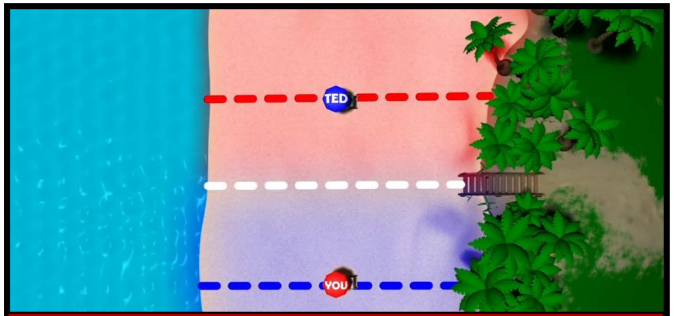
NORMAL-FORM GAME

- A game in normal form consists of:
    - Set of players $N = \{1, \dots, n\}$
    - Strategy set S
    - For each $i \in N$, utility function $u_i\colon S^n \rightarrow \mathbb{R}$: if each j $\in N$ plays the strategy $s_j \in S$, the utility of player i is $u_i(s_1, \dots, s_n)$
- Next example created by taking screenshots of http://youtu.be/jILgxeNBK_8

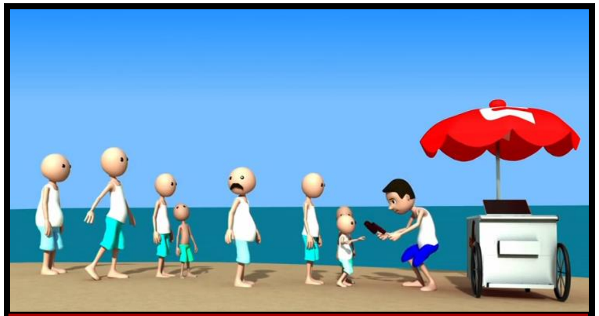
Selling ice cream at the beach.

One day your cousin Ted shows up.

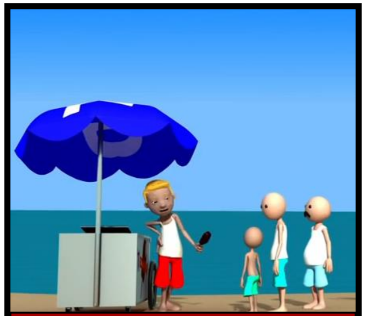
His ice cream is identical!



You split the beach in half; you set up at 1/4.

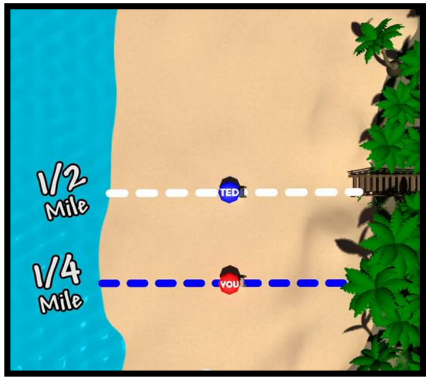


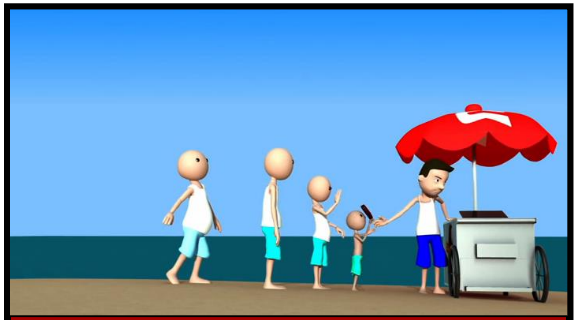
50% of the customers buy from you.



50% buy from Teddy.

One day Teddy sets up at the 1/2 point!





Now you serve only 37.5%!

THE ICE CREAM WARS

- $N = \{1,2\}$
- $S = [0,1]$
- $u_i(s_i, s_j) = \begin{cases} \frac{s_i+s_j}{2}, & s_i < s_j \\ 1 - \frac{s_i+s_j}{2}, & s_i > s_j \\ \frac{1}{2}, & s_i = s_j \end{cases}$
- To be continued...

THE PRISONER'S DILEMMA

- Two men are charged with a crime
- They are told that:
    - If one rats out and the other does not, the rat will be freed, other jailed for nine years
    - If both rat out, both will be jailed for six years
- They also know that if neither rats out, both will be jailed for one year

THE PRISONER'S DILEMMA

	Cooperate	Defect
Cooperate	-1,-1	-9,0
Defect	0,-9	-6,-6

What would you do?

UNDERSTANDING THE DILEMMA

- Defection is a dominant strategy
- But the players can do much better by cooperating
- Related to the tragedy of the commons

IN REAL LIFE

- Presidential elections
  - Cooperate = positive ads
  - Defect = negative ads
- Nuclear arms race
  - Cooperate = destroy arsenal
  - Defect = build arsenal
- Climate change
  - Cooperate = curb CO_2 emissions
  - Defect = do not curb

ON TV

http://youtu.be/S0qjK3TWZE8

THE PROFESSOR'S DILEMMA

		Class	
		Listen	Sleep
Professor	Make effort	10^6,10^6	-10,0
	Slack off	0,-10	0,0

Dominant strategies?

NASH EQUILIBRIUM

- Each player's strategy is a best response to strategies of others
- Formally, a Nash equilibrium is a vector of strategies $s = (s_1 \ldots, s_n) \in S^n$ such that $\forall i \in N, \forall s_i' \in S, u_i(s) \geq u_i(s_i', s_{-i})$

NASH EQUILIBRIUM

- **Poll 1:** How many Nash equilibria does the Professor's Dilemma have?

1. 0
2. 1
[✓ 3.] 2
4. 3

	Listen	Sleep
Make effort	10^6,10^6	-10,0
Slack off	0,-10	0,0

NASH EQUILIBRIUM

http://youtu.be/CemLiSI5ox8

RUSSEL CROWE WAS WRONG

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Turing's Invisible Hand

Computation, Economics, and Game Theory

« STOC Submissions: message from the PC Chair

Russell Crowe was wrong

October 30, 2012 by Ariel Procaccia | Edit

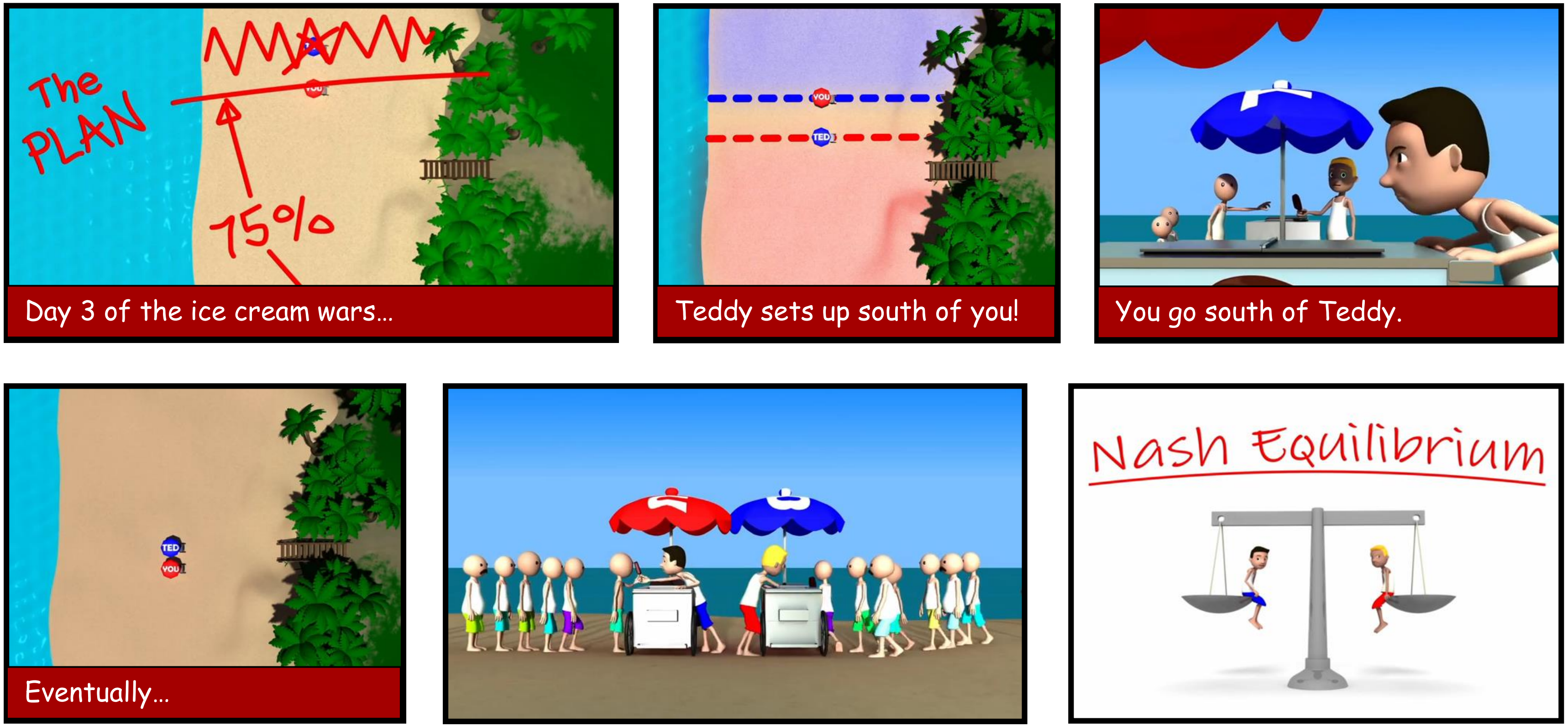
Yesterday I taught the first of five algorithmic economics lectures in my undergraduate AI course. This lecture just introduced the basic concepts of game theory, focusing on Nash equilibria. I was contemplating various way making the lecture more lively, and it occurred to me that I could stand on t shoulders of giants. Indeed, didn't Russell Crowe already explain Nash's ideas in A Beautiful Mind, complete with a 1940's-style male chauvinistic example?

The first and last time I watched the movie was when it was released in 2001. Back then I was an undergrad freshman, working for 20+ hours a week on the programming exercises of Hebrew U's Intro to CS course, which was taught by some guy called Noam Nisan. I didn't know anything about game theory, and Crowe's explanation made a lot of sense at the time.

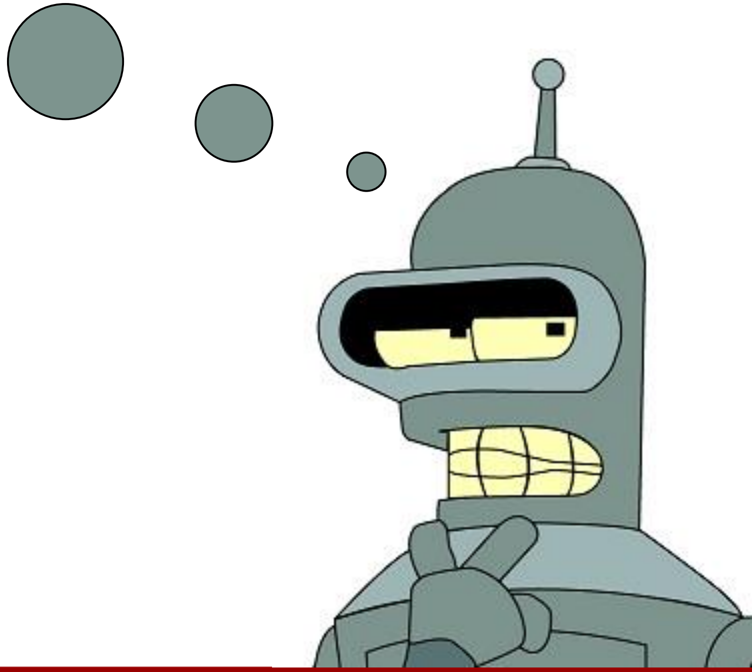
I easily found the relevant scene on youtube. In the scene, Nash's friends are trying to figure out how to seduce a beautiful blonde and her less beautiful friends. Then Nash/Crowe has an epiphany. The hubbub of the seedy Princeton bar is drowned by inspirational music, as Nash announces:

January 2012
December 2011
November 2011
October 2011
September 2011
August 2011
July 2011
June 2011

END OF THE ICE CREAM WARS



This is why competitors open their stores next to one another!



ROCK-PAPER-SCISSORS

	R	P	S
R	0,0	-1,1	1,-1
P	1,-1	0,0	-1,1
S	-1,1	1,-1	0,0

Nash equilibrium?

MIXED STRATEGIES

- A mixed strategy is a probability distribution over (pure) strategies
- The mixed strategy of player $i \in N$ is x_i, where

$$x_i(s_i) = \Pr[i \text{ plays } s_i]$$

- The utility of player $i \in N$ is

$$u_i(x_1, \dots, x_n) = \sum_{(s_1,\dots,s_n)\in S^n} u_i(s_1, \dots, s_n) \cdot \prod_{j=1}^{n} x_j(s_j)$$

EXERCISE: MIXED NE

- **Exercise:** player 1 plays $\left(\frac{1}{2},\frac{1}{2},0\right)$, player 2 plays $\left(0,\frac{1}{2},\frac{1}{2}\right)$. What is u_1?
- **Exercise:** Both players play $\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$. What is u_1?

	R	P	S
R	0,0	-1,1	1,-1
P	1,-1	0,0	-1,1
S	-1,1	1,-1	0,0

Exercise: Mixed NE

- Poll 2: Which is a NE?

1. $\left(\left(\frac{1}{2},\frac{1}{2},0\right),\left(\frac{1}{2},\frac{1}{2},0\right)\right)$
2. $\left(\left(\frac{1}{2},\frac{1}{2},0\right),\left(\frac{1}{2},0,\frac{1}{2}\right)\right)$
[✓ 3.] $\left(\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right),\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)\right)$
4. $\left(\left(\frac{1}{3},\frac{2}{3},0\right),\left(\frac{2}{3},0,\frac{1}{3}\right)\right)$

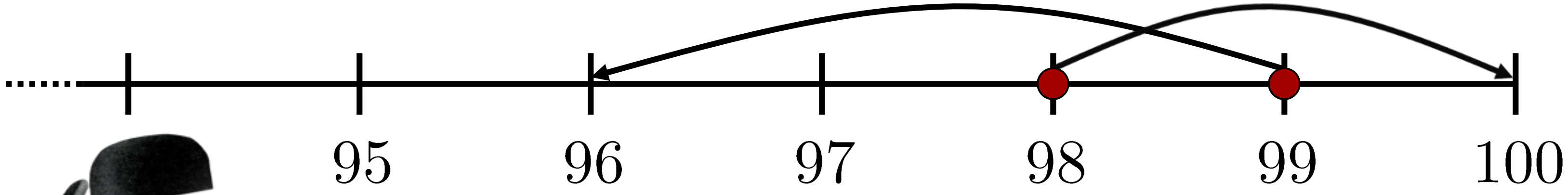
	R	P	S
R	0,0	-1,1	1,-1
P	1,-1	0,0	-1,1
S	-1,1	1,-1	0,0

NASH’S THEOREM

- **Theorem [Nash, 1950]:** In any (finite) game there exists at least one (possibly mixed) Nash equilibrium

DOES NE MAKE SENSE?

- Two players, strategies are $\{2, \dots, 100\}$
- If both choose the same number, that is what they get
- If one chooses s, the other t, and $s < t$, the former player gets $s + 2$, and the latter gets $s - 2$
- Poll 3: What would you choose?



SUMMARY

- Terminology:
    - Normal-form game
    - Nash equilibrium
    - Mixed strategies
- Nobel-prize-winning ideas:
    - Nash equilibrium ☺