CMU 15-781

Lecture 20:
Social Choice

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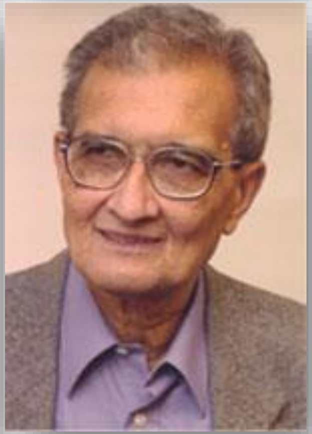
Social choice theory

- A mathematical theory that deals with aggregation of individual preferences
- Origins in ancient Greece
- Formal foundations: 18th Century (Condorcet and Borda)
- 19th Century: Charles Dodgson
- 20th Century: Nobel prizes to Arrow and Sen



The voting model

- Set of voters $N = \{1, \ldots, n\}$
- Set of alternatives A; denote $|A| = m$
- Each voter has a ranking over the alternatives
- Preference profile = collection of all voters' rankings

1	2	3
a	c	b
b	a	c
c	b	a

Voting rules

- Voting rule = function from preference profiles to alternatives that specifies the winner of the election
- Plurality
    - Each voter awards one point to top alternative
    - Alternative with most points wins
    - Used in almost all political elections

More voting rules

- Borda count
  - Each voter awards $m - k$ points to alternative ranked k'th
  - Alternative with most points wins
  - Proposed in the 18th Century by the chevalier de Borda
  - Used for elections to the national assembly of Slovenia
  - Similar to rule used in the Eurovision song contest

Lordi, Eurovision 2006 winners

More voting rules

- x beats y in a pairwise election if the majority of voters prefer x to y
- Plurality with runoff
  - First round: two alternatives with highest plurality scores survive
  - Second round: pairwise election between these two alternatives

More voting rules

- Single Transferable vote (STV)
  - $m - 1$ rounds
  - In each round, alternative with least plurality votes is eliminated
  - Alternative left standing is the winner
  - Used in Ireland, Malta, Australia, and New Zealand (and Cambridge, MA)

STV: EXAMPLE

2 voters	2 voters	1 voter
a	b	c
b	a	d
c	d	b
d	c	a

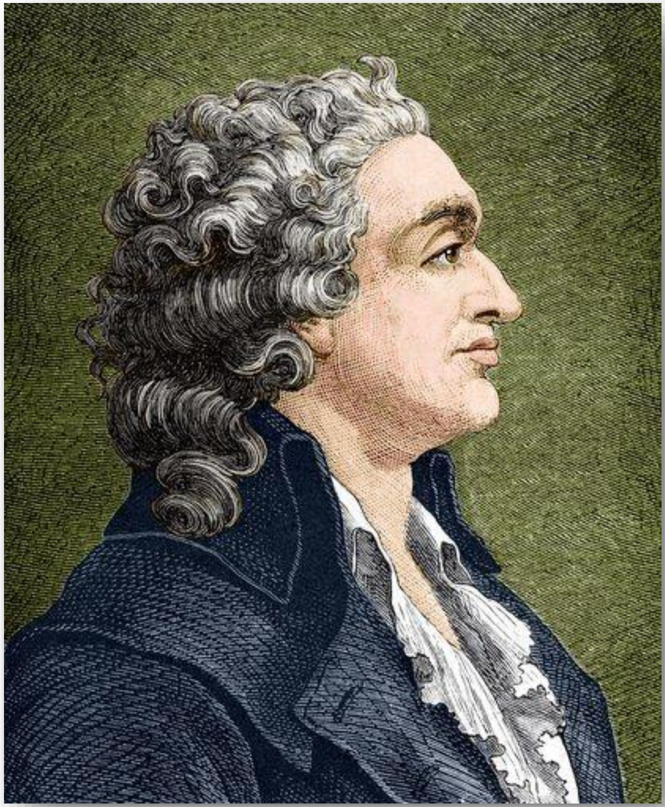
2 voters	2 voters	1 voter
a	b	c
b	a	b
c	c	a

2 voters	2 voters	1 voter
a	b	b
b	a	a

2 voters	2 voters	1 voter
b	b	b

Marquis de Condorcet

- 18th Century French Mathematician, philosopher, political scientist
- One of the leaders of the French revolution
- After the revolution became a fugitive
- His cover was blown and he died mysteriously in prison



Condorcet winner

- Recall: x beats y in a pairwise election if a majority of voters rank x above y
- Condorcet winner beats every other alternative in pairwise election
- Condorcet paradox = cycle in majority preferences

1	2	3
a	c	b
b	a	c
c	b	a

Condorcet consistency

- Condorcet consistency = select a Condorcet winner if one exists
- Poll 1: Which rule is Condorcet consistent?
  1. Plurality
  2. Borda count
  3. Both
  4. Neither

More voting rules

- Copeland: Alternative's score is #alternatives it beats in pairwise elections
- Why does Copeland satisfy the Condorcet criterion?
  - If x is a Condorcet winner, score $= m - 1$
  - Otherwise, score $< m - 1$

Dodgson's Rule

- Dodgson score of x = the number of swaps between adjacent alternatives needed to make x a Condorcet winner
- Dodgson's rule: select alternative that minimizes Dodgson score
- The problem of computing the Dodgson score is NP-complete!

Awesome example

- Plurality: a
- Borda: b
- Condorcet winner: c
- STV: d
- Plurality with runoff: e

33 voters	16 voters	3 voters	8 voters	18 voters	22 voters
a	b	c	c	d	e
b	d	d	e	e	c
c	c	b	b	c	b
d	e	a	d	b	d
e	a	e	a	a	a

Condorcet strikes again

- For Condorcet [1785], the purpose of voting is not merely to balance subjective opinions; it is a collective quest for the truth
- Enlightened voters try to judge which alternative best serves society
- For $m = 2$ the majority opinion will very likely be correct
- Realistic in trials by jury or the pooling of expert opinions — or in human computation!

EXAMPLE: ETERNA

- Developed at CMU (Adrien Treuille) and Stanford
- Choose 8 RNA designs to synthesize
- Some designs are truly more stable than others
- The goal of voting is to compare the alternatives by true quality

Condorcet's noise model

- True ranking of the alternatives
- Voting pairwise on alternatives, each comparison is correct with prob. $p > 1/2$
- Results are tallied in a voting matrix

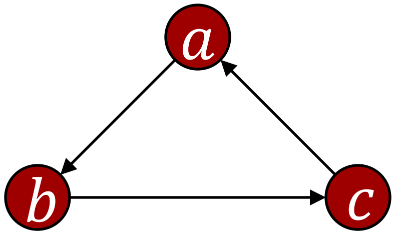
	a	b	c
a	-	8	6
b	5	-	11
c	7	2	-

- Poll 2: What is the Borda score of alternative b?
  1. 5
  2. 8
  3. 10
  4. 16

CONDORCET'S 'SOLUTION'

- Condorcet's goal: find "the most probable" ranking
- Condorcet suggested: take the majority opinion for each comparison; if a cycle forms, "successively delete the comparisons that have the least plurality"
- In example, we delete $c \succ a$ to get $a \succ b \succ c$

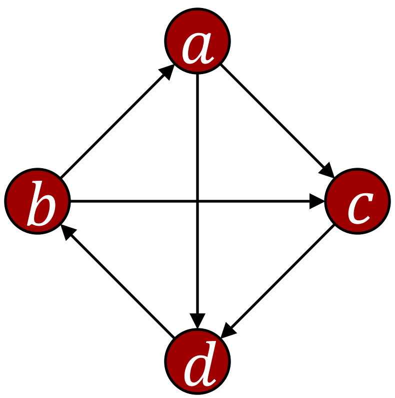
	a	b	c
a	-	8	6
b	5	-	11
c	7	2	-



Condorcet's 'solution'

- With four alternatives we get ambiguities
- In example, order of strength is $c \succ d$, $a \succ d$, $b \succ c$, $a \succ c$, $d \succ b$, $b \succ a$
- Delete $b \succ a \Rightarrow$ still cycle
- Delete $d \succ b \Rightarrow$ either a or b could be top-ranked

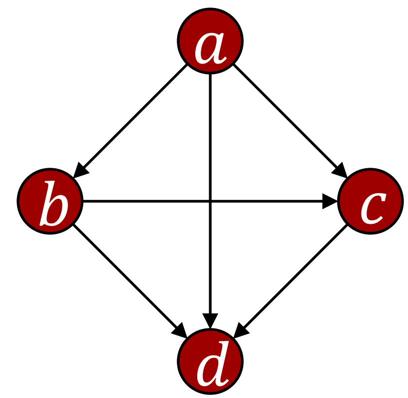
	a	b	c	d
a	-	12	15	17
b	13	-	16	11
c	10	9	-	18
d	8	14	7	-



Condorcet's 'solution'

- Did Condorcet mean we should reverse the weakest comparisons?
- Reverse $b \succ a$ and $d \succ b \Rightarrow$ we get $a \succ b \succ c \succ d$, with 89 votes
- $b \succ a \succ c \succ d$ has 90 votes (only reverse $d \succ b$)

	a	b	c	d
a	-	12	15	17
b	13	-	16	11
c	10	9	-	18
d	8	14	7	-



Exasperation?

- "The general rules for the case of any number of candidates as given by Condorcet are stated so briefly as to be hardly intelligible . . . and as no examples are given it is quite hopeless to find out what Condorcet meant" [Black 1958]
- "The obscurity and self-contradiction are without any parallel, so far as our experience of mathematical works extends ... no amount of examples can convey an adequate impression of the evils" [Todhunter 1949]

Young's solution

- M = matrix of votes
- Suppose true ranking is $a \succ b \succ c$; prob of observations $\Pr[M \mid \succ]$:
  $$\binom{13}{8} p^8(1-p)^5 \cdot \binom{13}{6} p^6(1-p)^7 \cdot \binom{13}{11} p^{11}(1-p)^2$$
- For $a \succ c \succ b$, $\Pr[M \mid \succ]$ is
  $$\binom{13}{8} p^8(1-p)^5 \cdot \binom{13}{6} p^6(1-p)^7 \cdot \binom{13}{2} p^2(1-p)^{11}$$
- Coefficients are identical, so
  $\Pr[M \mid \succ] \propto p^{\#agree}(1-p)^{\#disagree}$

	a	b	c
a	-	8	6
b	5	-	11
c	7	2	-

Young's solution

- $\Pr[\succ |M] = \frac{\Pr[M|\succ]\cdot\Pr[\succ]}{\Pr[M]}$
- Assume uniform prior over $\succ$, $\Pr[\succ] = \frac{1}{m!}$
- Must maximize $\Pr[M| \succ]$
- The optimal rule maximizes #agreements with voters on pairs of candidates
- This rule is called the Kemeny rule

The Kemeny rule

- Theorem [Bartholdi, Tovey, Trick 1989]: Computing the Kemeny ranking is NP-hard
- Typically formulated as an IP: for every $a, b \in A$, $x_{(a,b)} = 1$ iff a is ranked above b, and

$$w_{(a,b)} = |\{i \in N \mid a \succ_i b\}|$$

The Kemeny rule

Maximize $\sum_{(a,b)} x_{(a,b)} w_{(a,b)}$

Subject to

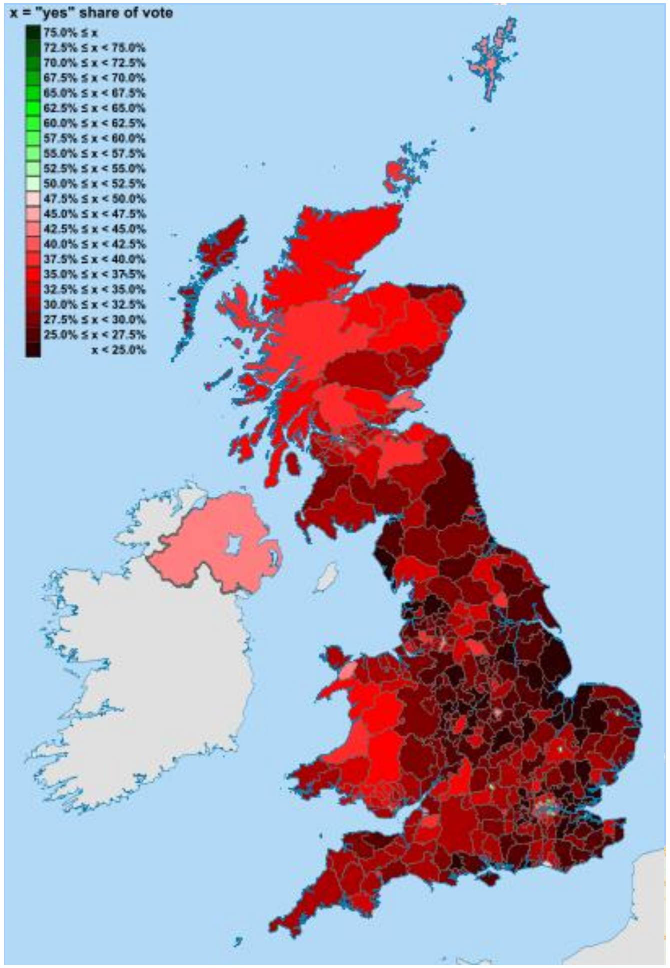
For all distinct $a, b \in A, x_{(a,b)} + x_{(b,a)} = 1$

For all distinct $a, b, c \in A, x_{(a,b)} + x_{(b,c)} + x_{(c,a)} \leq 2$

For all distinct $a, b \in A, x_{(a,b)} \in \{0,1\}$

Is social choice practical?

- UK referendum: Choose between plurality and STV as a method for electing MPs
- Academics agreed STV is better…
- … but STV seen as beneficial to the hated Nick Clegg
- Hard to change political elections!



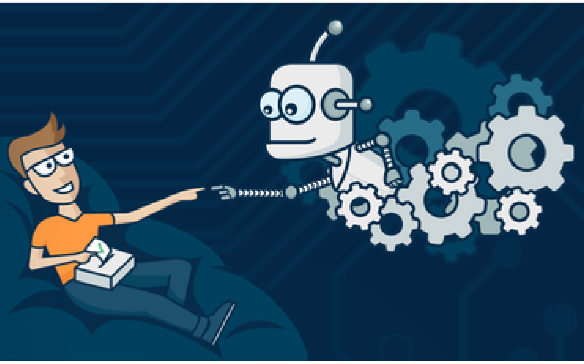
Computational social choice

- However:
  - in human computation systems...
  - in online voting systems...

  the designer is free to employ any voting rule!
- Computational social choice focuses on positive results through computational thinking

AI-Driven Decisions

RoboVote is a free service that helps users combine their preferences or opinions into optimal decisions. To do so, RoboVote employs state-of-the-art voting methods developed in artificial intelligence research.
Learn More



Poll Types

RoboVote offers two types of polls, which are tailored to different scenarios; it is up to users to indicate to RoboVote which scenario best fits the problem at hand.

Objective Opinions

In this scenario, some alternatives are objectively better than others, and the opinion of a participant reflects an attempt to estimate the correct order. RoboVote's proposed outcome is guaranteed to be as close as possible — based on the available information — to the best outcome. Examples include deciding which product prototype to develop, or which company to invest in, based on a metric such as projected revenue or market share. Try the demo.

Subjective Preferences

In this scenario participants' preferences reflect their subjective taste; RoboVote proposes an outcome that mathematically makes participants as happy as possible overall. Common examples include deciding which restaurant or movie to go to as a group, which destination to choose for a family vacation, or whom to elect as class president. Try the demo.

Ready to get started?

CREATE A POLL

SUMMARY

- Terminology:
  - Voting rules: plurality, Borda, plurality with runoff, STV, Copeland, Dodgson
  - The Condorcet noise model
  - The Kemeny rule
- Big ideas:
  - Voting rules as MLEs
  - When we build voting systems, we are not constrained by politics and tradition!