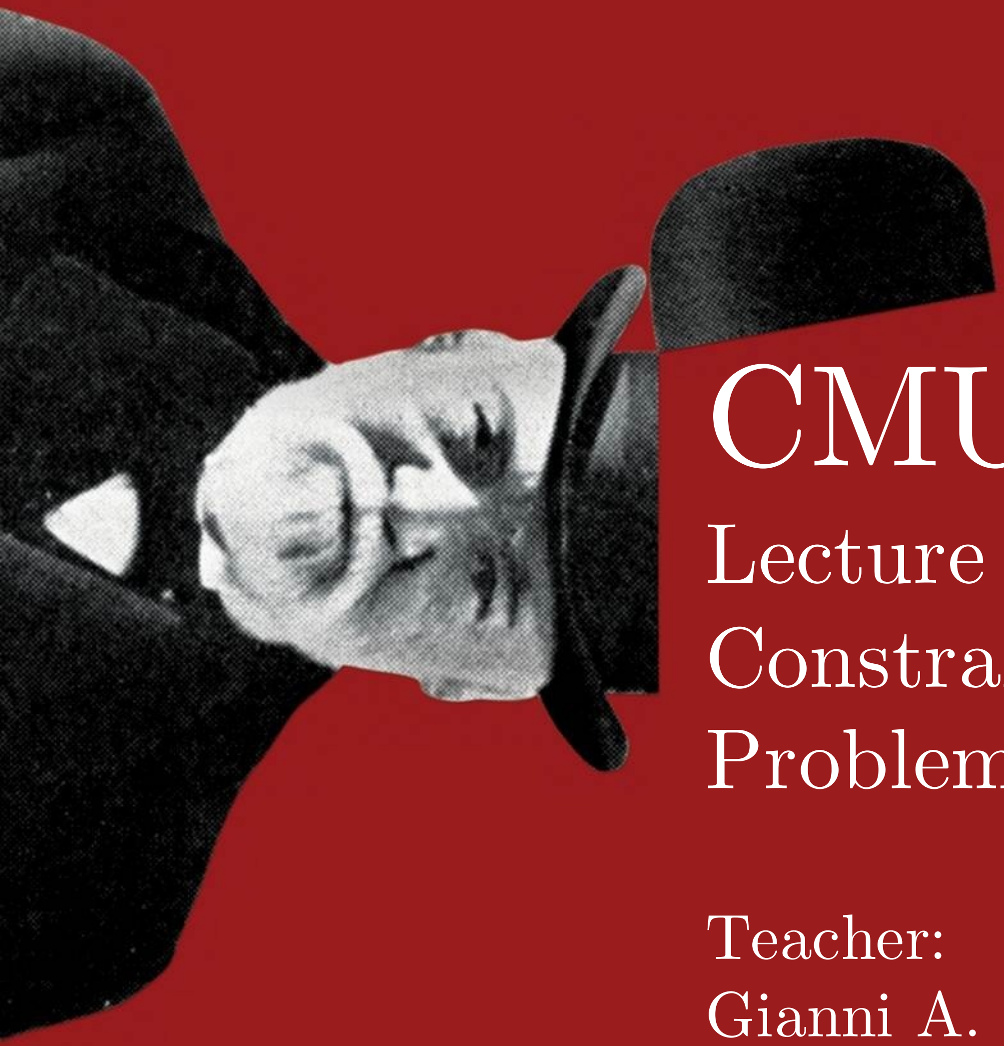

CMU 15-781

Lecture 3: Constraint Satisfaction Problems (CSPs)

Teacher:
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Overview

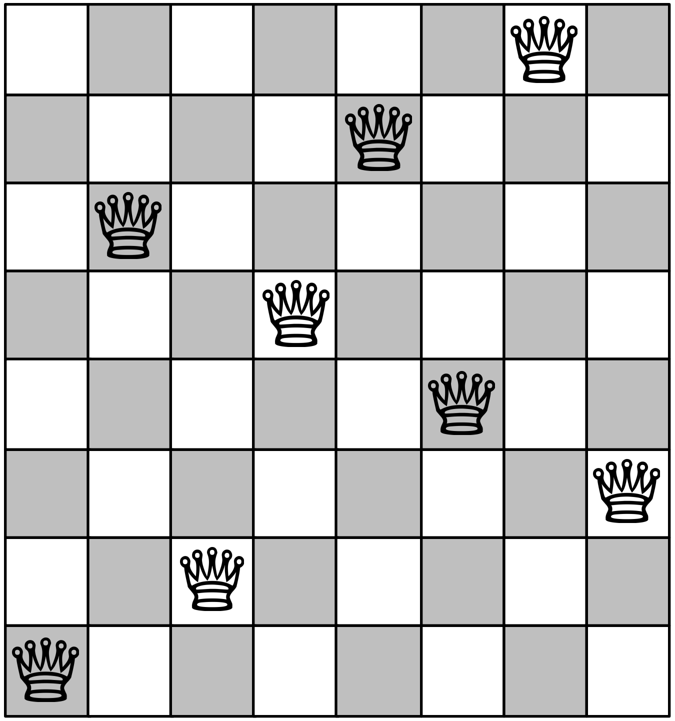
- **Definitions, toy and real-world examples**
- Basic algorithms for solving CSPs
- Pruning space through propagating information

Constraint Satisfaction Problems (CSP)

- Set of decision *Variables*: $V = \{V_1,..,V_N\}$
- *Domains*: Sets of D_i possible values for each variable Vi
- Set of *Constraints*: $C = \{C_1,..,C_K\}$ restricting the values the variables can simultaneously take
- A constraint consists of:
  - variable tuple
  - list of possible values for tuple (ex.$[(V_2, V_3),\{(R,B),(R,G)\}]$)
  - Or functional relation (ex. $V_2 \neq V_3$, $V_1 > V_4 + 5$)
- Allows useful general-purpose algorithms with more power than standard search algorithms

Example: N-Queens

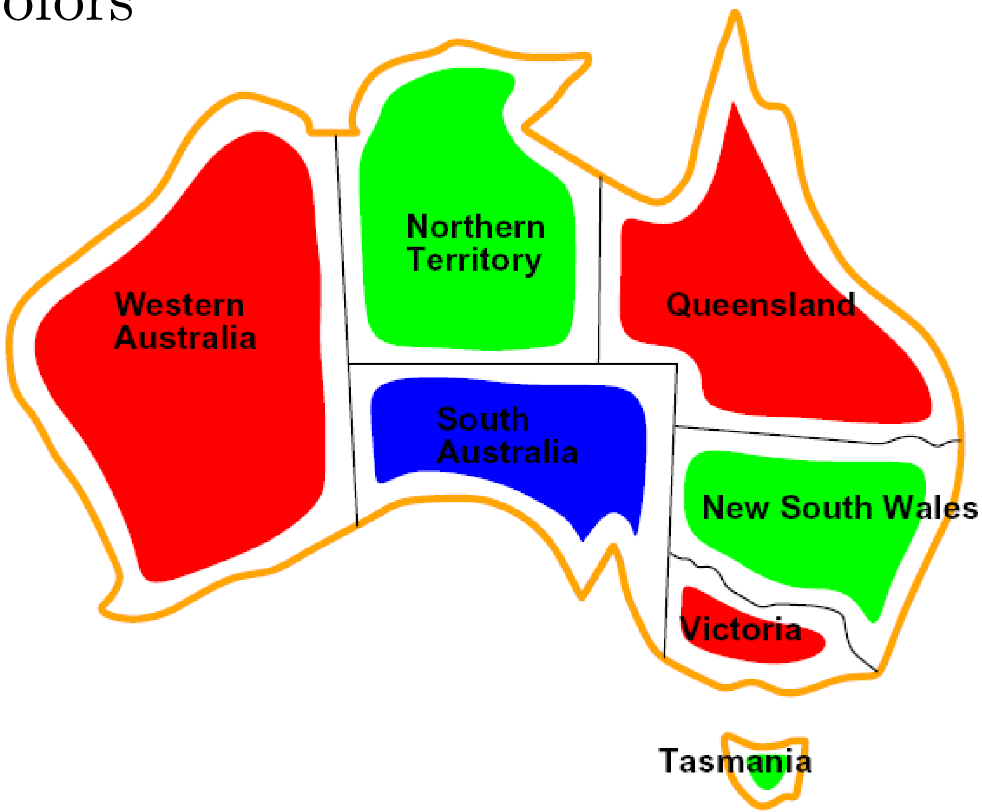
- Variables:
  - Q_i position of queen in column i
- Domains:
  - {1, ..., 8}
- Constraints:
  - No queen attack each other
  - $Q_i = k \Rightarrow Q_j \neq k, \forall\ j = 1,..8,\ j \neq i$
  - Similar constraints for diagonals



Alternative formulation?

Example: Map Coloring

Given n different colors, color a map so that adjacent areas are different colors



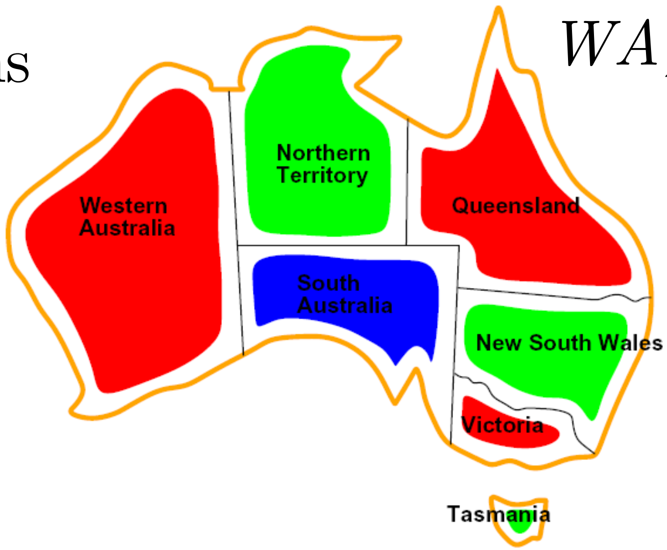
Map Coloring: Match!

Constraints $\{red, green, blue\}$

Variables $\{WA = red, NT = green, Q = red,$
$NSW = green, V = red, SA = blue,\ T = green\}$

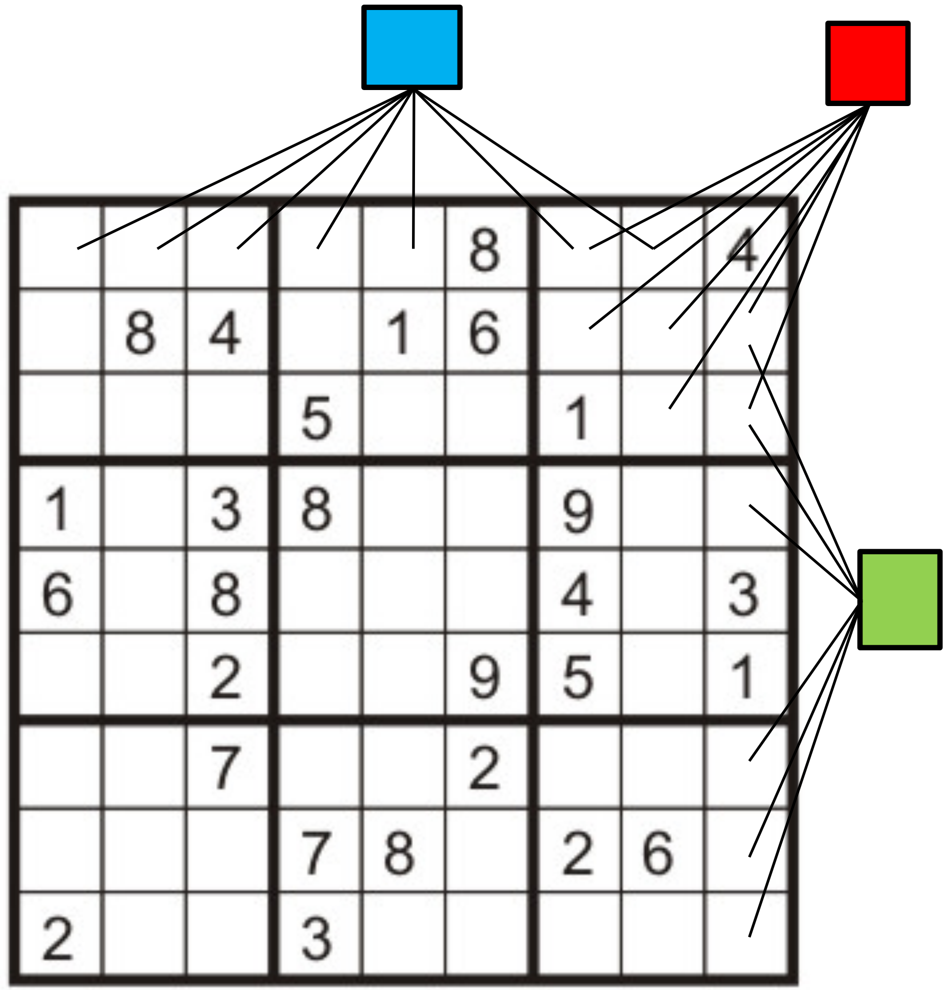
Domain $(WA, NT) \in \{(red, green), (red, blue), (green, red), \ldots\}$

Solutions $WA,\ NT,\ Q,\ NSW,\ V,\ T,\ SA$



Example: Sudoku

- Variables:
  - X_{ij}, each open square
- Domain:
  - {1:9}
- Constraints:
  - 9-way all diff col
  - 9-way all diff row
  - 9-way all diff box



Scheduling (Important Example)

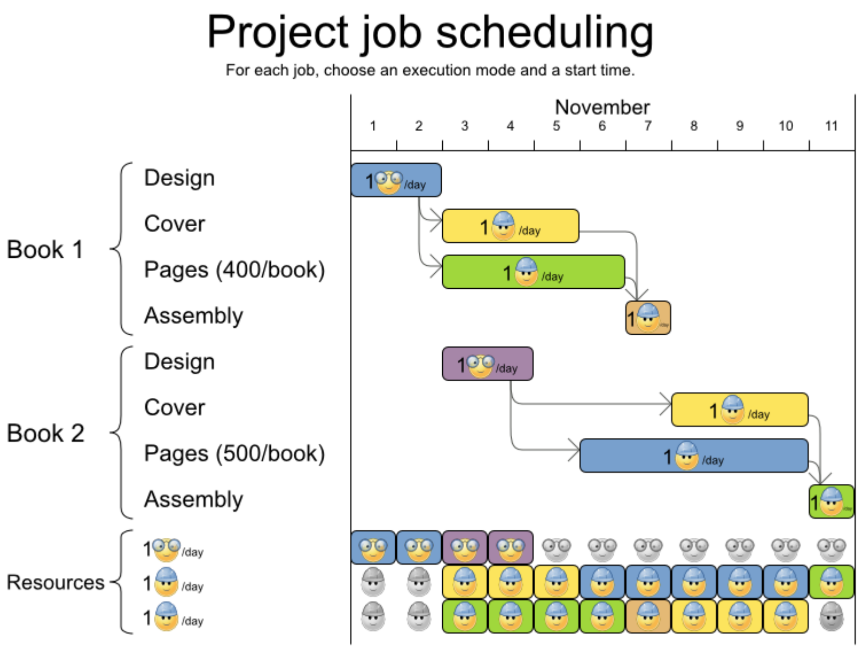
- Many industries. Many multi-million $ decisions. Used extensively for space mission planning. Military uses.
- People *really care* about improving scheduling algorithms! Problems with phenomenally huge state spaces. But for which solutions are needed very quickly
- Many kinds of scheduling problems e.g.:
  - *Job shop*: Discrete time; weird ordering of operations possible; set of separate jobs.
  - *Batch shop*: Discrete or continuous time; restricted operation of ordering; grouping is important.

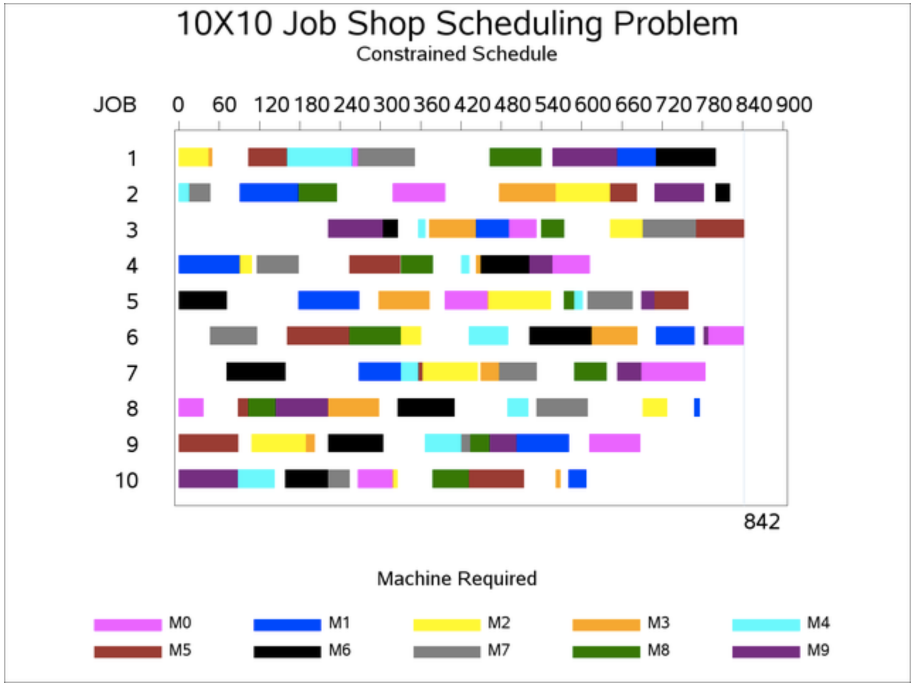
Job Scheduling

- A set of J *jobs*, $J_1, \ldots, J_n$
- A set of R *resources*, $R_1, R_2, \ldots, R_m$ to do the jobs
- Each job j is a sequence of *operations* $O^j_1, \ldots, O^j_{Lj}$ to be scheduled according to process plans: $O^j_1 < O^j_2 < O^j_3 \ldots$
- Each operation has a fixed processing time and requires the use of resources R_i, a resource can have capacity constraints
- Each job has a *ready time* and a *due time*
- A resource can only be used by a single operation at a time.
- All jobs must be completed by a due time.

- Problem: assign a start time to each job such that all jobs are completed by their due times respecting all constraints

Job Scheduling





Class Scheduling Woes

- **4 more required classes to graduate**
  - A: Algorithms
  - B: Bayesian Learning
  - C: Computer Programming
  - D: Distributed Computing
- **A few restrictions**
  - Algorithms must be taken same semester as Distributed computing
  - Computer programming is a prereq for Distributed computing and Bayesian learning, so it must be taken in an earlier semester
  - Advanced algorithms and Bayesian Learning are always offered at the same time, so they cannot be taken the same semester
- **3 semesters** (semester 1,2,3) when can take classes

Exercise: Define CSP

- 4 more required classes to graduate: A, B, C, D
- A must be taken same semester as D
- C is a prereq for D and B so must take C earlier than D & B
- A & B are always offered at the same time, so they cannot be taken the same semester
- 3 semesters (semester 1,2,3) when can take classes

Exercise: Define CSP

- 4 more required classes to graduate: A, B, C, D
- A must be taken same semester as D
- C is a prereq for D and B so must take C earlier than D & B
- A & B are always offered at the same time, so they cannot be taken the same semester
- 3 semesters (semester 1,2,3) when can take classes
- Variables: A,B,C,D
- Domain: {1,2,3}
- Constraints: $A \neq B$, $A=D$, $C < B$, $C < D$

Types of CSPs

- Discrete-domain variables
  - **Finite domains** (Map coloring, Sudoku, N-queens, SAT)

    ➔ Our focus!
  - **Infinite domains** (Integers or strings, deadline-free JSS)

    Constraint language is needed to understand relations $J_1 + d_1 \leq J_2$ without enumerating all tuples

    Integer programming methods deal effectively with (integer, binary) problems with *linear constraints*

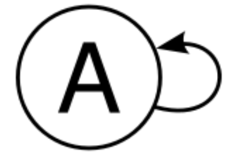
- Continuous variables (planning, blending, positioning,…)
  - *Linear/convex programming* for linear/convex constraints

Types of constraints

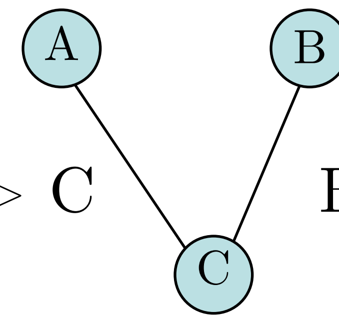
- Unary: involve a single variable
- Binary: involve two variables
- n-ary: involve n variables
- Soft constraints: violation incurs a cost, the problem becomes a constraint optimization one

CONSTRAINT GRAPH

- Variables → Vertices
- Constraints → Edges
  - Unary: Self-edges
  - Binary: regular edges
  - *n*-ary: hyperedges (hypergraphs)

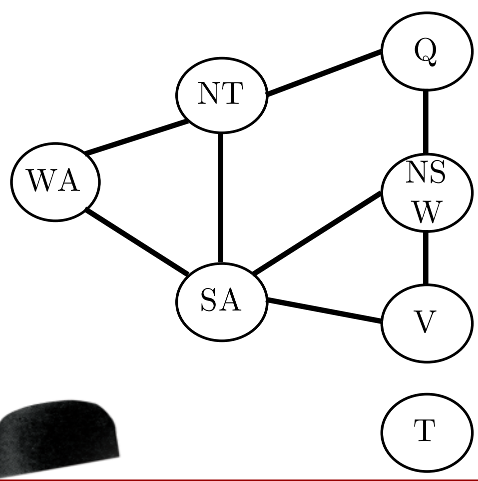


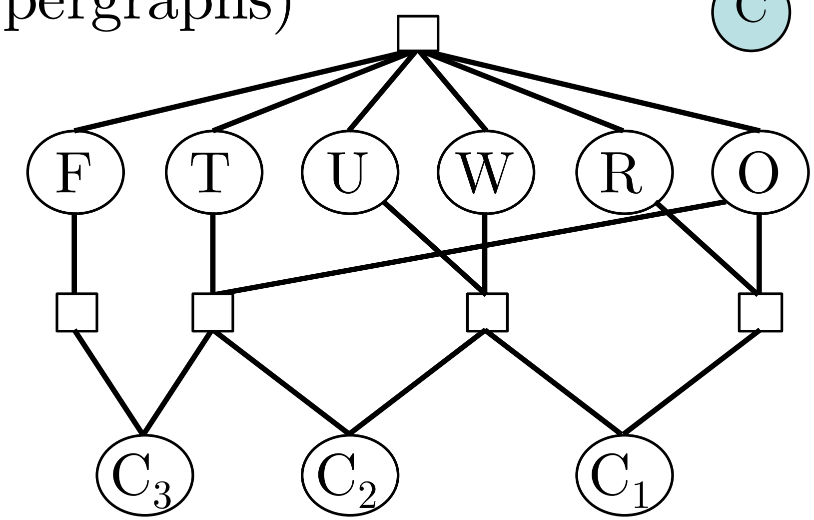
$A < 2$



$A > C$

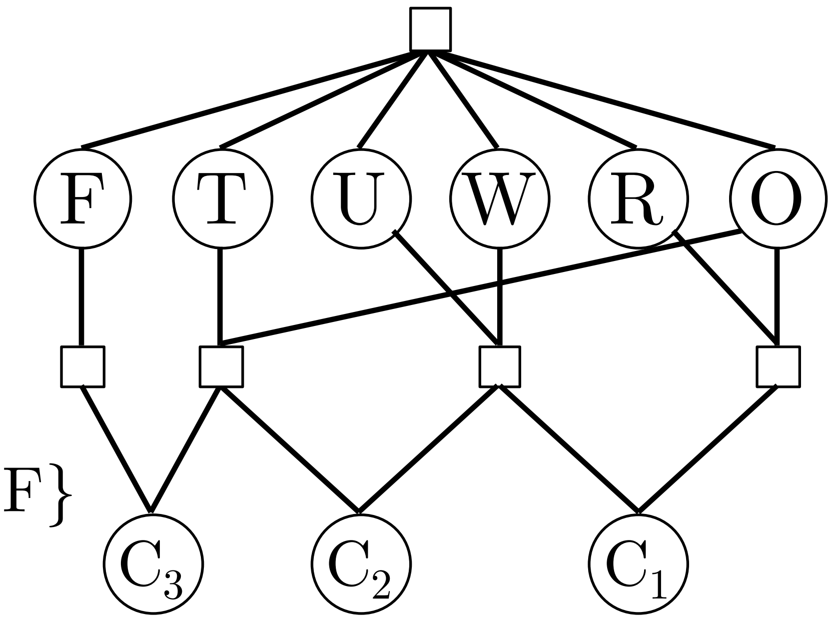
$B \neq C$





Cryptarithmetic puzzles

$$\begin{array}{r} \text{TWO} \\ +\ \text{TWO} \\ \hline \text{FOUR} \end{array} =$$



V = {O,W,T,R,U,F}

D = {0, ..., 9}

$$10^0(O+O) + 10^1(W+W)+10^2(T+T) = 10^0R+10^1U+10^2O+10^3F$$

$$\{O+O = R+10C_1,\ C_1+W+W=U+10C_2,\ C_2+T+T=O+10C_3,\ C_3 = F\}$$

$V = \{O,W,T,R,U,F, C_1, C_2, C_3\}$ Auxiliary vars

Binary constraint graphs

It's always possible to reduce a hypergraph to a binary constraint graph!

But this is not always the best thing to do ....

If you want to know more ...

On the Conversion between Non-Binary and Binary Constraint Satisfaction Problems. Bacchus, F. and van Beek, P. In*Proceedings of the 15th AAAI Conference on Artificial Intelligence (AAAI-1998)*, pages 310-318, 1998.

Overview

- Definitions, toy and real-world examples
- **Basic algorithms for solving CSPs**
- Pruning space through propagating information

Why not just do basic search algorithms from last time?

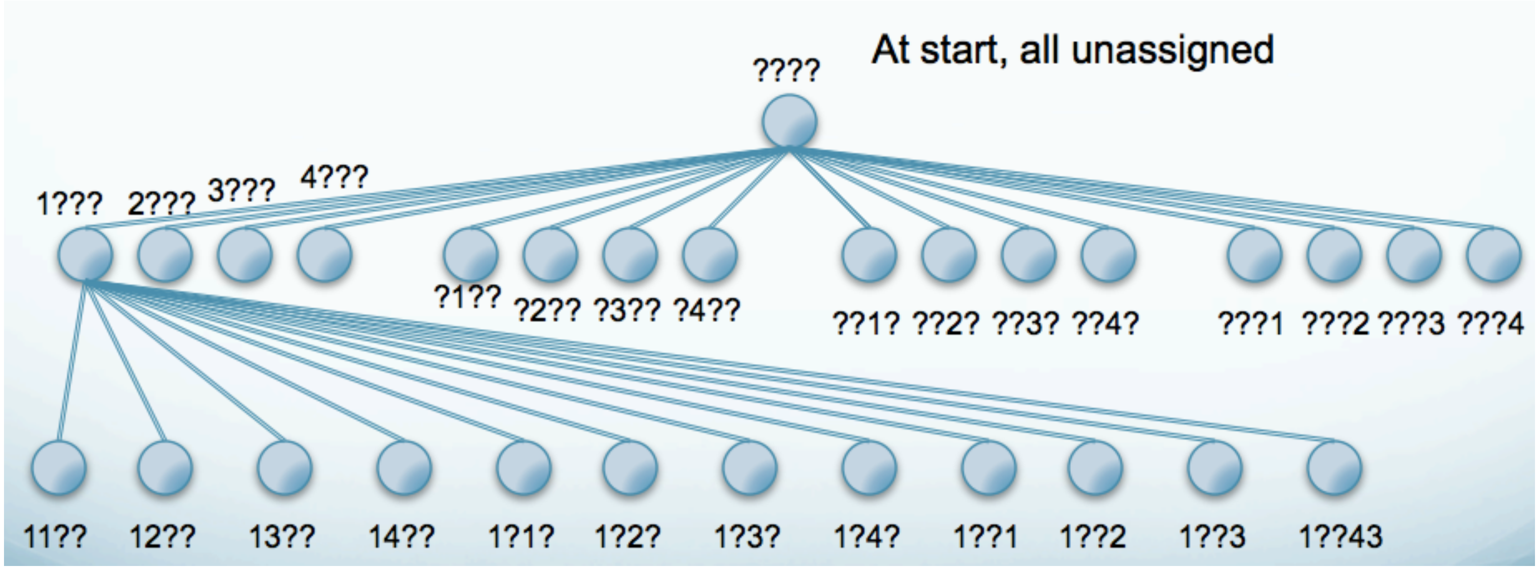
- **States:** Partial assignments to the n variables
- **Initial state:** Empty state
- **Action**: Select an unassigned variable i and assign a feasible value from its domain D_i to it
- **Goal test:** Assignnent consistent (no violations) and complete (all variables assigned)
- **Step cost:** Constant
- Solution is found at depth n, using **depth-limited DFS**
- **Size of the search tree?**

Why not just do basic search algorithms from last time?

$n = 4$ variables each taking $d = 4$ values

$b = nd$

$b=(n-1)d$



Generate a search tree of $n!d^n$ but there are only d^n possible assignments!

Commutativity!

- The order of assigning the variables has no effect on the final outcome
- CSPs are commutative: Regardless of the assignment order, the same partial solution is reached for a defined set of assignment values
- *Don't care about path!*
- → Only a single variable at each node in the search tree needs to be considered!! (can fix the order)
- → d^n number of leaves in the search tree!

Backtracking: DFS with single variable assignments

- Only consider a single variable at each point
- Don't care about path
- Order of variable assignment doesn't matter, so fix ordering
- Only consider values which do not conflict with assignment made so far
- *Depth-first search* for CSPs with these two improvements is called **backtracking search**

BACKTRACKING

- Function **Backtracking**(csp) returns solution or fail
  - Return Backtrack({},csp)
- Function **Backtrack***(assignment,csp)* returns solution or fail
  - If assignment is complete, return assignment
  - V_i ← *select_unassigned_var(csp)*
  - For each val in *order-domain-values(var, csp, assign)*

    If value is consistent with assignment

      Add [V_i = val] to assignment

      Result ← *Backtrack(assignment, csp)*

      If Result ≠ fail, return result

    Remove [V_i = val] from assignments
  - Return fail

Backtracking

- Function **Backtracking**(csp) returns soln or fail
  - Return Backtrack({},csp)
- Function ***Backtrack****(assignment,csp)* returns soln or fail
  - If assignment is complete, return assignment
  - V_i ← ***select_unassigned_var(csp)***
  - For each val in ***order-domain-values(var,csp,assign)***

    If value is consistent with assignment

    Add [V_i = val] to assignment

    Result ← *Backtrack(assignment,csp)*

    If Result ≠ fail, return result

    Remove [V_i = val] from assignments
  - Return fail

Think and discuss

- Does the variable/value order used affect how long backtracking takes to find a solution?
- Does the variable/value order used affect the solution found by backtracking?

Example

Variables: A,B,C,D   Domain: {1,2,3}

Constraints: $A \neq B$, $A = D$, $C < B$, $C < D$

Variable order: alphabetical   Value order: Descending

- (A=3)

Example

Variables: A,B,C,D   Domain: {1,2,3}

Constraints: $A \neq B$, $A=D$, $C < B$, $C < D$

Variable order: alphabetical   Value order: Descending

- (A=3)
- (A=3, B=3) inconsistent with $A \neq B$
- (A=3, B=2)
- (A=3, B=2, C=3) inconsistent with $C < B$
- (A=3, B=2, C=2) inconsistent with $C < B$
- (A=3, B=2, C=1)
- (A=3, B=2, C=1,D=3) VALID

Example

Variables: A,B,C,D   Domain: {1,2,3}

Constraints: $A \neq B$, $A=D$, $C < B$, $C < D$

Variable order: alphabetical   **Value order:** Ascending

- (A=1)

Example

Variables: A,B,C,D   Domain: {1,2,3}

Constraints: $A \neq B$, $A=D$, $C < B$, $C < D$

Variable order: alphabetical   **Value order:** Ascending

- (A=1)
- (A=1,B=1) inconsistent with $A \neq B$
- (A=1,B=2)
- (A=1,B=2,C=1)
- (A=1,B=2,C=1,D=1) inconsistent with $C < D$
- (A=1,B=2,C=1,D=2) inconsistent with $A=D$
- (A=1,B=2,C=1,D=3) inconsistent with $A=D$

Example

Variables: A,B,C,D Domain: {1,2,3}

Constraints: $A \neq B$, A=D, C < B, C < D

Variable order: alphabetical Value order: Ascending

- (A=1)
- (A=1,B=1) inconsistent with $A \neq B$
- (A=1,B=2)
- (A=1,B=2,C=1)
- (A=1,B=2,C=1,D=1) inconsistent with C < D
- (A=1,B=2,C=1,D=2) inconsistent with A=D
- (A=1,B=2,C=1,D=3) inconsistent with A=D
- No valid assignment for D, return result = fail
  - Backtrack to (A=1,B=2,C=)
- Try (A=1,B=2,C=2) but inconsistent with C < B
- Try (A=1,B=2,C=3) but inconsistent with C < B
- No other assignments for C, return result= fail
  - Backtrack to (A=1,B=)
- (A=1,B=3)
- (A=1,B=3,C=1)
- (A=1,B=3,C=1,D=1) inconsistent with C < D
- (A=1,B=3,C=1,D=2) inconsistent with A = D
- (A=1,B=3,C=1,D=3) inconsistent with A = D
- Return result = fail
  - Backtrack to (A=1,B=3,C=)
- (A=1,B=3,C=2) inconsistent with C < B
- (A=1,B=3,C=3) inconsistent with C < B
- No remaining assignments for C, return fail
  - Backtrack to (A=1,B=)
- No remaining assignments for B, return fail
  - Backtrack to A
- (A=2)
- (A=2,B=1)
- (A=2,B=1,C=1) inconsistent with C < B
- (A=2,B=1,C=2) inconsistent with C < B
- (A=2,B=1,C=3) inconsistent with C < B
- No remaining assignments for C, return fail
  - Backtrack to (A=2,B=?)
- (A=2,B=2) inconsistent with $A \neq B$
- (A=2,B=3)
- (A=2,B=3,C=1)
- (A=2,B=3,C=1,D=1) inconsistent with C < D
- (A=2,B=3,C=1,D=2) **ALL VALID**

Ordering Matters!

- Function Backtracking(csp) returns soln or fail
  - Return Backtrack({},csp)
- Function *Backtrack(assignment,csp)* returns soln or fail
  - If assignment is complete, return assignment
  - V_i ← ***select_unassigned_var(csp)***
  - For each val in ***order-domain-values(var,csp,assign****)*

    If value is consistent with assignment

        Add [V_i = val] to assignment

        Result ← *Backtrack(assignment,csp)*

        If Result ≠ fail, return result

    Remove [V_i = val] from assignments
  - Return fail

Ordering heuristics

- Next variable?
  - *Random or static*
  - Variable with the fewest legal values: *Minimum remaining values (MRV)* heuristic (aka the *most constrained* var, the *fail-first* var)
  - Variable with the largest number of constraints on other unassigned variables, reduces b on future choices (*Degree* heuristic)
- Variable's value?
  - Value that leaves most choices for the neighboring variables in the constraint graph, max flexibility (*least-constraining-value* heuristic), fail-last

(Test) Cost of Backtracking?

- d values per variable
- n variables
- Possible number of CSP assignments?

- A) $O(d^n)$
- B) $O(n^d)$
- C) $O(nd)$

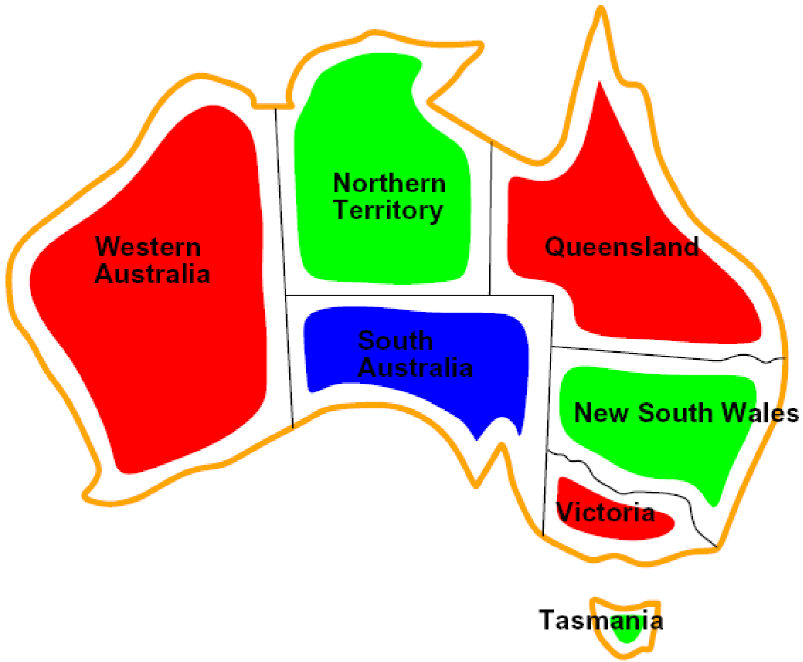
Overview

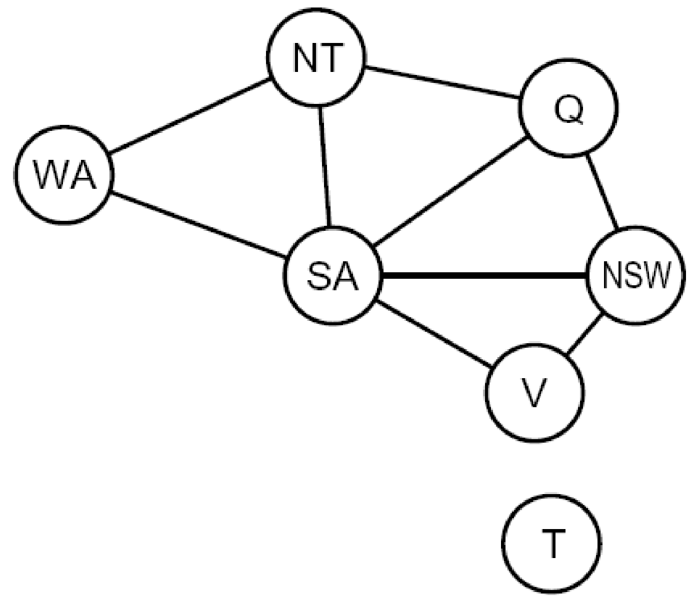
- Real world CSPs
- Basic algorithms for solving CSPs
- **Pruning space through propagating information**

Limitations of Backtracking

- Can inevitable failure be detected earlier?
- Can problem structure can be exploited?
- Can the search space be reduced to speed up computation?

Propagate information





- If we choose a value for one variable, that affects its neighbors
- And then potentially those neighbors…
- We can use this inference to prune the search space.

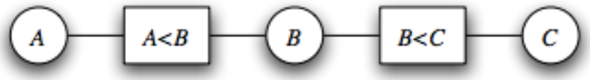
Arc Consistency

- Definition:
  - An "arc" (connection between two variables X → Y in constraint graph) is consistent if:
  - For every value could assign to X

    There exists some value of Y that could be assigned without violating a constraint

If a variable is not arc consistent with another one, it can be made so by removing some values from its domain. This can be done recursively → Form of constraint propagation that enforces arc consistency, maintains the problem solutions, and prunes the tree!

Arc Consistency in practice



- $dom(A) = \{1, 2, 3, 4\}; dom(B) = \{1, 2, 3, 4\}; dom(C) = \{1, 2, 3, 4\}$
- Suppose you first select the arc $\langle A, A < B \rangle$.
  - Remove $A = 4$ from the domain of A.
  - Add nothing to TDA. (To-Do-Arcs)
- Suppose that $\langle B, B < C \rangle$ is selected next.
  - Prune the value 4 from the domain of B.
  - Add $\langle A, A < B \rangle$ back into the TDA set (why?)
- Suppose that $\langle B, A < B \rangle$ is selected next.
  - Prune 1 from the domain of B.
  - Add no element to TDA (why?)
- Suppose the arc $\langle A, A < B \rangle$ is selected next
  - The value $A = 3$ can be pruned from the domain of A.
  - Add no element to TDA (why?)
- Select $\langle C, B < C \rangle$ next.
  - Remove 1 and 2 from the domain of C.
  - Add $\langle B, B < C \rangle$ back into the TDA set

The other two edges are arc consistent, so the algorithm terminates with $dom(A) = \{1, 2\}$, $dom(B) = \{2, 3\}$, $dom(C) = \{3, 4\}$.

Example from: Kevin Leyton-Brown

AC-3 Computational Complexity?

- Input: CSP
- Output: CSP, possible with reduced domains for variables, or inconsistent
- Local variables: queue, initially queue of all arcs (binary constraints in csp)
- While queue is not empty
- (X_i,X_j) = Remove-First(queue)
- [domainX_i, anyChangeToDomainX_i] = Revise(csp,X_i,X_j)
- if anyChangeToDomainX_i == true
- if size(domainX_i) = 0, return inconsistent
- else
- for each X_k in Neighbors(X_i) except X_j
- add (X_k,X_i) to queue
- Return csp
- --
- **Function Revise(csp,X_i,X_j)** returns DomainXi and anyChangeToDomainX_i
- anyChangeToDomainX_i = false
- for each x in Domain(X_i)
- if no value y in Domain(Xj) allows (x,y) to satisfy constraint between (X_i,X_j)
- delete x from Domain(X_i)
- anyChangeToDomainX_i = true

Have to add in arc for (X_i,X_j) and (X_j,X_i) for i,j constraint

D domain values
C binary constraints

Complexity of revise function? D^2

Number of times can put a constraint in queue? D

Total: CD^3

(Test) Sufficient?

- After we run AC-3 have we always found a solution? (aka only 1 value left for each variable)

- A) Yes
- B) No

AC-3 Example

- Variables: A,B,C,D
- Domain: {1,2,3}
- Constraints: $A \neq B$, $C < B$, $C < D$ (subset of constraints from before)

AC-3 EXAMPLE

- Variables: A,B,C,D
- Domain: {1,2,3}
- Constraints: A ≠ B, C < B, C < D (subset of constraints from before)
- Constraints both ways: A≠ B, B≠ A, C < B, B > C, C < D, D > C

AC-3 Example

- Variables: A,B,C,D
- Domain: {1,2,3}
- Constraints: $A \neq B$, $C < B$, $C < D$ (subset of constraints from before)
- Constraints both ways: $A \neq B$, $B \neq A$, $C < B$, $B > C$, $C < D$, $D > C$
- Queue: AB, BA, BC, CB, CD, DC

AC-3 EXAMPLE

- Variables: A,B,C,D
- Domain: {1,2,3}
- Constraints: $A \neq B$, $C < B$, $C < D$ (subset of constraints from before)
- Constraints both ways: $A \neq B$, $B \neq A$, $C < B$, $B > C$, $C < D$, $D > C$
- Queue: AB, BA, BC, CB, CD, DC
- Pop AB:
- *for each x in Domain(A)*

  if no value y in Domain(B) that allows (x,y) to satisfy constraint between (A,B)

  delete x from Domain(A)
- No change to domain of A

AC-3 Example

- Variables: A,B,C,D
- Domain: {1,2,3}
- Constraints: $A \neq B$, $C < B$, $C < D$ (subset of constraints from before)
- Constraints both ways: $A \neq B$, $B \neq A$, $C < B$, $B > C$, $C < D$, $D > C$
- Queue: AB, BA, BC, CB, CD, DC
- Pop AB
- Queue: BA, BC, CB, CD, DC

AC-3 EXAMPLE

- Variables: A,B,C,D
- Domain: {1,2,3}
- Constraints: $A \neq B$, $C < B$, $C < D$ (subset of constraints from before)
- Constraints both ways: $A \neq B$, $B \neq A$, $C < B$, $B > C$, $C < D$, $D > C$
- Queue: AB, BA, BC, CB, CD, DC
- Pop AB
- Queue: BA, BC, CB, CD, DC
- Pop BA
- *for each x in Domain(B)*

  if no value y in Domain(A) that allows (x,y) to satisfy

  constraint between (B,A)

  delete x from Domain(B)
- No change to domain of B

AC-3 EXAMPLE

- Variables: A,B,C,D
- Domain: {1,2,3}
- Constraints: $A \neq B$, $C < B$, $C < D$ (subset of constraints from before)
- Constraints both ways: $A \neq B$, $B \neq A$, $C < B$, $B > C$, $C < D$, $D > C$
- Queue: AB, BA, BC, CB, CD, DC
- Queue: BA, BC, CB, CD, DC
- Queue: BC, CB, CD, DC
- Pop BC
- *for each x in Domain(B)*
    - *if no value y in Domain(C) that allows (x,y) to satisfy constraint between (B,C)*
    - *delete x from Domain(B)*
- If B is 1, constraint $B > C$ cannot be satisfied. So delete 1 from B's domain, B={2,3}
- **Also have to add neighbors of B (except C) back to queue: AB**
- Queue: AB, CB, CD, DC

AC-3 Example

Variables: A,B,C,D
Domain: {1,2,3}
Constraints: $A \neq B$, $C < B$, $C < D$

- Queue: AB, BA, BC, CB, CD, DC   A-D = {1,2,3}
- Queue: BA, BC, CB, CD, DC   A-D = {1,2,3}
- Queue: BC, CB, CD, DC   A-D = {1,2,3}
- Queue: AB, CB, CD, DC   B={2,3}, A/C/D = {1,2,3}
- Pop AB
  - For every value of A is there a value of B such that A ≠ B?
  - Yes, so no change

AC-3 EXAMPLE

Variables: A,B,C,D
Domain: {1,2,3}
Constraints: $A \neq B$, $C < B$, $C < D$

- Queue: AB, BA, BC, CB, CD, DC,   A-D = {1,2,3}
- Queue: BA, BC, CB, CD, DC   A-D = {1,2,3}
- Queue: BC, CB, CD, DC   A-D = {1,2,3}
- Queue: AB, CB, CD, DC   B={2,3}, A/C/D = {1,2,3}
- Queue: CB, CD, DC   B={2,3}, A/C/D = {1,2,3}
- Pop CB
  - For every value of C is there a value of B such that C < B
  - If C = 3, no value of B that fits
  - So delete 3 from C's domain, C = {1,2}
  - **Also have to add neighbors of C (except B) back to queue: no change because already in**

AC-3 Example

Variables: A,B,C,D
Domain: {1,2,3}
Constraints: $A \neq B$, $C < B$, $C < D$

- Queue: AB, BA, BC, CB, CD, DC, A-D = {1,2,3}
- Queue: BA, BC, CB, CD, DC A-D = {1,2,3}
- Queue: BC, CB, CD, DC A-D = {1,2,3}
- Queue: AB, CB, CD, DC B={2,3}, A/C/D = {1,2,3}
- Queue: CB, CD, DC B={2,3}, A/C/D = {1,2,3}
- Queue: CD, DC B={2,3}, C = {1,2} A,D = {1,2,3}
- Pop CD
  - For every value of C, is there a value of D such that C < D?
  - Yes, so no change

AC-3 EXAMPLE

Variables: A,B,C,D
Domain: {1,2,3}
Constraints: $A \neq B$, $C < B$, $C < D$

- Queue: AB, BA, BC, CB, CD, DC   A-D = {1,2,3}
- Queue: BA, BC, CB, CD, DC   A-D = {1,2,3}
- Queue: BC, CB, CD, DC   A-D = {1,2,3}
- Queue: AB, CB, CD, DC   B={2,3}, A/C/D = {1,2,3}
- Queue: CB, CD, DC   B={2,3}, A/C/D = {1,2,3}
- Queue: CD, DC   B={2,3}, C = {1,2} A,D = {1,2,3}
- Queue: DC   B={2,3}, C = {1,2} A,D = {1,2,3}
- For every value of D is there a value of C such that $D > C$?
  - Not if D = 1
  - So D = {2,3}

AC-3 Example

Variables: A,B,C,D
Domain: {1,2,3}
Constraints: $A \neq B$, $C < B$, $C < D$

- Queue: AB, BA, BC, CB, CD, DC   A-D = {1,2,3}
- Queue: BA, BC, CB, CD, DC   A-D = {1,2,3}
- Queue: BC, CB, CD, DC   A-D = {1,2,3}
- Queue: AB, CB, CD, DC   B={2,3}, A/C/D = {1,2,3}
- Queue: CB, CD, DC   B={2,3}, A/C/D = {1,2,3}
- Queue: CD, DC   B={2,3}, C = {1,2} A,D = {1,2,3}
- Queue: DC   B={2,3}, C = {1,2} A,D = {1,2,3}
- A = {1,2,3} B={2,3}, C = {1,2} D = {2,3}

Forward Checking

- AC-3 can ran *before* the search begins to prune search tree; it operates on the entire search tree (expensive!)
- It's a form of *inference* (inferring reductions)
- What if, instead, we make inference at *run-time*?
- Forward checking: When assign a variable, make all of its neighbors arc-consistent (*purely local*)

Backtracking + Forward Checking

- Function *Backtrack(assignment,csp)* returns soln or fail
  - If assignment is complete, return assignment
  - V_i ← *select_unassigned_var(csp)*
  - For each val in *order-domain-values(var,csp,assign)*
    - If value is consistent with assignment
      - Add [V_i = val] to assignment
      - **Make domains of all neighbors of V_i arc-consistent with [V_i = val]**
      - Result ← *Backtrack(assignment,csp)*
      - If Result ≠ fail, return result
    - Remove [V_i = val] from assignments
  - Return fail

- Note: When backtracking, domains must be restored

Maintaining Arc Consistency

- Forward checking doesn't ensure all arcs are consistent, only the local ones, no look-ahead
- AC-3 can detect failure faster than forward checking
- The MAC algorithm includes AC-3 in the search, executing it from the arcs of the locally unassigned variables
- What's the downside? **Computation**

Maintaining Arc Consistency (MAC)

- Function *Backtrack(assignment,csp)* returns soln or fail
  - If assignment is complete, return assignment
  - V_i ← *select_unassigned_var(csp)*
  - For each val in *order-domain-values(var,csp,assign)*
    - If value is consistent with assignment
      - Add [V_i = val] to assignment
      - **Run AC-3 to make all variables arc-consistent with [V_i = val]. Initial queue is arcs (X_j,V_i) of neighbors of V_i that are unassigned, but add other arcs if these vars change domains.**
      - Result ← *Backtrack(assignment,csp)*
      - If Result ≠ fail, return result
    - Remove [V_i = val] from assignments
  - Return fail

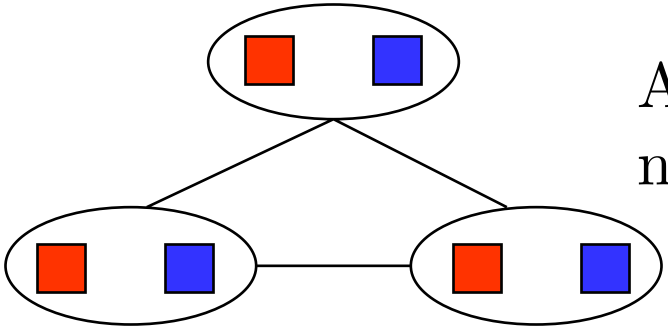
(Test) Sufficient to Avoid backtracking?

- If we maintain arc consistency, we will never have to backtrack while solving a CSP

- A) True
- B) False

AC Limitations

- After running AC-3
  - Can have one solution left
  - Can have multiple solutions left
  - Can have no solutions left (and not know it)



Arc-consistent but no feasible assignment

What went wrong here?

Complexity

- CSPs in general are NP-complete
- Valued, optimization version of CSPs are usually NP-hard
- Some structured domains, like those with a constraint tree, are easier and can be solved in polynomial time

Constraint Trees

- Constraint tree
  - Any 2 variables in constraint graph connected by $<=1$ path
- Can be solved in time **linear in # of variables**

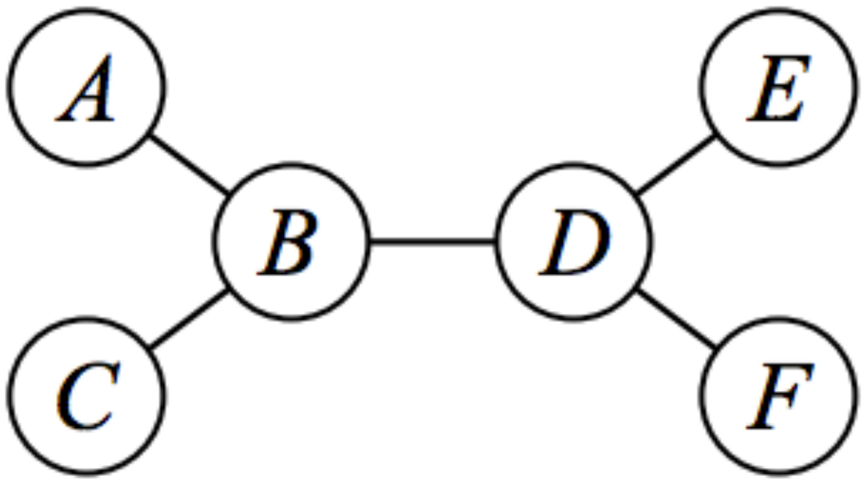
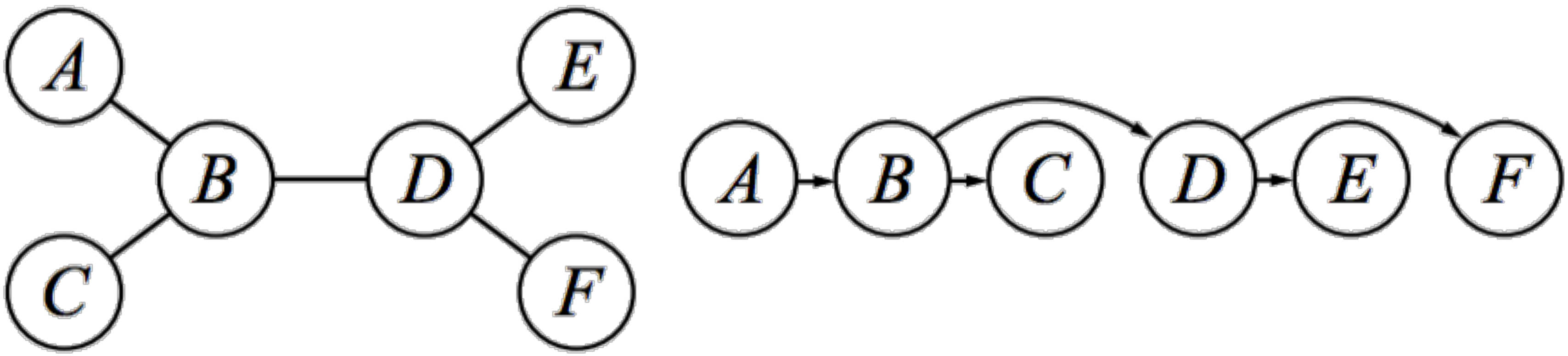


Figure from Russell & Norvig

Algorthm for CSP Trees

1) Choose any var as root and order vars such that every var's parents in constraint graph precede it in ordering

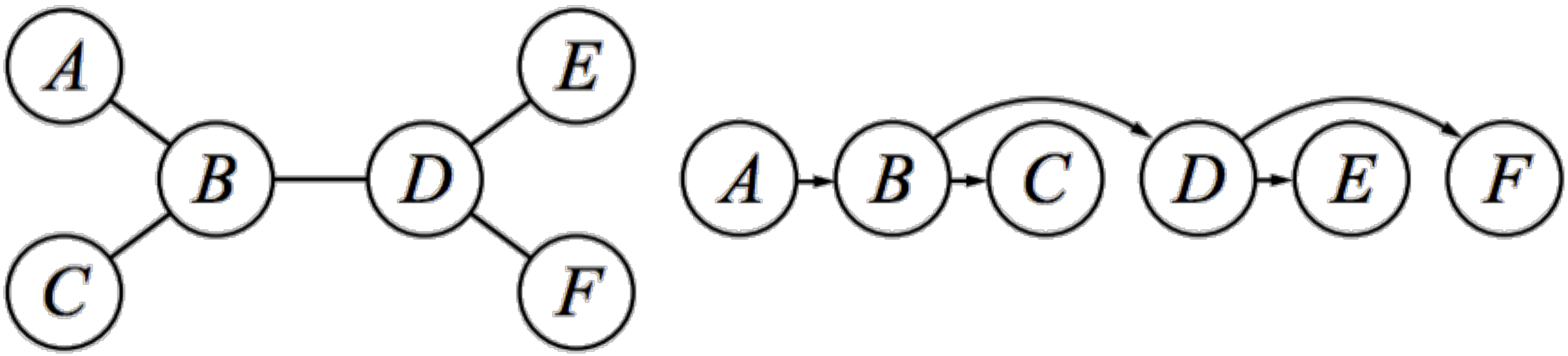


2) Let X_i be the parent of X_j in the new ordering
3) For j=n:2, run arc consistency to arc (X_i, X_j)
4) For j=1:n, assign val for X_j consistent w/val assigned for X_i

Figure from Russell & Norvig

Computational Complexity?

1) Choose any var as root and order vars such that every var's parents in constraint graph precede it in ordering



2) Let X_i be the parent of X_j in the new ordering
3) For j=n:2, run arc consistency to arc (X_i, X_j)
4) For j=1:n, assign val for X_j consistent w/val assigned for X_i

Figure from Russell & Norvig

Summary

- Be able to define real world CSPs
- Understand basic algorithm (backtracking)
  - Complexity relative to basic search algorithms
  - Doesn't require problem specific heuristics
  - Ideas shaping search (ordering heuristics)
- Pruning space through propagating information
  - Arc consistency
  - Tradeoffs: + reduces search space, - computation costs
- Computational complexity and special cases (tree)
- Relevant reading: R&N Chapter 6