CMU 15-781

Lecture 22:
Game Theory I

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GAME THEORY

- Game theory is the formal study of conflict and cooperation in (rational) multi-agent systems
- Decision-making where several players must make choices that potentially affect the interests of other players: **the effect of the actions of several agents are interdependent** (and agents are aware of it)

Psychology:
Theory of social situations

ELEMENTS OF A GAME

- The players: how many players are there? Does nature/chance play a role?
- A complete description of what the players can do: the set of all possible actions.
- The information that players have available when choosing their actions
- A description of the payoff / consequences for each player for every possible combination of actions chosen by all players playing the game.
- A description of all players' preferences over payoffs

INFORMATION

- **Complete information game:** Utility functions, payoffs, strategies and "types" of players are *common knowledge*
- **Incomplete information game:** players may not possess full information about their opponents (e.g., in auctions, each player knows its utility but not that of the other players)

- **Perfect information game:** each player, when making any decision, is perfectly informed of all the events that have previously occurred (e.g., chess)
- **Imperfect information game:** not all information is accessible to the player (e.g., poker, prisoner's dilemma)

STRATEGIES

- Strategy: tells a player what to do for every possible situation throughout the game (complete algorithm for playing the game). It can be deterministic or stochastic
- Strategy set: what strategies are available for the players to play. The set can be finite or infinite (e.g., beach war game)
- Strategy profile: a set of strategies for all players which fully specifies all actions in a game. A strategy profile must include one and only one strategy for every player
- Pure strategy: one specific element from the strategy set, a single strategy which is played 100% of the time
- Mixed strategy: assignment of a probability to each pure strategy. Pure strategy ≡ degenerate case of a mixed strategy

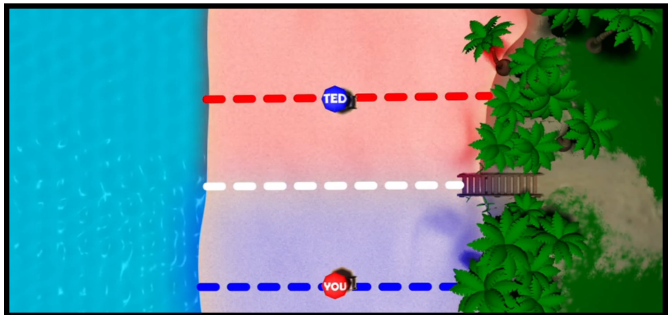
(STRATEGIC-) NORMAL-FORM GAME

- A game in normal form consists of:
  - Set of **players** $N = \{1, \dots, n\}$
  - **Strategy set** S
  - For each $i \in N$, a **utility function** u_i defined over the set of all possible *strategy profiles*, $u_i: S^n \rightarrow \mathbb{R}$, such that if each $j \in N$ plays the strategy $s_j \in S$, the utility of player i is $u_i(s_1, \dots, s_n)$ (i.e., $u_i(s_1, \dots, s_n)$ is player *i*'s payoff when strategy profile $(s_1, \dots, s_n)$ is chosen)
- Next example created by taking screenshots of http://youtu.be/jILgxeNBK_8

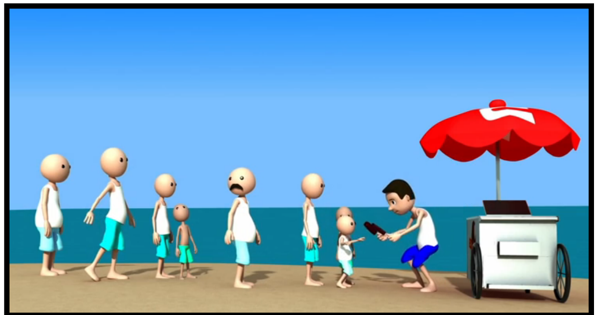
Selling ice cream at the beach.

One day your cousin Ted shows up.

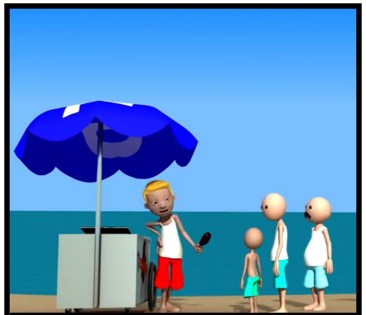
His ice cream is identical!



You split the beach in half; you set up at 1/4.

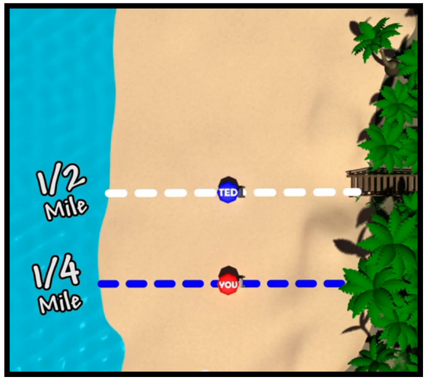


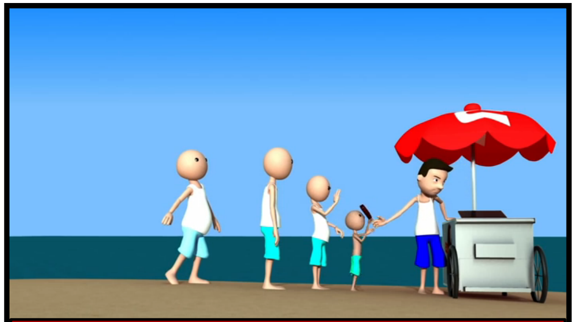
50% of the customers buy from you.



50% buy from Teddy.

One day Teddy sets up at the 1/2 point!





Now you serve only 37.5%!

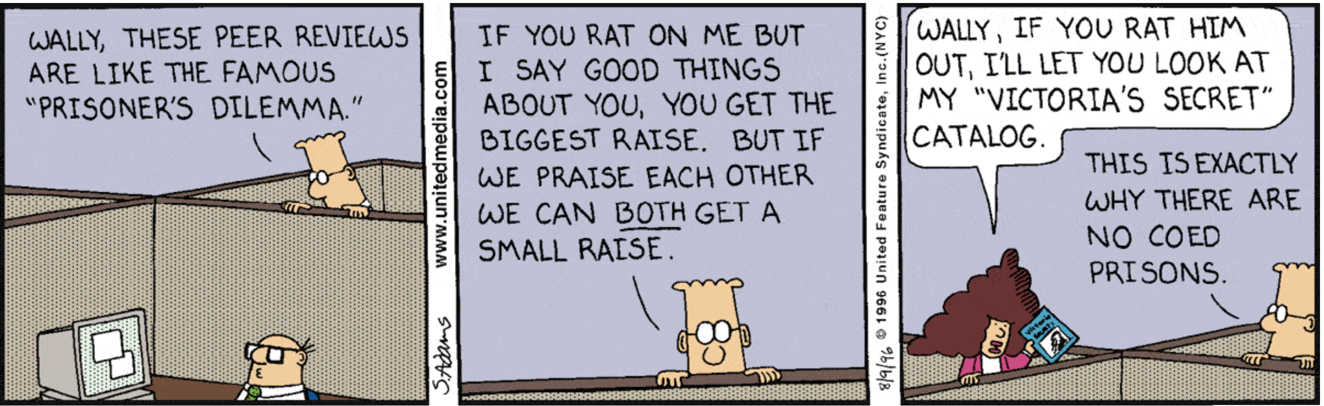
THE ICE CREAM WARS

- $N = \{1,2\}$
- $S = [0,1]$
- s_i is the fraction of beach
- $u_i(s_i, s_j) = \begin{cases} \frac{s_i + s_j}{2}, & s_i < s_j \\ 1 - \frac{s_i + s_j}{2}, & s_i > s_j \\ \frac{1}{2}, & s_i = s_j \end{cases}$
- To be continued…

THE PRISONER'S DILEMMA (1962)

- Two men are charged with a crime
- They can't communicate with each other
- They are told that:
    - If one rats out and the other does not, the rat will be freed, other jailed for 9 years
    - If both rat out, both will be jailed for 6 years
- They also know that if neither rats out, both will be jailed for 1 year

THE PRISONER'S DILEMMA (1962)



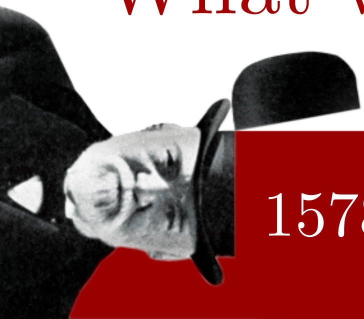
PRISONER'S DILEMMA: PAYOFF MATRIX

Don't confess = Cooperate:
Don't rat out, cooperate
with each other

Confess = Defect:
Don't cooperate to
each other, act
selfishly!

		B	
		Don't Confess	Confess
A	Don't Confess	-1,-1	-9,0
	Confess	0,-9	-6,-6

What would you do?



PRISONER'S DILEMMA: PAYOFF MATRIX

		B	
		Don't Confess	Confess
A	Don't Confess	-1,-1	-9,0
	Confess	0,-9	-6,-6

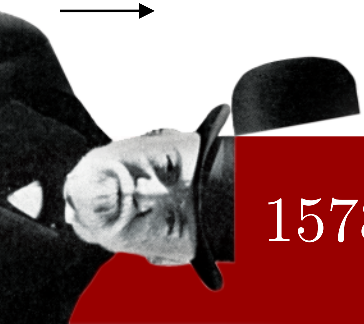
B Don't confess:

- If *A* don't confess, *B* gets -1
- If A confess, *B* gets -9

B Confess:

- If *A* don't confess, *B* gets 0
- If A confess, *B* gets -6

Rational agent B opts to *confess*



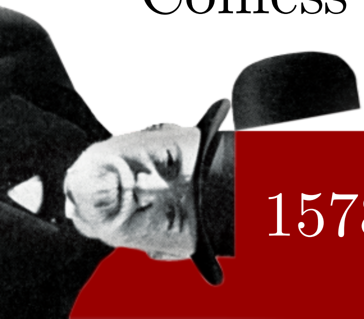
PRISONER'S DILEMMA

- Confess (Defection, Acting selfishly) is a dominant strategy for *B*: no matters what A plays, the best reply strategy is always to confess

- **(Strictly) dominant strategy**: yields a player strictly higher payoff, no matter which decision(s) the other player(s) choose. Weakly: ties in some cases

- Confess is a dominant strategy also for *A*

- *A* will reason as follows: *B*'s dominant strategy is to Confess, therefore, given that we are both rational agents, *B* will also Confess and we will both get 6 years.



PRISONER’S DILEMMA

- But, is the dominant strategy the best strategy?
- **Pareto optimality**: an outcome such that there is no other outcome that makes every player at least as well off and at least one player strictly better off → Outcome (-1,-1)
- Being selfish is a dominant strategy
- But the players can do much better by cooperating: (-1,-1), which is the Pareto-optimal outcome
- A strategy profile forms an **equilibrium** if no player can benefit by switching strategies, *given that every other player sticks with the same strategy*, which is the case of (C,C)
- An equilibrium is a **local optimum in** the space of the policies

UNDERSTANDING THE DILEMMA

- Self-interested rational agents would choose a strategy that does not bring the maximal reward
- The ***dilemma*** *is* that the equilibrium outcome is worse for both players than the outcome they would get if both refuse to confess
- Related to the tragedy of the commons

IN REAL LIFE

- Presidential elections
  - Cooperate = positive ads
  - Defect = negative ads
- Nuclear arms race
  - Cooperate = destroy arsenal
  - Defect = build arsenal
- Climate change
  - Cooperate = curb CO_2 emissions
  - Defect = do not curb

ON TV: GOLDEN BALLS

- If both choose Split, they each receive half the jackpot.
- If one chooses Steal and the other chooses Split, the Steal contestant wins the entire jackpot.
- If both choose Steal, neither contestant wins any money.

http://youtu.be/S0qjK3TWZE8

THE PROFESSOR'S DILEMMA

		Class	
		Listen	Sleep
Professor	Make effort	$10^6,10^6$	-10,0
	Slack off	0,-10	0,0

Dominant strategies?

NASH EQUILIBRIUM (1951)

- Each player's strategy is a best response to strategies of others

- Formally, a Nash equilibrium is *strategy profile* $s = (s_1 \ldots, s_n) \in S^n$ such that

$$\forall i \in N, \forall s_i' \in S, u_i(s) \geq u_i(s_i', s_{-i})$$

NASH EQUILIBRIUM

- In equilibrium, each player is playing the strategy that is a "**best response**" to the strategies of the other players. No one has an *incentive* to change his strategy given the strategy choices of the others
- A NE is an equilibrium where each player's strategy is optimal *given the strategies of all other players*.
- A Nash Equilibrium exists when there is no unilateral profitable deviation from any of the players involved
- Nash Equilibria are *self-enforcing*: when players are at a Nash Equilibrium they have no desire to move because they will be worse off → **Equilibrium** in the policy space

NASH EQUILIBRIUM

Equilibrium is *not*:

- The best possible outcome of the game. Equilibrium in the one-shot prisoners' dilemma is for both players to confess, which is not the best possible outcome (not Pareto optimal)
- A situation where players always choose the same action. Sometimes equilibrium will involve changing action choices (*mixed strategy* equilibrium).

NASH EQUILIBRIUM

- Poll 1: How many Nash equilibria does the Professor's Dilemma have?

1. 0
2. 1

→ 3. 2

4. 3

	Listen	Sleep
Make effort	$10^6,10^6$	-10,0
Slack off	0,-10	0,0

NASH EQUILIBRIUM

- *Nash equilibrium:* A play of the game where each strategy is a best reply to the given strategy of the other. Let's examine all the possible pure strategy profiles and check if for a profile (X,Y) one player could improve its payoff given the strategy of the other

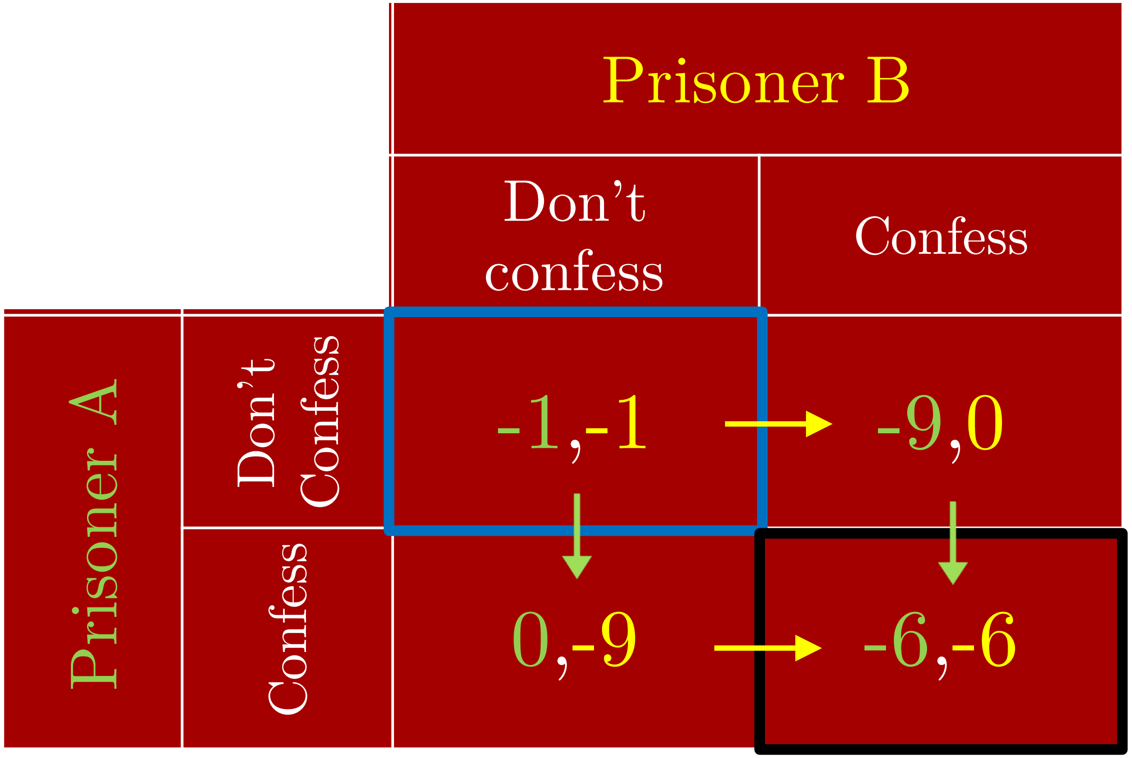
✓(M, L)? If Prof plays M, then L is the best reply given M. Neither player can increase its the payoff by choosing a different action

o(S,L)? If Prof plays S, S is the best reply given S, not L.

o(M, S)? If Prof plays M, then L is the best reply given M, not S

✓(S,S)? If Prof plays S, then S is the best reply given S. Neither player can increase its the payoff by choosing a different action

NASH EQUILIBRIUM FOR PRISONER'S DILEMMA



		Prisoner B	
		Don't confess	Confess
Prisoner A	Don't Confess	-1,-1	-9,0
	Confess	0,-9	-6,-6

(NOT) NASH EQUILIBRIUM

http://youtu.be/CemLiSI5ox8

RUSSEL CROWE WAS WRONG

« STOC Submissions: message from the PC Chair

Russell Crowe was wrong

October 30, 2012 by Ariel Procaccia | Edit

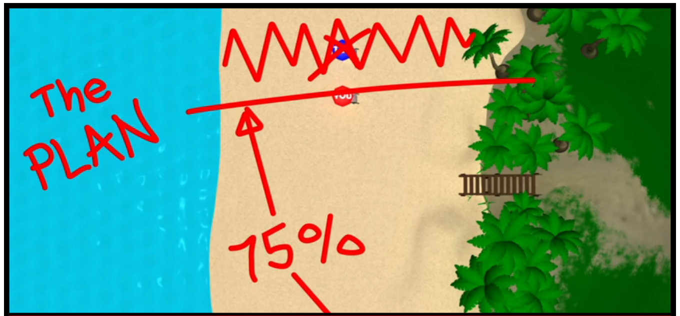
Yesterday I taught the first of five algorithmic economics lectures in my undergraduate AI course. This lecture just introduced the basic concepts of game theory, focusing on Nash equilibria. I was contemplating various way making the lecture more lively, and it occurred to me that I could stand on t shoulders of giants. Indeed, didn't Russell Crowe already explain Nash's ideas in A Beautiful Mind, complete with a 1940's-style male chauvinistic example?

The first and last time I watched the movie was when it was released in 2001. Back then I was an undergrad freshman, working for 20+ hours a week on the programming exercises of Hebrew U' Intro to CS course, which was taught by some guy called Noam Nisan. I didn't know anything about game theory, and Crowe's explanation made a lot of sense at the time.

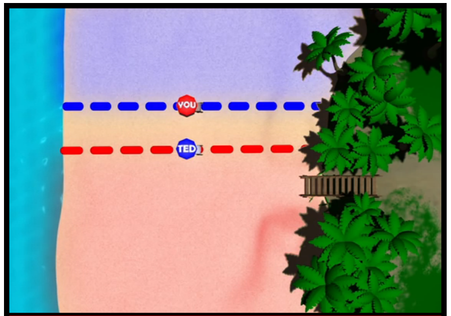
I easily found the relevant scene on youtube. In the scene, Nash's friends are trying to figure out how to seduce a beautiful blonde and her less beautiful friends. Then Nash/Crowe has an epiphany. The hubbub of the seedy Princeton bar is drowned by inspirational music, as Nash announces:

January 2012
December 2011
November 2011
October 2011
September 2011
August 2011
July 2011
June 2011

END OF THE ICE CREAM WARS

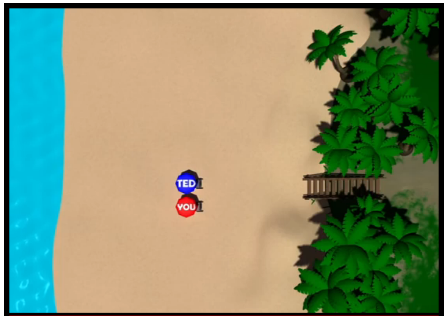


Day 3 of the ice cream wars...

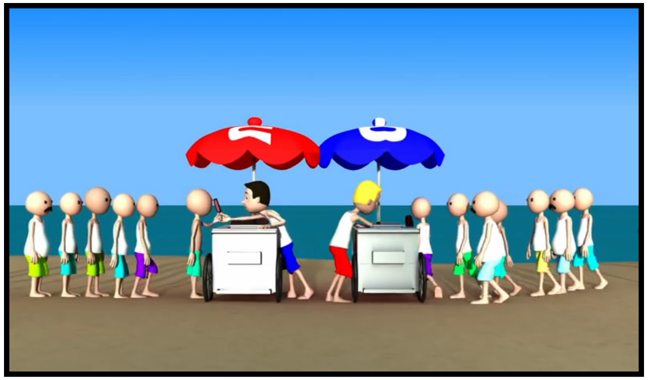


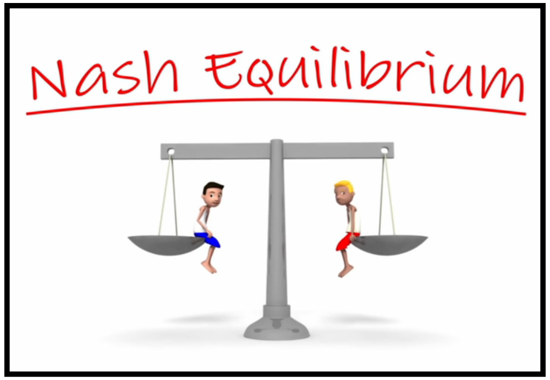
Teddy sets up south of you!

You go south of Teddy.

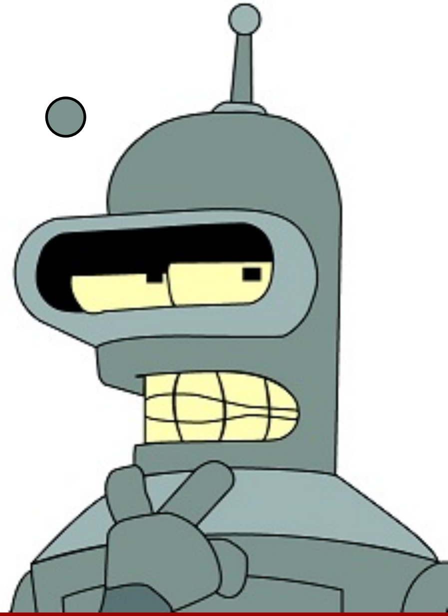


Eventually...





This is why competitors open their stores next to one another!



ROCK-PAPER-SCISSORS

	R	P	S
R	0,0	-1,1	1,-1
P	1,-1	0,0	-1,1
S	-1,1	1,-1	0,0

Nash equilibrium?

Is there a pure strategy as best response?

ROCK-PAPER-SCISSORS

	R	P	S
R	0,0	-1,1	1,-1
P	1,-1	0,0	-1,1
S	-1,1	1,-1	0,0

No (pure) Nash equilibria:
Best response: randomize!

- For every pure strategy (X,Y), there is a different strategy choice that increases the payoff of a player
- E.g., for strategy (P,R), player B can get a higher payoff playing strategy S instead R
- E.g., for strategy (S,R), player A can get a higher payoff playing strategy P instead S
- No strategy equilibrium can be settled, players have the incentive to keep switching their strategy

MIXED STRATEGIES

- A mixed strategy is a probability distribution over (*pure*) strategies
- The mixed strategy of player $i \in N$ is x_i, where $x_i(s_i) = \Pr[i \text{ plays } s_i]$ (e.g., $x_i(R) = 0.3$, $x_i(P) = 0.5$, $x_i(S) = 0.2$)
- The (expected) **utility** of player $i \in N$ is

$$u_i(\underbrace{x_1, \dots, x_n}_{\text{Mixed strategy profile}}) = \sum_{\underbrace{(s_1,\dots,s_n)}_{\text{Pure strategy profile}} \in S^n} \underbrace{u_i(s_1, \dots, s_n)}_{\text{Utility of pure strategy profile}} \cdot \underbrace{\prod_{j=1}^{n} x_j\ (s_j)}_{\text{Joint probability of the pure strategy profile given the mixed profile}}$$

EXERCISE: MIXED NE

- Exercise: player 1 plays $\left(\frac{1}{2},\frac{1}{2},0\right)$, player 2 plays $\left(0,\frac{1}{2},\frac{1}{2}\right)$. What is u_1?
- Exercise: Both players play $\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$. What is u_1?

	R	P	S
R	0,0	-1,1	1,-1
P	1,-1	0,0	-1,1
S	-1,1	1,-1	0,0

Exercise: Mixed NE

$$u_1\Big(x_1(R,P,S), x_2(R,P,S)\Big) =$$

$$u_1(R,R)p(R,R|x_1,x_2) + u_1(R,P)p(R,P|x_1,x_2) + u_1(R,S)p(R,S|x_1,x_2)$$

$$u_1(P,R)p(P,R|x_1,x_2) + u_1(P,P)p(P,P|x_1,x_2) + u_1(P,S)p(P,S|x_1,x_2)$$

$$u_1(S,R)p(S,R|x_1,x_2) + u_1(S,P)p(S,P|x_1,x_2) + u_1(S,S)p(S,S|x_1,x_2)$$

$$= 0 \cdot (\tfrac{1}{2} \cdot 0) + (-1) \cdot (\tfrac{1}{2} \cdot \tfrac{1}{2}) + 1 \cdot (\tfrac{1}{2} \cdot \tfrac{1}{2})$$

$$+1 \cdot (\tfrac{1}{2} \cdot 0) + 0 \cdot (\tfrac{1}{2} \cdot \tfrac{1}{2}) + (-1) \cdot (\tfrac{1}{2} \cdot \tfrac{1}{2})$$

$$+(-1) \cdot (0 \cdot 0) + 1 \cdot (0 \cdot \tfrac{1}{2}) + 0 \cdot (0 \cdot \tfrac{1}{2})$$

$$= -\tfrac{1}{4}$$

In the second case, because of symmetry, the utility is zero: It's a *zero-sum game*

	R	P	S
R	0,0	-1,1	1,-1
P	1,-1	0,0	-1,1
S	-1,1	1,-1	0,0

MIXED STRATEGIES NASH EQUILIBRIUM

- The mixed strategy profile x^* in a strategic game is a **mixed strategy Nash equilibrium** if

$$u_i(x_i^*, x_{-i}^*) \geq u_i(x_i\ , x_{-i}^*)\ \forall\ x_i \text{ and } i$$

- $u_i(x)$ is player *i*'s expected utility with mixed strategy profile x
- Same definition as in the case f pure strategies, where u_i was the utility of a pure strategy instead of a mixed strategy

MIXED STRATEGIES NASH EQUILIBRIUM

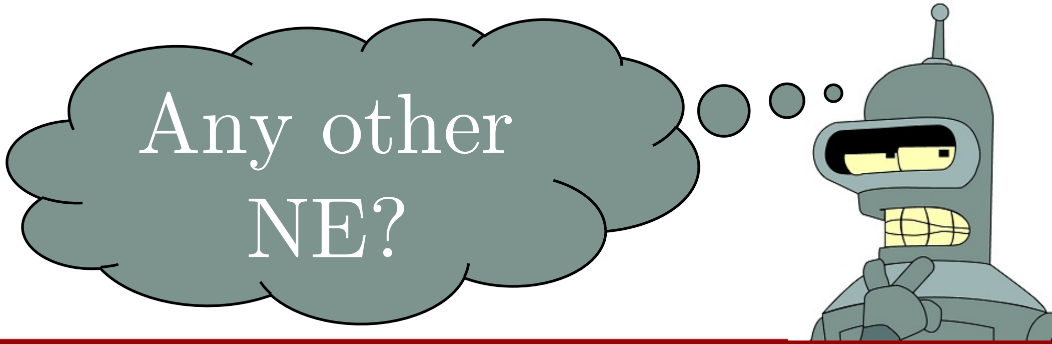
- Using *best response* functions, x^* is a mixed strategy NE iff x_i^* is the best response for every player i.
- If a mixed strategy x^* is a best response, then each of the *pure strategies in the mix must be best responses*: they must yield the same expected payoff (otherwise it would just make sense to choose the one with the better payoff)
- → If a mixed strategy is a best response for player i, then the player must be indifferent among the pure strategies in the mix
- E.g., in the RPS game, if the mixed strategy of player i assigns non-zero probabilities p_{R} for playing R and p_{P} for playing P, then i's expected utility for playing R or P has to be the same

Exercise: Mixed NE

- Poll 2: Which is a NE?

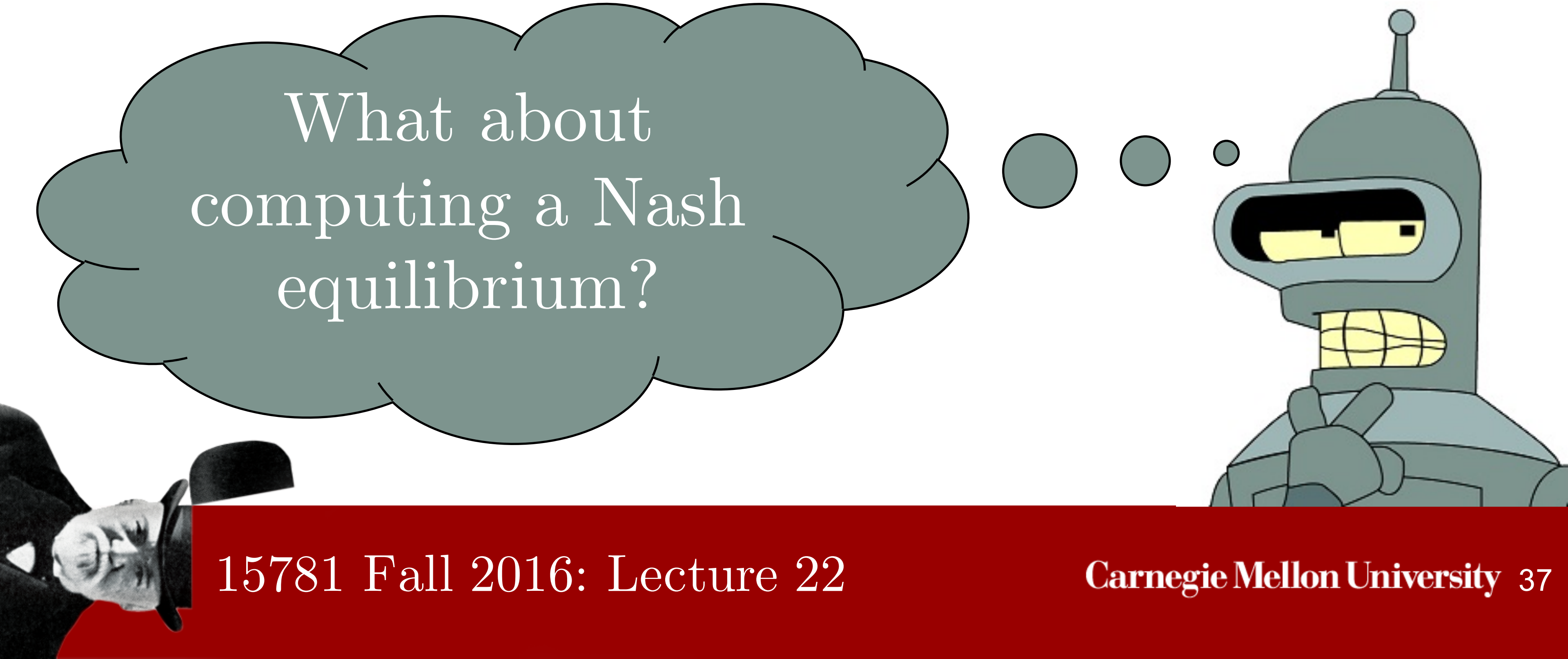
1. $\left(\left(\frac{1}{2},\frac{1}{2},0\right),\left(\frac{1}{2},\frac{1}{2},0\right)\right)$
2. $\left(\left(\frac{1}{2},\frac{1}{2},0\right),\left(\frac{1}{2},0,\frac{1}{2}\right)\right)$
3. [✓ 3.] $\left(\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right),\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)\right)$
4. $\left(\left(\frac{1}{3},\frac{2}{3},0\right),\left(\frac{2}{3},0,\frac{1}{3}\right)\right)$

	R	P	S
R	0,0	-1,1	1,-1
P	1,-1	0,0	-1,1
S	-1,1	1,-1	0,0



NASH'S THEOREM

- Theorem [Nash, 1950]: In any game with finite number of strategies there exists at least one (possibly mixed) Nash equilibrium



COMPUTATION OF MS NE

Player B

		Left	Right
Player A	Up	1,2	0,4
	Down	0,5	3,2

This game has no pure strategy Nash equilibria but it does have a Nash equilibrium in mixed strategies. How is it computed?

Example slides from Ted Bergstrom

Computation of MS NE

		Player B	
		Left	Right
Player A	Up	1,2	0,4
	Down	0,5	3,2

In a *mixed strategy:*

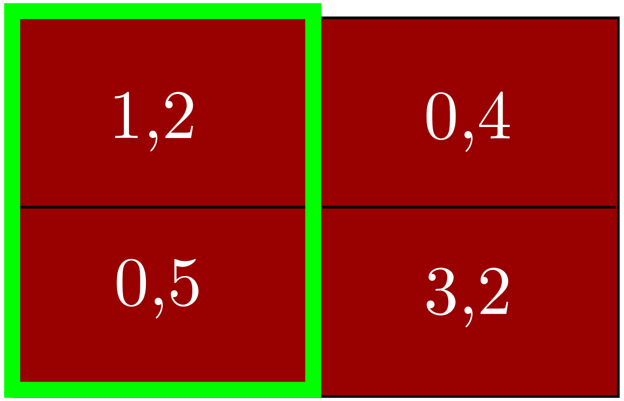
Player A plays Up with probability π_U and plays Down with probability 1-π_U Player B plays Left with probability π_L and plays Right with probability 1-π_L.

COMPUTATION OF MS NE

Player B

		L,π_L	R,1-π_L
Player A	U,π_U	1,2	0,4
	D,1-π_U	0,5	3,2

COMPUTATION OF MS NE



		Player B	
		L,π_L	R,1-π_L
Player A	U,π_U	1,2	0,4
	D,1-π_U	0,5	3,2

If B plays Left, its expected utility is

$$2\pi_U + 5(1 - \pi_U)$$

COMPUTATION OF MS NE

Player B

		L, π_L	$R, 1-\pi_L$
Player A	U, π_U	1,2	0,4
	$D, 1-\pi_U$	0,5	3,2

If B plays Right, its expected utility is

$$4\pi_U + 2(1-\pi_U).$$

COMPUTATION OF MS NE

		Player B	
		L,π_L	R,1-π_L
Player A	U,π_U	1,2	0,4
	D,1-π_U	0,5	3,2

If $2\pi_U + 5(1-\pi_U) > 4\pi_U + 2(1-\pi_U)$ Then B would play only Left. But there are no (pure) Nash equilibria in which B plays only Left

COMPUTATION OF MS NE

		Player B	
		L,π_L	R,$1-\pi_L$
Player A	U,π_U	1,2	0,4
	D,$1-\pi_U$	0,5	3,2

If $2\pi_U + 5(1-\pi_U) < 4\pi_U + 2(1-\pi_U)$ then

B would play only Right. But there are no (pure) Nash equilibria in which B plays only Right

COMPUTATION OF MS NE

Player B

		L,π_L	R,$1-\pi_L$
Player A	U,π_U	1,2	0,4
	D,$1-\pi_U$	0,5	3,2

For there to exist a MS Nash equilibrium, B must be indifferent between playing Left or Right:

$$2\pi_U + 5(1-\pi_U) = 4\pi_U + 2(1-\pi_U)$$

COMPUTATION OF MS NE

Player B

		L,π_L	R,1-π_L
Player A	U,π_U	1,2	0,4
	D,1-π_U	0,5	3,2

$$2\pi_U + 5(1 - \pi_U) = 4\pi_U + 2(1 - \pi_U)$$

$$\Rightarrow \quad \pi_U = 3/5.$$

COMPUTATION OF MS NE

Player B

		L,π_L	R,1-π_L
Player A	U, $\frac{3}{5}$	1,2	0,4
	D, $\frac{2}{5}$	0,5	3,2

$$\pi_U = \frac{3}{5} \qquad 1-\pi_U = \frac{2}{5}$$

COMPUTATION OF MS NE

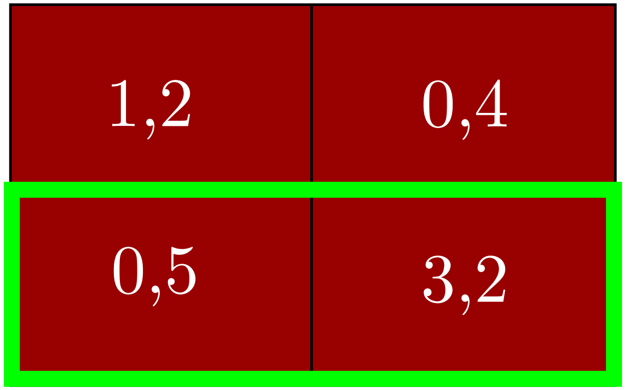
Player B

		L,π_L	R,1-π_L
Player A	U, $\frac{3}{5}$	1,2	0,4
	D, $\frac{2}{5}$	0,5	3,2

If A plays Up its expected payoff is

$$1 \times \pi_L + 0 \times (1 - \pi_L) = \pi_L .$$

COMPUTATION OF MS NE



		Player B	
		L, π_L	R, $1-\pi_L$
Player A	U, $\frac{3}{5}$	1,2	0,4
	D, $\frac{2}{5}$	0,5	3,2

If A plays Down his expected payoff is

$$0 \times \pi_L + 3 \times (1 - \pi_L) = 3(1 - \pi_L).$$

COMPUTATION OF MS NE

Player B

		L, π_L	R, $1-\pi_L$
Player A	U, $\frac{3}{5}$	1,2	0,4
	D, $\frac{2}{5}$	0,5	3,2

If $\pi_L > 3(1-\pi_L)$ then A would play only Up

But there are no Nash equilibria in which A plays only Up

COMPUTATION OF MS NE

Player B

		L,π_L	R,1-π_L
Player A	U, $\frac{3}{5}$	1,2	0,4
	D, $\frac{2}{5}$	0,5	3,2

If $\pi_L < 3(1-\pi_L)$ then A would play only Down

But there are no Nash equilibria in which A plays only Down

COMPUTATION OF MS NE

Player B

		L, π_L	$R, 1-\pi_L$
Player A	$U, \frac{3}{5}$	1,2	0,4
	$D, \frac{2}{5}$	0,5	3,2

For there to exist a Nash equilibrium, A must be indifferent between playing Up or Down:

$$\pi_L = 3(1-\pi_L)$$

COMPUTATION OF MS NE

Player B

Player A		L, π_L	$R, 1-\pi_L$
U,	$\frac{3}{5}$	1,2	0,4
D,	$\frac{2}{5}$	0,5	3,2

$$\pi_L = 3(1-\pi_L) \quad \Rightarrow \quad \pi_L = 3/4.$$

Computation of MS NE

Player B

		L, $\frac{3}{4}$	R, $\frac{1}{4}$
Player A	U, $\frac{3}{5}$	1,2	0,4
	D, $\frac{2}{5}$	0,5	3,2

$\pi_L = \frac{3}{4}$ $\quad 1-\pi_L = \frac{1}{4}$

COMPUTATION OF MS NE

		Player B	
		L, $\frac{3}{4}$	R, $\frac{1}{4}$
Player A	U, $\frac{3}{5}$	1,2	0,4
	D, $\frac{2}{5}$	0,5	3,2

Game's only Nash equilibrium has A playing the mixed strategy $\left(\frac{3}{5}, \frac{2}{5}\right)$ and B playing the mixed strategy $\left(\frac{3}{4}, \frac{1}{4}\right)$

COMPUTATION OF MS NE

Player B

		L, $\frac{3}{4}$	R, $\frac{1}{4}$
Player A	U, $\frac{3}{5}$	1,2	0,4
	D, $\frac{2}{5}$	0,5	3,2

Payoffs:

- (1,2) with probability $(\frac{3}{5} \times \frac{3}{4}) = \frac{9}{20}$
- (0,4) with probability $(\frac{3}{5} \times \frac{1}{4}) = \frac{3}{20}$
- (0,5) with probability $(\frac{2}{5} \times \frac{3}{4}) = \frac{6}{20}$
- (3,2) with probability $(\frac{2}{5} \times \frac{1}{4}) = \frac{2}{20}$

COMPUTATION OF MS NE

		Player B	
		L, $\frac{3}{4}$	R, $\frac{1}{4}$
Player A	U, $\frac{3}{5}$	1,2	0,4
	D, $\frac{2}{5}$	0,5	3,2

A's expected Nash equilibrium payoff:

$$1\times\frac{9}{20} + 0\times\frac{3}{20} + 0\times\frac{6}{20} + 3\times\frac{2}{20} = \frac{3}{4}.$$

COMPUTATION OF MS NE

Player B

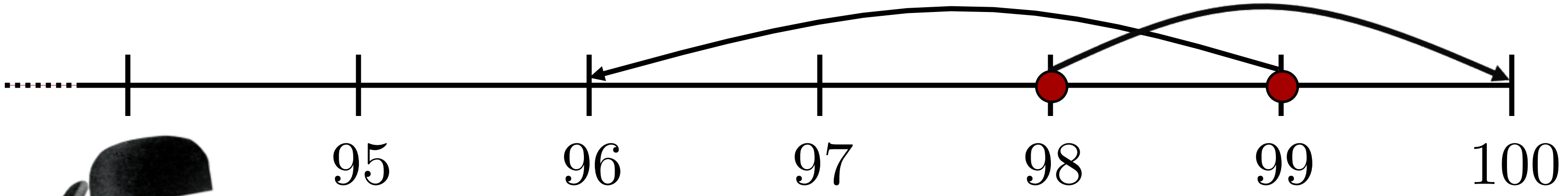
		L, $\frac{3}{4}$	R, $\frac{1}{4}$
Player A	U, $\frac{3}{5}$	1,2	0,4
	D, $\frac{2}{5}$	0,5	3,2

B's expected Nash equilibrium payoff:

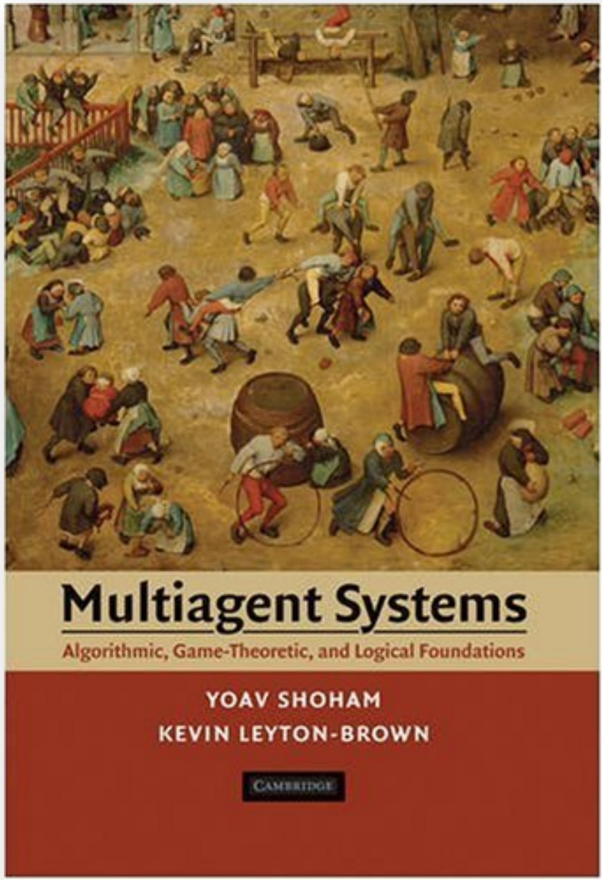
$$\mathbf{2 \times \frac{9}{20} + 4 \times \frac{3}{20} + 5 \times \frac{6}{20} + 2 \times \frac{2}{20} = \frac{16}{5}.}$$

DOES NE MAKE SENSE?

- Two players, strategies are $\{2, \ldots, 100\}$
- If both choose the same number, that is what they get
- If one chooses s, the other t, and $s < t$, the former player gets $s + 2$, and the latter gets $s - 2$
- Poll 3: What would you choose?



MULTIAGENT SYSTEMS



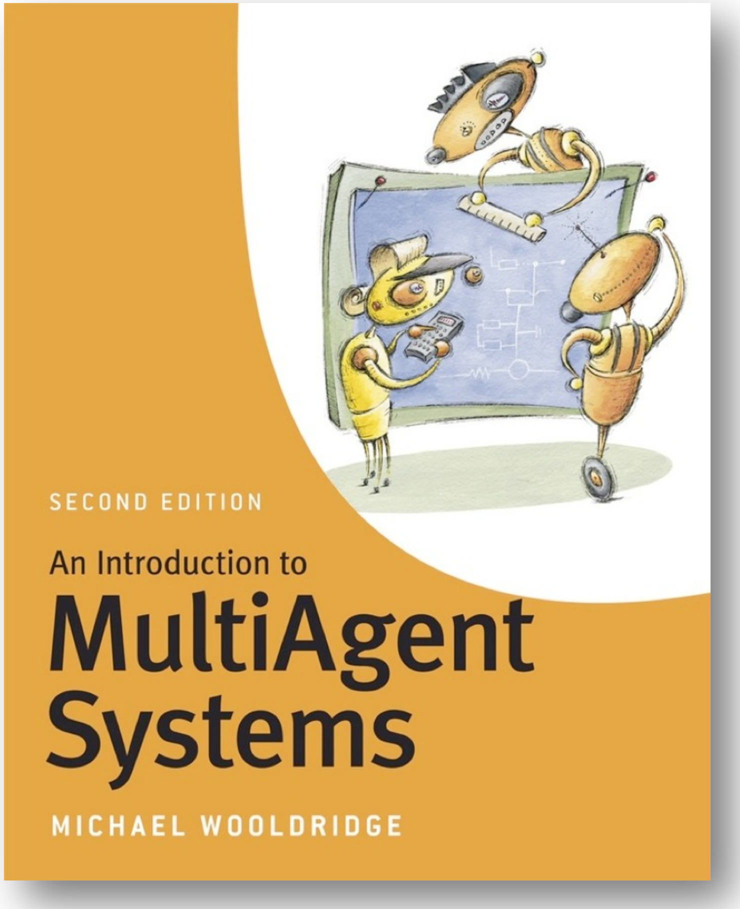
Multiagent Systems

Algorithmic, Game-Theoretic, and Logical Foundations

YOAV SHOHAM

KEVIN LEYTON-BROWN

CAMBRIDGE



SECOND EDITION

An Introduction to

MultiAgent Systems

MICHAEL WOOLDRIDGE

MULTIAGENT SYSTEMS

Chapters of the Shoham and Leyton-Brown book:

1. Distributed constraint satisfaction
2. Distributed optimization
3. Games in normal form
4. Computing solution concepts of normal-form games
5. Games with sequential actions
6. Beyond the normal and extensive forms
7. Learning and teaching
8. Communication
9. Social choice
10. Mechanism design
11. Auctions
12. Coalitional game theory
13. Logics of knowledge and belief
14. Probability, dynamics, and intention

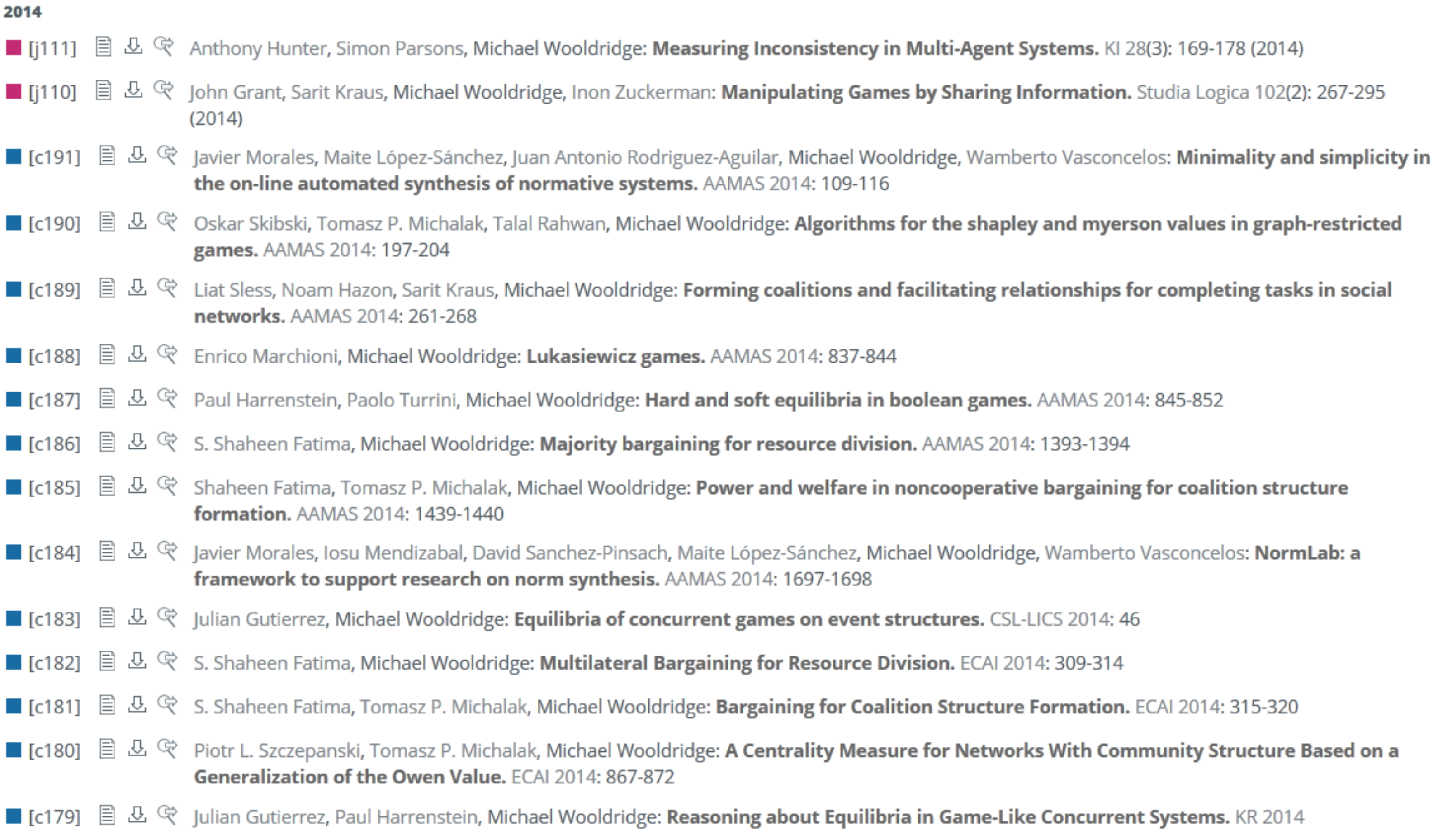
Legend:

🟥 "Game theory"

⬛ Not "game theory"

MULTIAGENT SYSTEMS

Mike Wooldridge's 2014 publications:



2014

- [j111] Anthony Hunter, Simon Parsons, Michael Wooldridge: **Measuring Inconsistency in Multi-Agent Systems.** KI 28(3): 169-178 (2014)
- [j110] John Grant, Sarit Kraus, Michael Wooldridge, Inon Zuckerman: **Manipulating Games by Sharing Information.** Studia Logica 102(2): 267-295 (2014)
- [c191] Javier Morales, Maite López-Sánchez, Juan Antonio Rodriguez-Aguilar, Michael Wooldridge, Wamberto Vasconcelos: **Minimality and simplicity in the on-line automated synthesis of normative systems.** AAMAS 2014: 109-116
- [c190] Oskar Skibski, Tomasz P. Michalak, Talal Rahwan, Michael Wooldridge: **Algorithms for the shapley and myerson values in graph-restricted games.** AAMAS 2014: 197-204
- [c189] Liat Sless, Noam Hazon, Sarit Kraus, Michael Wooldridge: **Forming coalitions and facilitating relationships for completing tasks in social networks.** AAMAS 2014: 261-268
- [c188] Enrico Marchioni, Michael Wooldridge: **Lukasiewicz games.** AAMAS 2014: 837-844
- [c187] Paul Harrenstein, Paolo Turrini, Michael Wooldridge: **Hard and soft equilibria in boolean games.** AAMAS 2014: 845-852
- [c186] S. Shaheen Fatima, Michael Wooldridge: **Majority bargaining for resource division.** AAMAS 2014: 1393-1394
- [c185] Shaheen Fatima, Tomasz P. Michalak, Michael Wooldridge: **Power and welfare in noncooperative bargaining for coalition structure formation.** AAMAS 2014: 1439-1440
- [c184] Javier Morales, Iosu Mendizabal, David Sanchez-Pinsach, Maite López-Sánchez, Michael Wooldridge, Wamberto Vasconcelos: **NormLab: a framework to support research on norm synthesis.** AAMAS 2014: 1697-1698
- [c183] Julian Gutierrez, Michael Wooldridge: **Equilibria of concurrent games on event structures.** CSL-LICS 2014: 46
- [c182] S. Shaheen Fatima, Michael Wooldridge: **Multilateral Bargaining for Resource Division.** ECAI 2014: 309-314
- [c181] S. Shaheen Fatima, Tomasz P. Michalak, Michael Wooldridge: **Bargaining for Coalition Structure Formation.** ECAI 2014: 315-320
- [c180] Piotr L. Szczepanski, Tomasz P. Michalak, Michael Wooldridge: **A Centrality Measure for Networks With Community Structure Based on a Generalization of the Owen Value.** ECAI 2014: 867-872
- [c179] Julian Gutierrez, Paul Harrenstein, Michael Wooldridge: **Reasoning about Equilibria in Game-Like Concurrent Systems.** KR 2014

SUMMARY

- Terminology:
  - Normal-form game
  - Nash equilibrium
  - Mixed strategies
- Nobel-prize-winning ideas:
  - Nash equilibrium ☺