

HOMEWORK 8  
DUE DECEMBER 1ST IN CLASS

---

1. Let  $G = (V, E)$  be any graph and let  $M$  be any matching in  $G$ . A *subgraph*  $H$  of  $G$  is any graph whose set of vertices and edges are a subset of the set of vertices and edges of  $G$  respectively. Show that  $G$  contains a subgraph  $H$ , where  $H$  is bipartite and contains at least  $\frac{1}{2}(|E(G)| + |M|)$  edges, where  $|E(G)|$  is the number of edges in  $G$ , and  $|M|$  is the number of edges in the matching  $M$ .
2. A *hospital cover* of a graph  $G = (V, E)$  is a subset of the vertices  $H \subseteq V$  such that every  $v \in V \setminus H$  has at least one neighbor in  $H$ . Show that if every vertex in  $G$  has degree at least  $d > 1$ , then there is a hospital cover of  $G$  of size at most  $|V|^{\frac{1+\ln(d+1)}{d+1}}$ .

Hint: This has similar flavor to showing that an independent set of a certain size must exist (which we proved in class).